

6.7 Triple Product

598. Scalar Triple Product

$$[\vec{u}\vec{v}\vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w}) = \vec{v} \cdot (\vec{w} \times \vec{u}) = \vec{w} \cdot (\vec{u} \times \vec{v})$$

$$599. \quad [\vec{u}\vec{v}\vec{w}] = [\vec{w}\vec{u}\vec{v}] = [\vec{v}\vec{w}\vec{u}] = -[\vec{v}\vec{u}\vec{w}] = -[\vec{w}\vec{v}\vec{u}] = -[\vec{u}\vec{w}\vec{v}]$$

$$600. \quad k\vec{u} \cdot (\vec{v} \times \vec{w}) = k[\vec{u}\vec{v}\vec{w}]$$

601. Scalar Triple Product in Coordinate Form

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ X_3 & Y_3 & Z_3 \end{vmatrix},$$

where

$$\vec{u} = (X_1, Y_1, Z_1), \quad \vec{v} = (X_2, Y_2, Z_2), \quad \vec{w} = (X_3, Y_3, Z_3).$$

602. Volume of Parallelepiped

$$V = |\vec{u} \cdot (\vec{v} \times \vec{w})|$$

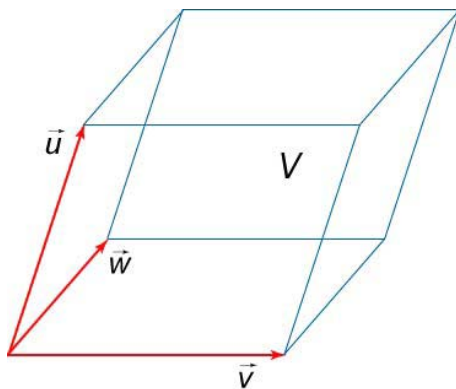


Figure 84.

603. Volume of Pyramid

$$V = \frac{1}{6} |\vec{u} \cdot (\vec{v} \times \vec{w})|$$

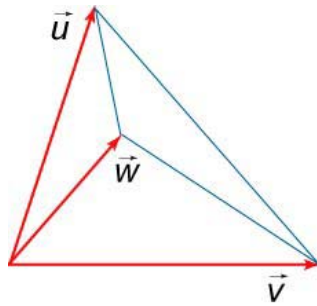


Figure 85.

- 604.** If $\vec{u} \cdot (\vec{v} \times \vec{w}) = 0$, then the vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ are **linearly dependent**, so $\vec{w} = \lambda \vec{u} + \mu \vec{v}$ for some scalars λ and μ .
- 605.** If $\vec{u} \cdot (\vec{v} \times \vec{w}) \neq 0$, then the vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ are **linearly independent**.
- 606.** Vector Triple Product

$$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$$