9.5 Stellar evolution

Evolutionary timescales

Free-fall timescale[a]	$\tau_{\rm ff} = \left(\frac{3\pi}{32G\rho_0}\right)^{1/2}$	(9.53)	$\tau_{\rm ff}$ free-fall timescale G constant of gravitation ρ_0 initial mass density
Kelvin–Helmholtz timescale	$\tau_{\rm KH} = \frac{-U_{\rm g}}{L}$	(9.54)	$\tau_{\rm KH}$ Kelvin–Helmholtz timescale $U_{\rm g}$ gravitational potential energy M body's mass
	$\simeq \frac{GM^2}{R_0 L}$	(9.55)	R_0 body's initial radius L body's luminosity

[a]For the gravitational collapse of a uniform sphere.

Star formation

Jeans length[a]	$\lambda_{\rm J} = \left(\frac{\pi}{G\rho}\frac{{\rm d}p}{{\rm d}\rho}\right)^{1/2}$	(9.56)	$\lambda_{\rm J}$ Jeans length G constant of gravitation ρ cloud mass density p pressure
Jeans mass	$M_{\rm J} = \frac{\pi}{6}\rho\lambda_{\rm J}^3$	(9.57)	$M_{\rm J}$ (spherical) Jeans mass
Eddington limiting luminosity[b]	$L_{\rm E} = \frac{4\pi G M m_{\rm p} c}{\sigma_{\rm T}}$	(9.58)	$L_{\rm E}$ Eddington luminosity M stellar mass $M_\odot$ solar mass
	$\simeq 1.26\times10^{31}\frac{M}{M_\odot}$ W	(9.59)	$m_{\rm p}$ proton mass c speed of light $\sigma_{\rm T}$ Thomson cross section

[a]Note that $({\rm d}p/{\rm d}\rho)^{1/2}$ is the sound speed in the cloud.
[b]Assuming the opacity is mostly from Thomson scattering.

Stellar theory[a]

Conservation of mass	$\frac{{\rm d}M_r}{{\rm d}r} = 4\pi\rho r^2$	(9.60)	r radial distance M_r mass interior to r ρ mass density
Hydrostatic equilibrium	$\frac{{\rm d}p}{{\rm d}r} = \frac{-G\rho M_r}{r^2}$	(9.61)	p pressure G constant of gravitation
Energy release	$\frac{{\rm d}L_r}{{\rm d}r} = 4\pi\rho r^2\epsilon$	(9.62)	L_r luminosity interior to r ϵ power generated per unit mass
Radiative transport	$\frac{{\rm d}T}{{\rm d}r} = \frac{-3}{16\sigma}\frac{\langle\kappa\rangle\rho}{T^3}\frac{L_r}{4\pi r^2}$	(9.63)	T temperature σ Stefan–Boltzmann constant $\langle\kappa\rangle$ mean opacity
Convective transport	$\frac{{\rm d}T}{{\rm d}r} = \frac{\gamma-1}{\gamma}\frac{T}{p}\frac{{\rm d}p}{{\rm d}r}$	(9.64)	γ ratio of heat capacities, c_p/c_V

[a]For stars in static equilibrium with adiabatic convection. Note that ρ is a function of r. κ and ϵ are functions of temperature and composition.

Stellar fusion processes[a]

PP I chain	PP II chain	PP III chain
$p^+ + p^+ \rightarrow {}^2_1H + e^+ + \nu_e$ ${}^2_1H + p^+ \rightarrow {}^3_2He + \gamma$ ${}^3_2He + {}^3_2He \rightarrow {}^4_2He + 2p^+$	$p^+ + p^+ \rightarrow {}^2_1H + e^+ + \nu_e$ ${}^2_1H + p^+ \rightarrow {}^3_2He + \gamma$ ${}^3_2He + {}^4_2He \rightarrow {}^7_4Be + \gamma$ ${}^7_4Be + e^- \rightarrow {}^7_3Li + \nu_e$ ${}^7_3Li + p^+ \rightarrow 2{}^4_2He$	$p^+ + p^+ \rightarrow {}^2_1H + e^+ + \nu_e$ ${}^2_1H + p^+ \rightarrow {}^3_2He + \gamma$ ${}^3_2He + {}^4_2He \rightarrow {}^7_4Be + \gamma$ ${}^7_4Be + p^+ \rightarrow {}^8_5B + \gamma$ ${}^8_5B \rightarrow {}^8_4Be + e^+ + \nu_e$ ${}^8_4Be \rightarrow 2{}^4_2He$
CNO cycle	**triple-α process**	
${}^{12}_6C + p^+ \rightarrow {}^{13}_7N + \gamma$ ${}^{13}_7N \rightarrow {}^{13}_6C + e^+ + \nu_e$ ${}^{13}_6C + p^+ \rightarrow {}^{14}_7N + \gamma$ ${}^{14}_7N + p^+ \rightarrow {}^{15}_8O + \gamma$ ${}^{15}_8O \rightarrow {}^{15}_7N + e^+ + \nu_e$ ${}^{15}_7N + p^+ \rightarrow {}^{12}_6C + {}^4_2He$	${}^4_2He + {}^4_2He \rightleftharpoons {}^8_4Be + \gamma$ ${}^8_4Be + {}^4_2He \rightleftharpoons {}^{12}_6C^*$ ${}^{12}_6C^* \rightarrow {}^{12}_6C + \gamma$	γ photon p^+ proton e^+ positron e^- electron ν_e electron neutrino

[a] All species are taken as fully ionised.

Pulsars

Braking index	$\dot{\omega} \propto -\omega^n$	(9.65)	ω rotational angular velocity P rotational period ($=2\pi/\omega$) n braking index
	$n = 2 - \frac{P\ddot{P}}{\dot{P}^2}$	(9.66)	
Characteristic age[a]	$T = \frac{1}{n-1}\frac{P}{\dot{P}}$	(9.67)	T characteristic age L luminosity μ_0 permeability of free space c speed of light m pulsar magnetic dipole moment R pulsar radius B_p magnetic flux density at magnetic pole θ angle between magnetic and rotational axes
Magnetic dipole radiation	$L = \frac{\mu_0 \|\ddot{m}\|^2 \sin^2\theta}{6\pi c^3}$	(9.68)	
	$= \frac{2\pi R^6 B_p^2 \omega^4 \sin^2\theta}{3c^3\mu_0}$	(9.69)	
Dispersion measure	$\mathrm{DM} = \int_0^D n_e \, \mathrm{d}l$	(9.70)	DM dispersion measure D path length to pulsar $\mathrm{d}l$ path element n_e electron number density
Dispersion[b]	$\frac{\mathrm{d}\tau}{\mathrm{d}\nu} = \frac{-e^2}{4\pi^2\epsilon_0 m_e c \nu^3}\mathrm{DM}$	(9.71)	τ pulse arrival time $\Delta\tau$ difference in pulse arrival time ν_i observing frequencies m_e electron mass
	$\Delta\tau = \frac{e^2}{8\pi^2\epsilon_0 m_e c}\left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right)\mathrm{DM}$	(9.72)	

[a] Assuming $n \neq 1$ and that the pulsar has already slowed significantly. Usually n is assumed to be 3 (magnetic dipole radiation), giving $T = P/(2\dot{P})$.
[b] The pulse arrives first at the higher observing frequency.

Compact objects and black holes

Schwarzschild radius	$r_s = \frac{2GM}{c^2} \simeq 3\frac{M}{M_\odot}$ km	(9.73)	r_s Schwarzschild radius; G constant of gravitation; M mass of body; c speed of light; $M_\odot$ solar mass
Gravitational redshift	$\frac{\nu_\infty}{\nu_r} = \left(1 - \frac{2GM}{rc^2}\right)^{1/2}$	(9.74)	r distance from mass centre; ν_∞ frequency at infinity; ν_r frequency at r
Gravitational wave radiation[a]	$L_g = \frac{32}{5}\frac{G^4}{c^5}\frac{m_1^2 m_2^2 (m_1+m_2)}{a^5}$	(9.75)	m_i orbiting masses; a mass separation; L_g gravitational luminosity
Rate of change of orbital period	$\dot{P} = -\frac{96}{5}(4\pi^2)^{4/3}\frac{G^{5/3}}{c^5}\frac{m_1 m_2 P^{-5/3}}{(m_1+m_2)^{1/3}}$	(9.76)	P orbital period
Neutron star degeneracy pressure (nonrelativistic)	$p = \frac{(3\pi^2)^{2/3}}{5}\frac{\hbar^2}{m_n}\left(\frac{\rho}{m_n}\right)^{5/3} = \frac{2}{3}u$	(9.77)	p pressure; $\hbar$ (Planck constant)/(2π); m_n neutron mass; ρ density
Relativistic[b]	$p = \frac{\hbar c (3\pi^2)^{1/3}}{4}\left(\frac{\rho}{m_n}\right)^{4/3} = \frac{1}{3}u$	(9.78)	u energy density
Chandrasekhar mass[c]	$M_{Ch} \simeq 1.46 M_\odot$	(9.79)	M_{Ch} Chandrasekhar mass
Maximum black hole angular momentum	$J_m = \frac{GM^2}{c}$	(9.80)	J_m maximum angular momentum
Black hole evaporation time	$\tau_e \sim \frac{M^3}{M_\odot^3} \times 10^{66}$ yr	(9.81)	τ_e evaporation time
Black hole temperature	$T = \frac{\hbar c^3}{8\pi G M k} \simeq 10^{-7}\frac{M_\odot}{M}$ K	(9.82)	T temperature; k Boltzmann constant

[a]From two bodies, m_1 and m_2, in circular orbits about their centre of mass. Note that the frequency of the radiation is twice the orbital frequency.
[b]Particle velocities $\sim c$.
[c]Upper limit to mass of a white dwarf.