

3.4 Scalene Triangle

(A triangle with no two sides equal)

Sides of a triangle: a, b, c

Semiperimeter: $p = \frac{a+b+c}{2}$

Angles of a triangle: α, β, γ

Altitudes to the sides a, b, c : h_a, h_b, h_c

Medians to the sides a, b, c : m_a, m_b, m_c

Bisectors of the angles α, β, γ : t_a, t_b, t_c

Radius of circumscribed circle: R

Radius of inscribed circle: r

Area: S

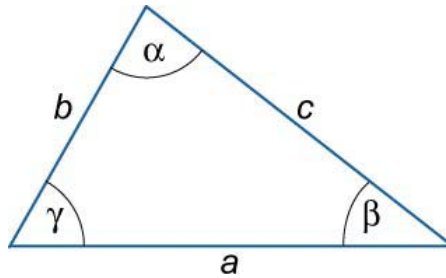


Figure 13.

181. $\alpha + \beta + \gamma = 180^\circ$

182. $a + b > c,$
 $b + c > a,$
 $a + c > b.$

183. $|a - b| < c,$
 $|b - c| < a,$
 $|a - c| < b.$

184. Midline

$$q = \frac{a}{2}, q \parallel a.$$

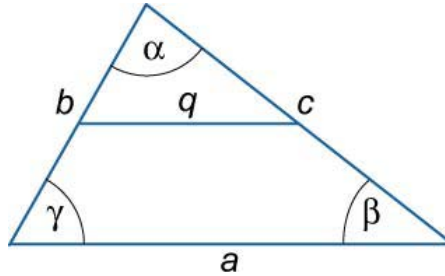


Figure 14.

185. Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos \alpha,$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta,$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma.$$

186. Law of Sines

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2R,$$

where R is the radius of the circumscribed circle.

$$\mathbf{187.} \quad R = \frac{a}{2 \sin \alpha} = \frac{b}{2 \sin \beta} = \frac{c}{2 \sin \gamma} = \frac{bc}{2h_a} = \frac{ac}{2h_b} = \frac{ab}{2h_c} = \frac{abc}{4S}$$

$$\mathbf{188.} \quad r^2 = \frac{(p-a)(p-b)(p-c)}{p},$$

$$\frac{1}{r} = \frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c}.$$

$$189. \quad \sin \frac{\alpha}{2} = \sqrt{\frac{(p-b)(p-c)}{bc}},$$

$$\cos \frac{\alpha}{2} = \sqrt{\frac{p(p-a)}{bc}},$$

$$\tan \frac{\alpha}{2} = \sqrt{\frac{(p-b)(p-c)}{p(p-a)}}.$$

$$190. \quad h_a = \frac{2}{a} \sqrt{p(p-a)(p-b)(p-c)},$$

$$h_b = \frac{2}{b} \sqrt{p(p-a)(p-b)(p-c)},$$

$$h_c = \frac{2}{c} \sqrt{p(p-a)(p-b)(p-c)}.$$

$$191. \quad h_a = b \sin \gamma = c \sin \beta,$$

$$h_b = a \sin \gamma = c \sin \alpha,$$

$$h_c = a \sin \beta = b \sin \alpha.$$

$$192. \quad m_a^2 = \frac{b^2 + c^2}{2} - \frac{a^2}{4},$$

$$m_b^2 = \frac{a^2 + c^2}{2} - \frac{b^2}{4},$$

$$m_c^2 = \frac{a^2 + b^2}{2} - \frac{c^2}{4}.$$

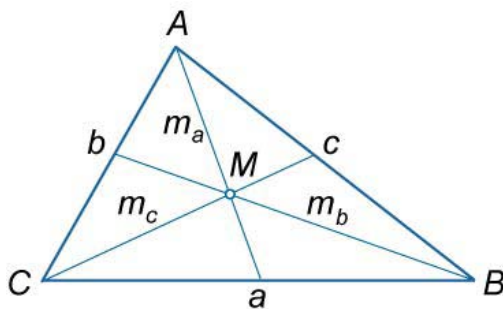


Figure 15.

193. $AM = \frac{2}{3}m_a$, $BM = \frac{2}{3}m_b$, $CM = \frac{2}{3}m_c$ (Fig.15).

194.
$$t_a^2 = \frac{4bcp(p-a)}{(b+c)^2},$$

$$t_b^2 = \frac{4acp(p-b)}{(a+c)^2},$$

$$t_c^2 = \frac{4abp(p-c)}{(a+b)^2}.$$

195.
$$S = \frac{ah_a}{2} = \frac{bh_b}{2} = \frac{ch_c}{2},$$

$$S = \frac{ab \sin \gamma}{2} = \frac{ac \sin \beta}{2} = \frac{bc \sin \alpha}{2},$$

$$S = \sqrt{p(p-a)(p-b)(p-c)} \text{ (Heron's Formula),}$$

$$S = pr,$$

$$S = \frac{abc}{4R},$$

$$S = 2R^2 \sin \alpha \sin \beta \sin \gamma,$$

$$S = p^2 \tan \frac{\alpha}{2} \tan \frac{\beta}{2} \tan \frac{\gamma}{2}.$$