

4.8 Relations between Trigonometric Functions

$$\begin{aligned} 396. \quad \sin \alpha &= \pm \sqrt{1 - \cos^2 \alpha} = \pm \sqrt{\frac{1}{2}(1 - \cos 2\alpha)} = 2 \cos^2 \left(\frac{\alpha}{2} - \frac{\pi}{4} \right) - 1 \\ &= \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} \end{aligned}$$

$$\begin{aligned} 397. \quad \cos \alpha &= \pm \sqrt{1 - \sin^2 \alpha} = \pm \sqrt{\frac{1}{2}(1 + \cos 2\alpha)} = 2 \cos^2 \frac{\alpha}{2} - 1 \\ &= \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} \end{aligned}$$

$$\begin{aligned} 398. \quad \tan \alpha &= \frac{\sin \alpha}{\cos \alpha} = \pm \sqrt{\sec^2 \alpha - 1} = \frac{\sin 2\alpha}{1 + \cos 2\alpha} = \frac{1 - \cos 2\alpha}{\sin 2\alpha} \\ &= \pm \sqrt{\frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}} = \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} \end{aligned}$$

$$\begin{aligned} 399. \quad \cot \alpha &= \frac{\cos \alpha}{\sin \alpha} = \pm \sqrt{\csc^2 \alpha - 1} = \frac{1 + \cos 2\alpha}{\sin 2\alpha} = \frac{\sin 2\alpha}{1 - \cos 2\alpha} \\ &= \pm \sqrt{\frac{1 + \cos 2\alpha}{1 - \cos 2\alpha}} = \frac{1 - \tan^2 \frac{\alpha}{2}}{2 \tan \frac{\alpha}{2}} \end{aligned}$$

$$400. \quad \sec \alpha = \frac{1}{\cos \alpha} = \pm \sqrt{1 + \tan^2 \alpha} = \frac{1 + \tan^2 \frac{\alpha}{2}}{1 - \tan^2 \frac{\alpha}{2}}$$

$$401. \quad \csc \alpha = \frac{1}{\sin \alpha} = \pm \sqrt{1 + \cot^2 \alpha} = \frac{1 + \tan^2 \frac{\alpha}{2}}{2 \tan \frac{\alpha}{2}}$$