

6.6 Vector Product

587. Vector Product of Vectors $\vec{u}$ and $\vec{v}$

$\vec{u} \times \vec{v} = \vec{w}$, where

- $|\vec{w}| = |\vec{u}| \cdot |\vec{v}| \cdot \sin \theta$, where $0 \leq \theta \leq \frac{\pi}{2}$;
- $\vec{w} \perp \vec{u}$ and $\vec{w} \perp \vec{v}$;
- Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$ form a right-handed screw.

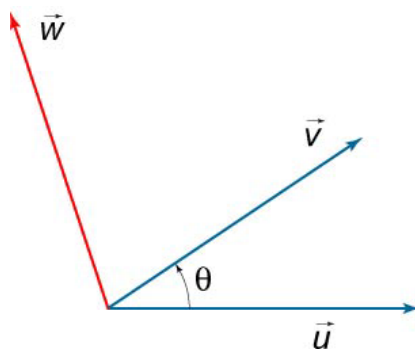


Figure 83.

$$588. \quad \vec{w} = \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix}$$

$$589. \quad \vec{w} = \vec{u} \times \vec{v} = \left(\begin{vmatrix} Y_1 & Z_1 \\ Y_2 & Z_2 \end{vmatrix}, -\begin{vmatrix} X_1 & Z_1 \\ X_2 & Z_2 \end{vmatrix}, \begin{vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{vmatrix} \right)$$

$$590. \quad S = |\vec{u} \times \vec{v}| = |\vec{u}| \cdot |\vec{v}| \cdot \sin \theta \text{ (Fig.83)}$$

591. Angle Between Two Vectors (Fig.83)

$$\sin \theta = \frac{|\vec{u} \times \vec{v}|}{|\vec{u}| \cdot |\vec{v}|}$$

592. Noncommutative Property

$$\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$$

593. Associative Property

$$(\lambda \vec{u}) \times (\mu \vec{v}) = \lambda \mu \vec{u} \times \vec{v}$$

594. Distributive Property

$$\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$$

595. $\vec{u} \times \vec{v} = \vec{0}$ if $\vec{u}$ and $\vec{v}$ are parallel ($\theta = 0$).

596. $\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = \vec{0}$

597. $\vec{i} \times \vec{j} = \vec{k}$, $\vec{j} \times \vec{k} = \vec{i}$, $\vec{k} \times \vec{i} = \vec{j}$