1.4 Complex Numbers

Natural number: n
Imaginary unit: i
Complex number: z
Real part: a, c
Imaginary part: bi, di
Modulus of a complex number: r, r_1, r_2
Argument of a complex number: φ, φ_1, φ_2

46.

$i^1 = i$	$i^5 = i$	$i^{4n+1} = i$
$i^2 = -1$	$i^6 = -1$	$i^{4n+2} = -1$
$i^3 = -i$	$i^7 = -i$	$i^{4n+3} = -i$
$i^4 = 1$	$i^8 = 1$	$i^{4n} = 1$

47. $z = a + bi$

48. Complex Plane

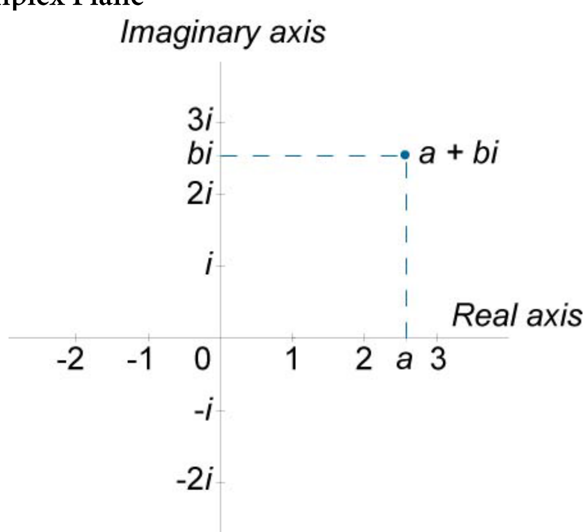

Figure 6.

49. $(a + bi) + (c + di) = (a + c) + (b + d)i$

50. $(a+bi)-(c+di)=(a-c)+(b-d)i$

51. $(a+bi)(c+di)=(ac-bd)+(ad+bc)i$

52. $\dfrac{a+bi}{c+di}=\dfrac{ac+bd}{c^2+d^2}+\dfrac{bc-ad}{c^2+d^2}\cdot i$

53. Conjugate Complex Numbers

$\overline{a+bi}=a-bi$

54. $a=r\cos\varphi,\ b=r\sin\varphi$

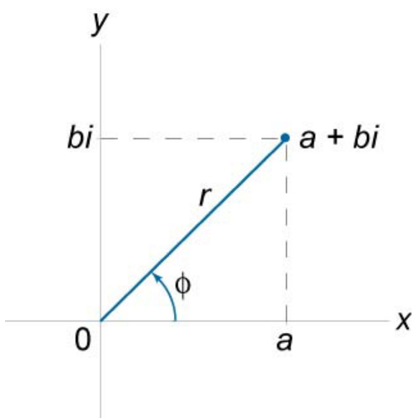

Figure 7.

55. Polar Presentation of Complex Numbers

$a+bi=r(\cos\varphi+i\sin\varphi)$

56. Modulus and Argument of a Complex Number

If $a+bi$ is a complex number, then

$r=\sqrt{a^2+b^2}$ (modulus),

$\varphi=\arctan\dfrac{b}{a}$ (argument).

57. Product in Polar Representation

$$z_1 \cdot z_2 = r_1(\cos\varphi_1 + i\sin\varphi_1)\cdot r_2(\cos\varphi_2 + i\sin\varphi_2)$$
$$= r_1 r_2[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)]$$

58. Conjugate Numbers in Polar Representation

$$\overline{r(\cos\varphi + i\sin\varphi)} = r[\cos(-\varphi) + i\sin(-\varphi)]$$

59. Inverse of a Complex Number in Polar Representation

$$\frac{1}{r(\cos\varphi + i\sin\varphi)} = \frac{1}{r}[\cos(-\varphi) + i\sin(-\varphi)]$$

60. Quotient in Polar Representation

$$\frac{z_1}{z_2} = \frac{r_1(\cos\varphi_1 + i\sin\varphi_1)}{r_2(\cos\varphi_2 + i\sin\varphi_2)} = \frac{r_1}{r_2}[\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)]$$

61. Power of a Complex Number

$$z^n = [r(\cos\varphi + i\sin\varphi)]^n = r^n[\cos(n\varphi) + i\sin(n\varphi)]$$

62. Formula “De Moivre”

$$(\cos\varphi + i\sin\varphi)^n = \cos(n\varphi) + i\sin(n\varphi)$$

63. Nth Root of a Complex Number

$$\sqrt[n]{z} = \sqrt[n]{r(\cos\varphi + i\sin\varphi)} = \sqrt[n]{r}\left(\cos\frac{\varphi + 2\pi k}{n} + i\sin\frac{\varphi + 2\pi k}{n}\right),$$

where

$k = 0, 1, 2, \ldots, n-1$.

64. Euler’s Formula

$$e^{ix} = \cos x + i\sin x$$