## 9.6 Cosmology

### Cosmological model parameters

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|---|---|---|---|
| Hubble law | $v_r = Hd$ | (9.83) | $v_r$ radial velocity<br>$H$ Hubble parameter<br>$d$ proper distance |
| Hubble parameter[a] | $H(t) = \frac{\dot{R}(t)}{R(t)}$ (9.84)<br>$H(z) = H_0[\Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0} + (1-\Omega_{m0}-\Omega_{\Lambda 0})(1+z)^2]^{1/2}$ | (9.85) | $_0$ present epoch<br>$R$ cosmic scale factor<br>$t$ cosmic time<br>$z$ redshift |
| Redshift | $z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} = \frac{R_0}{R(t_{em})} - 1$ | (9.86) | $\lambda_{obs}$ observed wavelength<br>$\lambda_{em}$ emitted wavelength<br>$t_{em}$ epoch of emission |
| Robertson–Walker metric[b] | $ds^2 = c^2\,dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)\right]$ | (9.87) | $ds$ interval<br>$c$ speed of light<br>$r,\theta,\phi$ comoving spherical polar coordinates |
| Friedmann equations[c] | $\ddot{R} = -\frac{4\pi}{3}GR\left(\rho + 3\frac{p}{c^2}\right) + \frac{\Lambda R}{3}$ (9.88)<br>$\dot{R}^2 = \frac{8\pi}{3}G\rho R^2 - kc^2 + \frac{\Lambda R^2}{3}$ | (9.89) | $k$ curvature parameter<br>$G$ constant of gravitation<br>$p$ pressure<br>$\Lambda$ cosmological constant |
| Critical density | $\rho_{crit} = \frac{3H^2}{8\pi G}$ | (9.90) | $\rho$ (mass) density<br>$\rho_{crit}$ critical density |
| Density parameters | $\Omega_m = \frac{\rho}{\rho_{crit}} = \frac{8\pi G\rho}{3H^2}$ (9.91)<br>$\Omega_\Lambda = \frac{\Lambda}{3H^2}$ (9.92)<br>$\Omega_k = -\frac{kc^2}{R^2H^2}$ (9.93)<br>$\Omega_m + \Omega_\Lambda + \Omega_k = 1$ | (9.94) | $\Omega_m$ matter density parameter<br>$\Omega_\Lambda$ lambda density parameter<br>$\Omega_k$ curvature density parameter |
| Deceleration parameter | $q_0 = -\frac{R_0\ddot{R}_0}{\dot{R}_0^2} = \frac{\Omega_{m0}}{2} - \Omega_{\Lambda 0}$ | (9.95) | $q_0$ deceleration parameter |

[a]Often called the Hubble "constant." At the present epoch, $60 \lesssim H_0 \lesssim 80\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \equiv 100h\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, where $h$ is a dimensionless scaling parameter. The Hubble time is $t_H = 1/H_0$. Equation (9.85) assumes a matter dominated universe and mass conservation.

[b]For a homogeneous, isotropic universe, using the $(-1,1,1,1)$ metric signature. $r$ is scaled so that $k = 0, \pm 1$. Note that $ds^2 \equiv (ds)^2$ etc.

[c]$\Lambda = 0$ in a Friedmann universe. Note that the cosmological constant is sometimes defined as equalling the value used here divided by $c^2$.

## Cosmological distance measures

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| Look-back time | $t_{\mathrm{lb}}(z) = t_0 - t(z)$ | (9.96) |
| Proper distance | $d_{\mathrm{p}} = R_0 \int_0^r \frac{\mathrm{d}r}{(1-kr^2)^{1/2}} = cR_0 \int_t^{t_0} \frac{\mathrm{d}t}{R(t)}$ | (9.97) |
| Luminosity distance[a] | $d_{\mathrm{L}} = d_{\mathrm{p}}(1+z) = c(1+z) \int_0^z \frac{\mathrm{d}z}{H(z)}$ | (9.98) |
| Flux density–redshift relation | $F(\nu) = \frac{L(\nu')}{4\pi d_{\mathrm{L}}^2(z)}$ where $\nu' = (1+z)\nu$ | (9.99) |
| Angular diameter distance[d] | $d_{\mathrm{a}} = d_{\mathrm{L}}(1+z)^{-2}$ | (9.100) |

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| $t_{\mathrm{lb}}(z)$ | light travel time from an object at redshift $z$ |
| $t_0$ | present cosmic time |
| $t(z)$ | cosmic time at $z$ |
| $d_{\mathrm{p}}$ | proper distance |
| $R$ | cosmic scale factor |
| $c$ | speed of light |
| $_0$ | present epoch |
| $d_{\mathrm{L}}$ | luminosity distance |
| $z$ | redshift |
| $H$ | Hubble parameter[b] |
| $F$ | spectral flux density |
| $\nu$ | frequency |
| $L(\nu)$ | spectral luminosity[c] |
| $d_{\mathrm{a}}$ | angular diameter distance |
| $k$ | curvature parameter |

[a]Assuming a flat universe ($k=0$). The apparent flux density of a source varies as $d_{\mathrm{L}}^{-2}$.
[b]See Equation (9.85).
[c]Defined as the output power of the body per unit frequency interval.
[d]True for all $k$. The angular diameter of a source varies as $d_{\mathrm{a}}^{-1}$.

## Cosmological models[a]

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|---|---|---|
| Einstein – de Sitter model ($\Omega_k = 0$, $\Lambda = 0$, $p = 0$ and $\Omega_{\mathrm{m}0} = 1$) | $d_{\mathrm{p}} = \frac{2c}{H_0}[1-(1+z)^{-1/2}]$ | (9.101) |
| | $H(z) = H_0(1+z)^{3/2}$ | (9.102) |
| | $q_0 = 1/2$ | (9.103) |
| | $t(z) = \frac{2}{3H(z)}$ | (9.104) |
| | $\rho = (6\pi G t^2)^{-1}$ | (9.105) |
| | $R(t) = R_0(t/t_0)^{2/3}$ | (9.106) |
| Concordance model ($\Omega_k = 0$, $\Lambda = 3(1-\Omega_{\mathrm{m}0})H_0^2$, $p = 0$ and $\Omega_{\mathrm{m}0} < 1$) | $d_{\mathrm{p}} = \frac{c}{H_0} \int_0^z \frac{\Omega_{\mathrm{m}0}^{-1/2}\,\mathrm{d}z'}{[(1+z')^3 - 1 + \Omega_{\mathrm{m}0}^{-1}]^{1/2}}$ | (9.107) |
| | $H(z) = H_0[\Omega_{\mathrm{m}0}(1+z)^3 + (1-\Omega_{\mathrm{m}0})]$ | (9.108) |
| | $q_0 = 3\Omega_{\mathrm{m}0}/2 - 1$ | (9.109) |
| | $t(z) = \frac{2}{3H_0}(1-\Omega_{\mathrm{m}0})^{-1/2} \operatorname{arsinh}\left[\frac{(1-\Omega_{\mathrm{m}0})^{1/2}}{(1+z)^{3/2}}\right]$ | (9.110) |

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|---|---|
| $d_{\mathrm{p}}$ | proper distance |
| $H$ | Hubble parameter |
| $_0$ | present epoch |
| $z$ | redshift |
| $c$ | speed of light |
| $q$ | deceleration parameter |
| $t(z)$ | time at redshift $z$ |
| $R$ | cosmic scale factor |
| $\Omega_{\mathrm{m}0}$ | present mass density parameter |
| $G$ | constant of gravitation |
| $\rho$ | mass density |

[a]Currently popular.