

7.10 Straight Line in Space

Point coordinates: $x, y, z, x_1, y_1, z_1, \dots$

Direction cosines: $\cos \alpha, \cos \beta, \cos \gamma$

Real numbers: $A, B, C, D, a, b, c, a_1, a_2, t, \dots$

Direction vectors of a line: $\vec{s}, \vec{s}_1, \vec{s}_2$

Normal vector to a plane: $\vec{n}$

Angle between two lines: φ

690. Point Direction Form of the Equation of a Line

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c},$$

where the point $P_1(x_1, y_1, z_1)$ lies on the line, and (a, b, c) is the direction vector of the line.

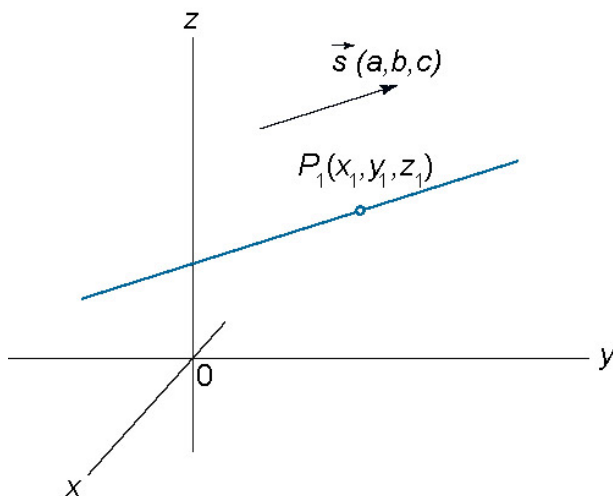


Figure 136.

691. Two Point Form

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

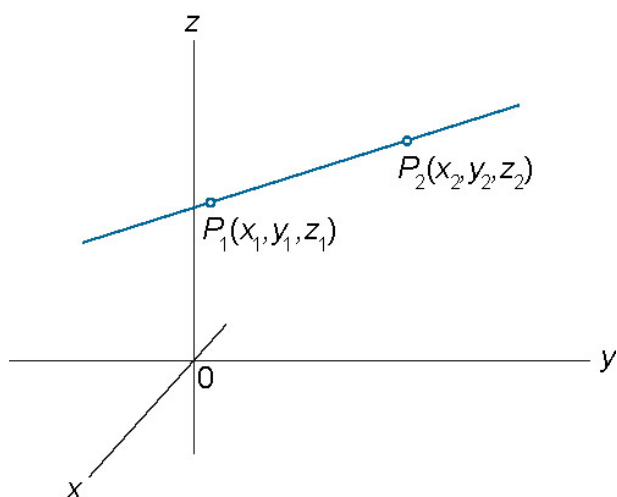


Figure 137.

692. Parametric Form

$$\begin{cases} x = x_1 + t \cos \alpha \\ y = y_1 + t \cos \beta \\ z = z_1 + t \cos \gamma \end{cases}$$

where the point $P_1(x_1, y_1, z_1)$ lies on the straight line, $\cos \alpha$, $\cos \beta$, $\cos \gamma$ are the direction cosines of the direction vector of the line, the parameter t is any real number.

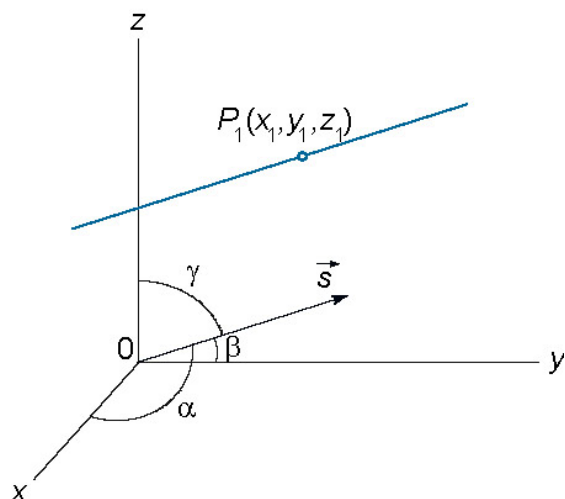


Figure 138.

693. Angle Between Two Straight Lines

$$\cos \varphi = \frac{\vec{s}_1 \cdot \vec{s}_2}{|\vec{s}_1| \cdot |\vec{s}_2|} = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

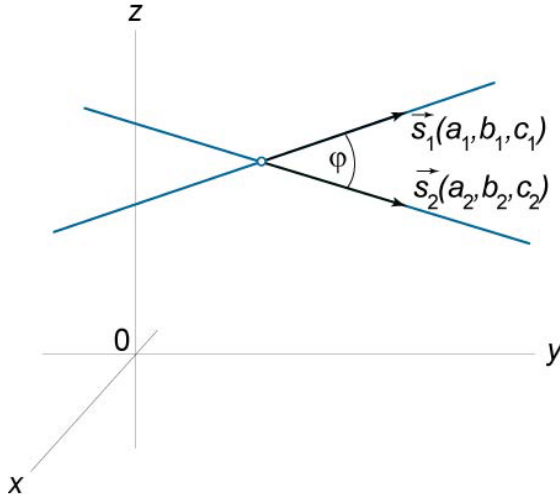


Figure 139.

694. Parallel Lines

Two lines are parallel if

$$\vec{s}_1 \parallel \vec{s}_2,$$

or

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

695. Perpendicular Lines

Two lines are perpendicular if

$$\vec{s}_1 \cdot \vec{s}_2 = 0,$$

or

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0.$$

696. Intersection of Two Lines

Two lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ intersect if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$$

697. Parallel Line and Plane

The straight line $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ and the plane

$Ax + By + Cz + D = 0$ are parallel if

$$\vec{n} \cdot \vec{s} = 0,$$

or

$$Aa + Bb + Cc = 0.$$

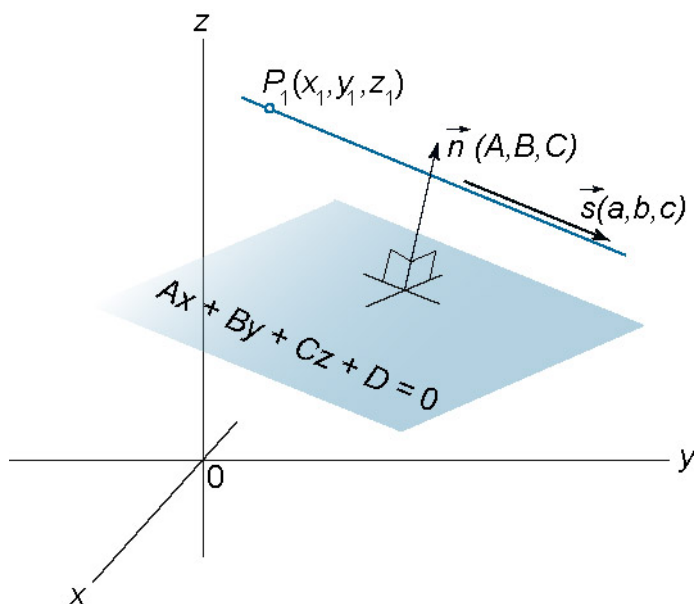


Figure 140.

698. Perpendicular Line and Plane

The straight line $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ and the plane

$Ax + By + Cz + D = 0$ are perpendicular if

$$\vec{n} \parallel \vec{s},$$

or

$$\frac{A}{a} = \frac{B}{b} = \frac{C}{c}.$$

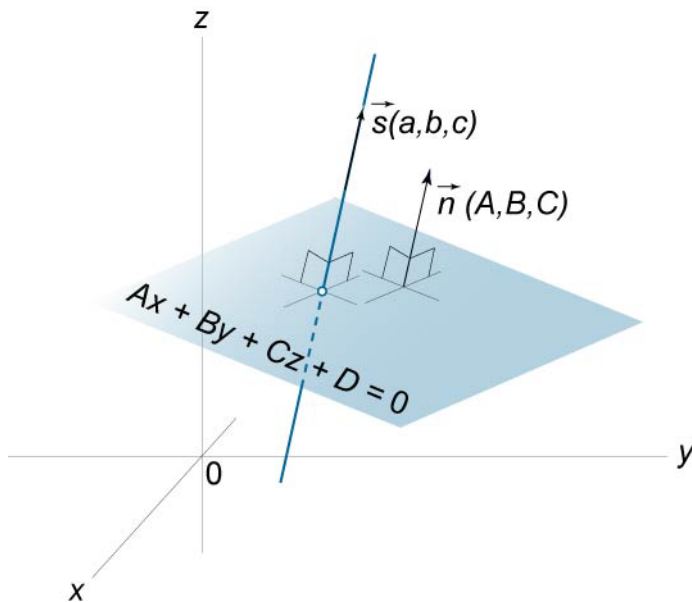


Figure 141.