7.2 Two-Dimensional Coordinate System

Point coordinates: x_0, x_1, x_2, y_0, y_1, y_2
Polar coordinates: r, φ
Real number: λ
Positive real numbers: a, b, c,
Distance between two points: d
Area: S

610. Distance Between Two Points

$$d = AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

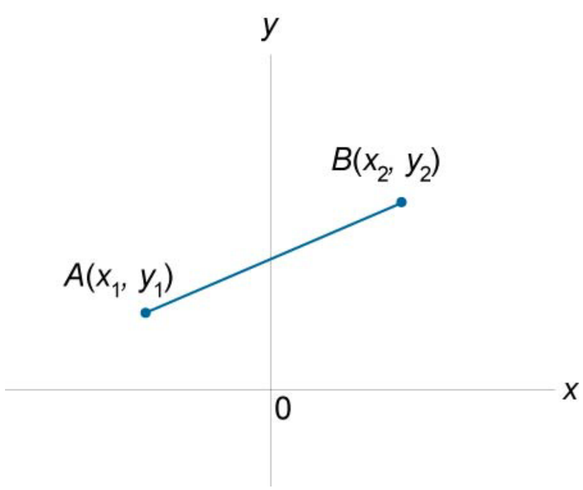
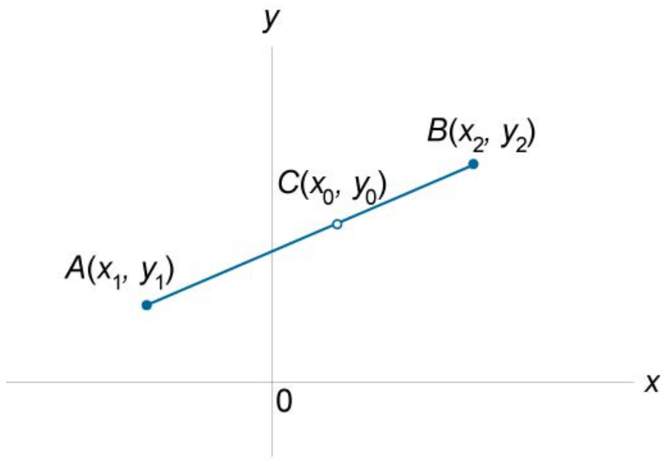


Figure 88.

611. Dividing a Line Segment in the Ratio λ

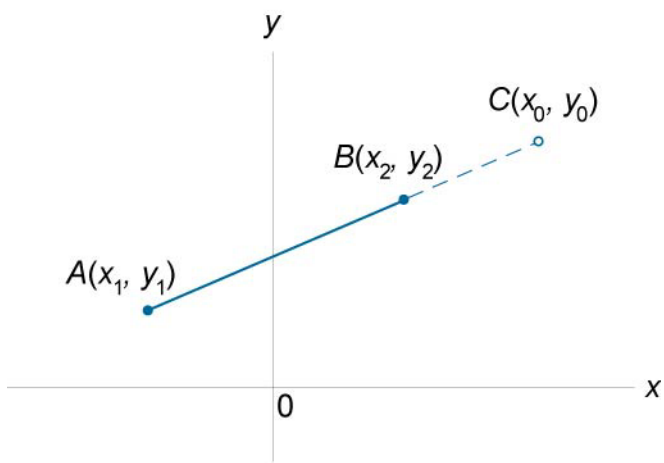
$$x_0 = \frac{x_1 + \lambda x_2}{1 + \lambda}, \; y_0 = \frac{y_1 + \lambda y_2}{1 + \lambda},$$

$$\lambda = \frac{AC}{CB}, \; \lambda \neq -1.$$



$\lambda > 0$

Figure 89.



$\lambda < 0$

Figure 90.

612. Midpoint of a Line Segment

$$x_0 = \frac{x_1 + x_2}{2},\ y_0 = \frac{y_1 + y_2}{2},\ \lambda = 1.$$

613. Centroid (Intersection of Medians) of a Triangle

$$x_0 = \frac{x_1 + x_2 + x_3}{3},\ y_0 = \frac{y_1 + y_2 + y_3}{3},$$

where $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ are vertices of the triangle ABC.

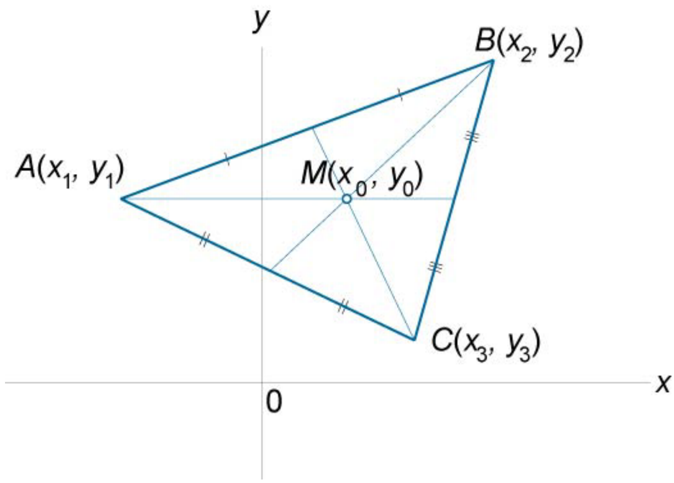


Figure 91.

614. Incenter (Intersection of Angle Bisectors) of a Triangle

$$x_0 = \frac{ax_1 + bx_2 + cx_3}{a + b + c}, \; y_0 = \frac{ay_1 + by_2 + cy_3}{a + b + c},$$

where $a = BC$, $b = CA$, $c = AB$.

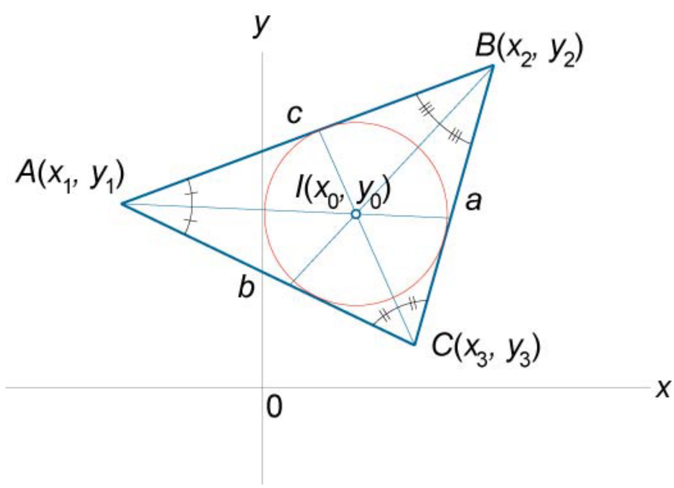


Figure 92.

615. **Circumcenter (Intersection of the Side Perpendicular Bisectors) of a Triangle**

$$x_0 = \frac{\begin{vmatrix} x_1^2+y_1^2 & y_1 & 1 \\ x_2^2+y_2^2 & y_2 & 1 \\ x_3^2+y_3^2 & y_3 & 1 \end{vmatrix}}{2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}, \quad y_0 = \frac{\begin{vmatrix} x_1 & x_1^2+y_1^2 & 1 \\ x_2 & x_2^2+y_2^2 & 1 \\ x_3 & x_3^2+y_3^2 & 1 \end{vmatrix}}{2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}$$

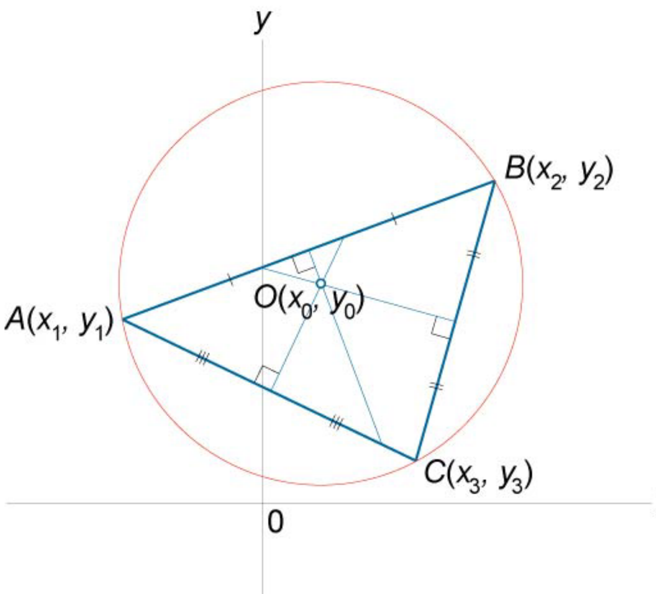


Figure 93.

616. Orthocenter (Intersection of Altitudes) of a Triangle

$$x_0 = \frac{\begin{vmatrix} y_1 & x_2x_3 + y_1^2 & 1 \\ y_2 & x_3x_1 + y_2^2 & 1 \\ y_3 & x_1x_2 + y_3^2 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}, \quad y_0 = \frac{\begin{vmatrix} x_1^2 + y_2y_3 & x_1 & 1 \\ x_2^2 + y_3y_1 & x_2 & 1 \\ x_3^2 + y_1y_2 & x_3 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}$$

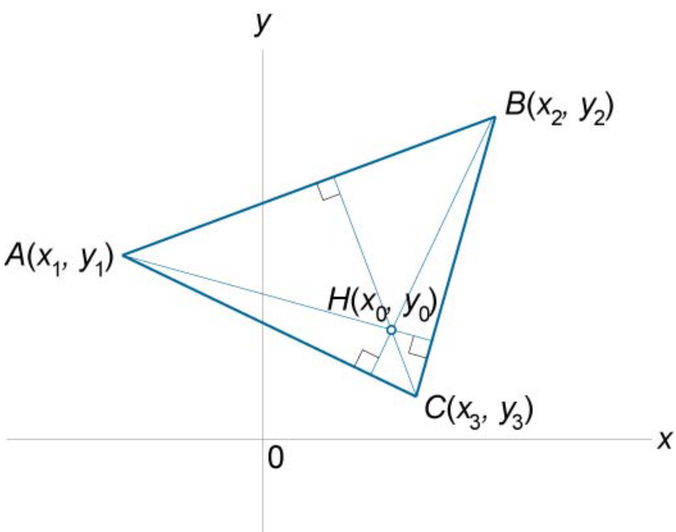


Figure 94.

617. Area of a Triangle

$$S = (\pm)\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = (\pm)\frac{1}{2}\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix}$$

618. Area of a Quadrilateral

$$S = (\pm)\frac{1}{2}[(x_1 - x_2)(y_1 + y_2) + (x_2 - x_3)(y_2 + y_3) + \\ + (x_3 - x_4)(y_3 + y_4) + (x_4 - x_1)(y_4 + y_1)]$$

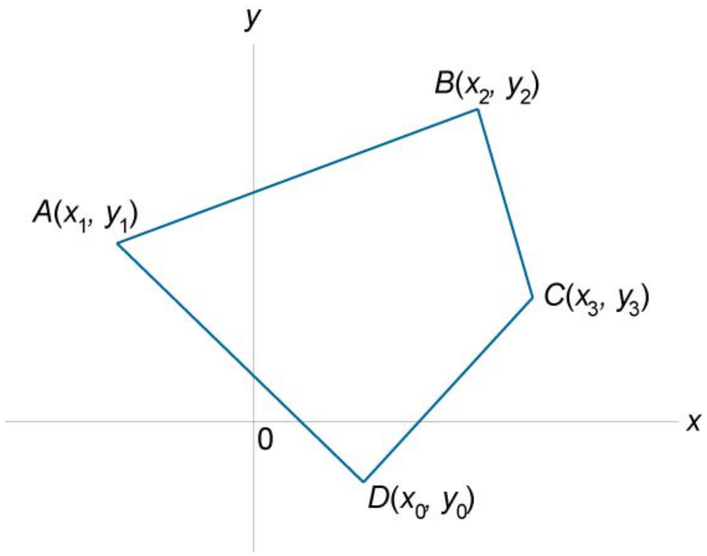


Figure 95.

Note: In formulas 617, 618 we choose the sign (+) or (–) so that to get a positive answer for area.

619. Distance Between Two Points in Polar Coordinates

$$d = AB = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\varphi_2 - \varphi_1)}$$

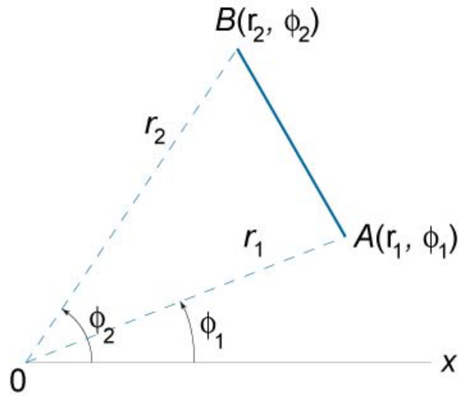


Figure 96.

620. **Converting Rectangular Coordinates to Polar Coordinates**
$x = r\cos\varphi, \ y = r\sin\varphi.$

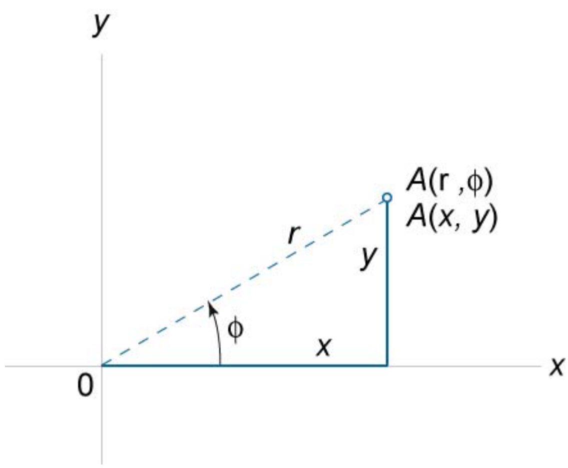


Figure 97.

621. **Converting Polar Coordinates to Rectangular Coordinates**
$r = \sqrt{x^2 + y^2}, \ \tan\varphi = \frac{y}{x}.$