

8.1 Functions and Their Graphs

723. Even Function

$$f(-x) = f(x)$$

724. Odd Function

$$f(-x) = -f(x)$$

725. Periodic Function

$$f(x + nT) = f(x)$$

726. Inverse Function

$y = f(x)$ is any function, $x = g(y)$ or $y = f^{-1}(x)$ is its inverse function.

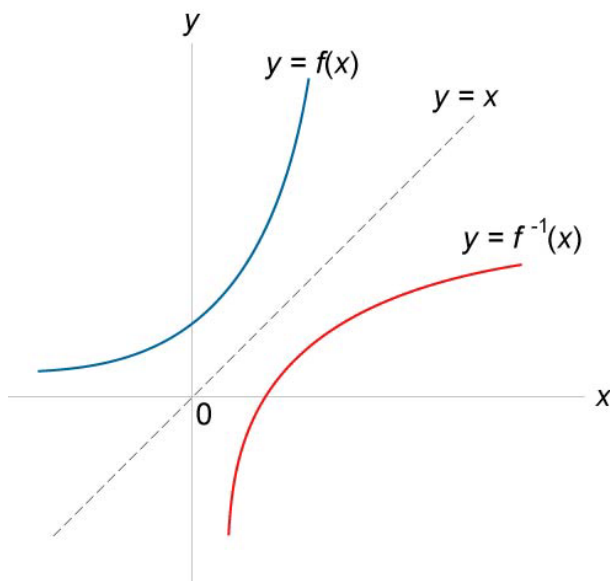


Figure 152.

727. Composite Function

$y = f(u)$, $u = g(x)$, $y = f(g(x))$ is a composite function.

728. Linear Function

$y = ax + b$, $x \in \mathbb{R}$, $a = \tan \alpha$ is the slope of the line, b is the y-intercept.

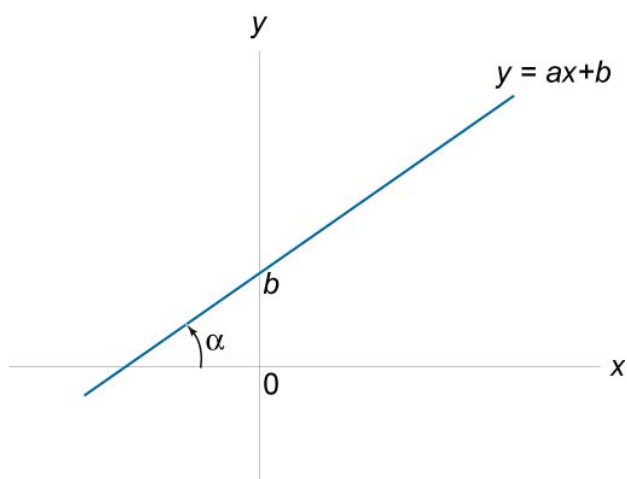


Figure 153.

729. Quadratic Function
 $y = x^2$, $x \in \mathbb{R}$.

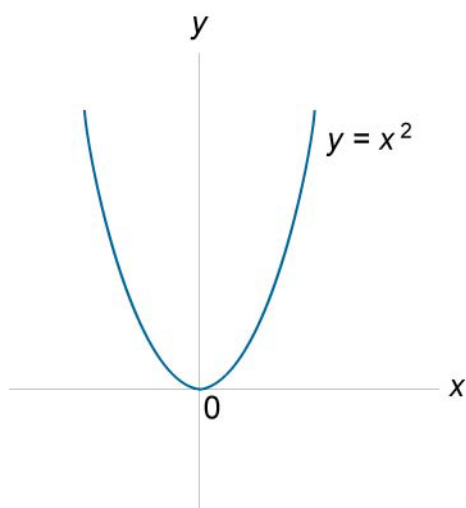


Figure 154.

730. $y = ax^2 + bx + c, x \in \mathbb{R}.$

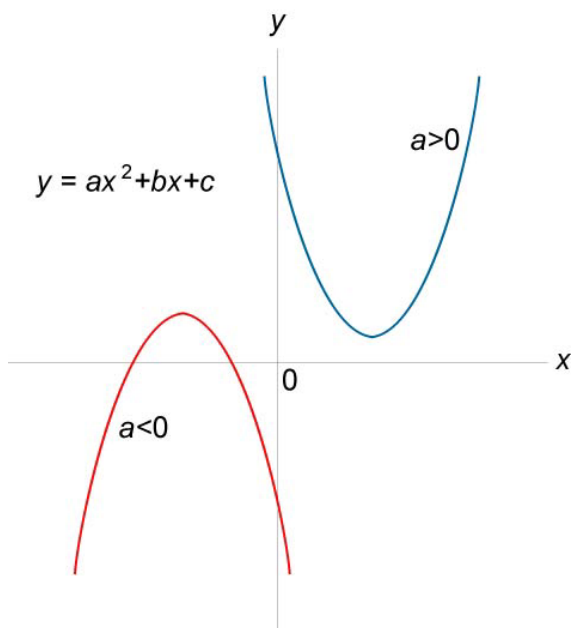


Figure 155.

731. Cubic Function
 $y = x^3, x \in \mathbb{R}.$

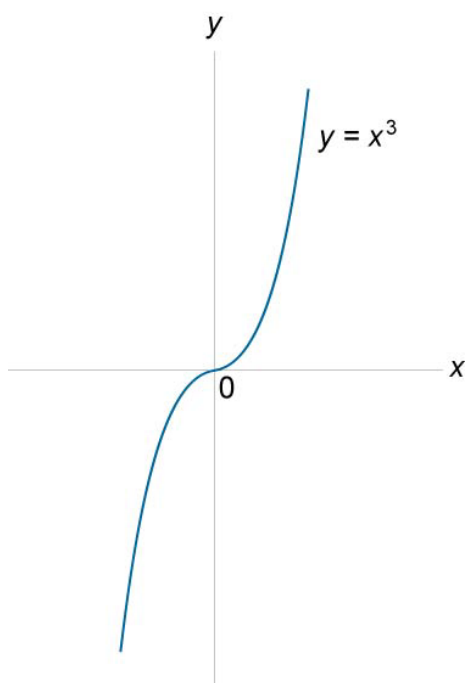


Figure 156.

732. $y = ax^3 + bx^2 + cx + d, x \in \mathbb{R}.$

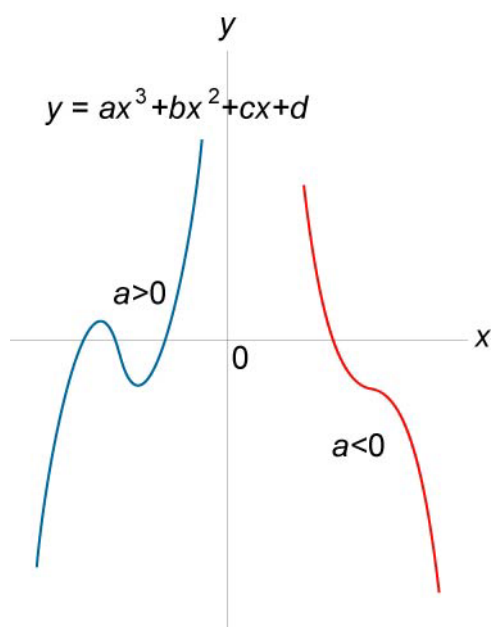


Figure 157.

733. Power Function
 $y = x^n, n \in \mathbb{N}.$

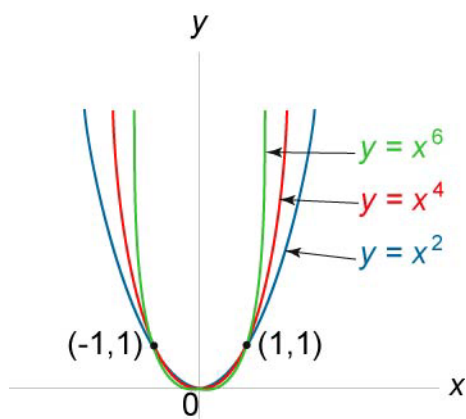


Figure 158.

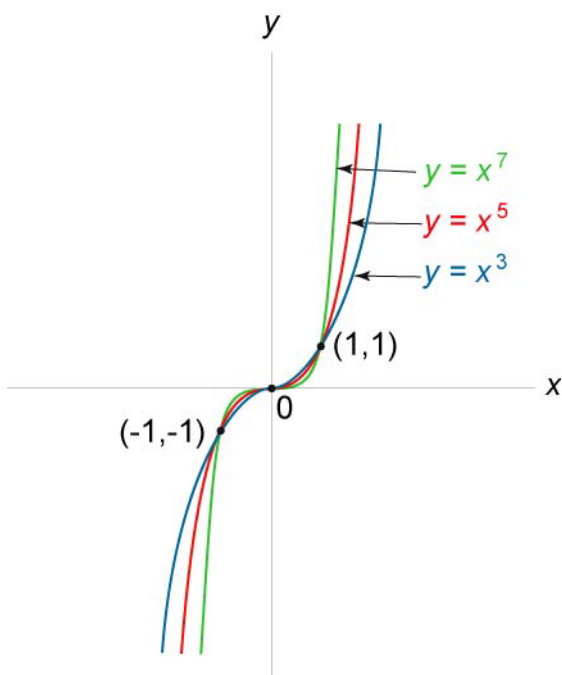


Figure 159.

734. Square Root Function

$y = \sqrt{x}$, $x \in [0, \infty)$.

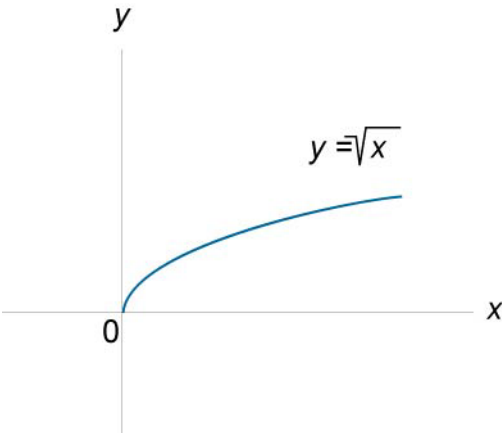


Figure 160.

735. Exponential Functions

$y = a^x$, $a > 0$, $a \neq 1$,

$y = e^x$ if $a = e$, $e = 2.71828182846\dots$

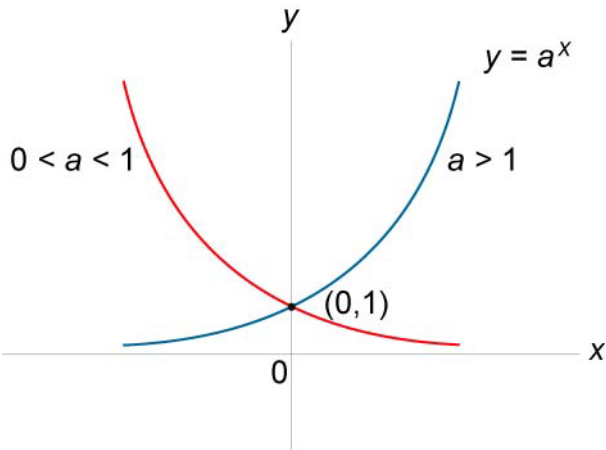


Figure 161.

736. Logarithmic Functions

$$y = \log_a x, \quad x \in (0, \infty), \quad a > 0, \quad a \neq 1,$$

$$y = \ln x \quad \text{if } a = e, \quad x > 0.$$

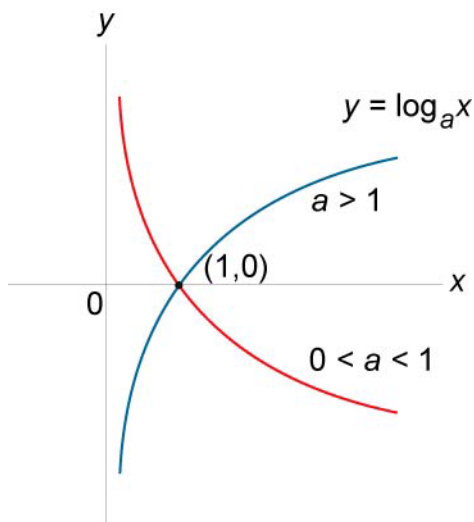


Figure 162.

737. Hyperbolic Sine Function

$$y = \sinh x, \quad \sinh x = \frac{e^x - e^{-x}}{2}, \quad x \in \mathbb{R}.$$

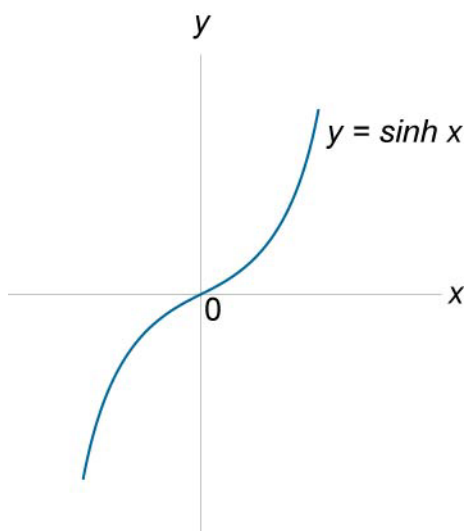


Figure 163.

738. Hyperbolic Cosine Function

$$y = \cosh x, \quad \cosh x = \frac{e^x + e^{-x}}{2}, \quad x \in \mathbb{R}.$$

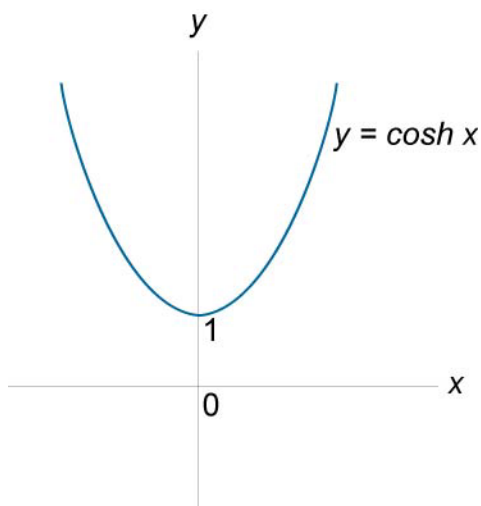


Figure 164.

739. Hyperbolic Tangent Function

$$y = \tanh x, \quad y = \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}, \quad x \in \mathbb{R}.$$

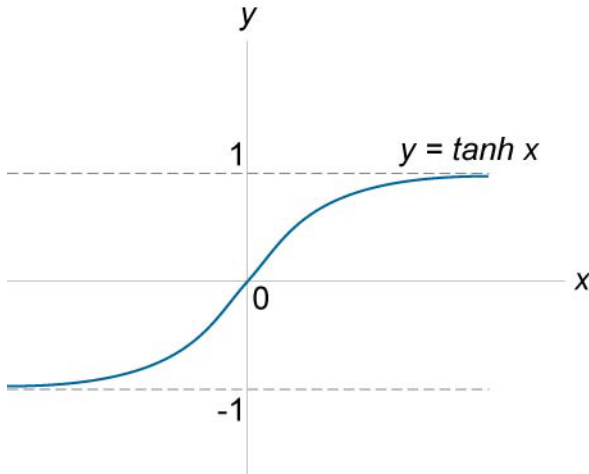


Figure 165.

740. Hyperbolic Cotangent Function

$$y = \coth x, \quad y = \coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}, \quad x \in \mathbb{R}, \quad x \neq 0.$$

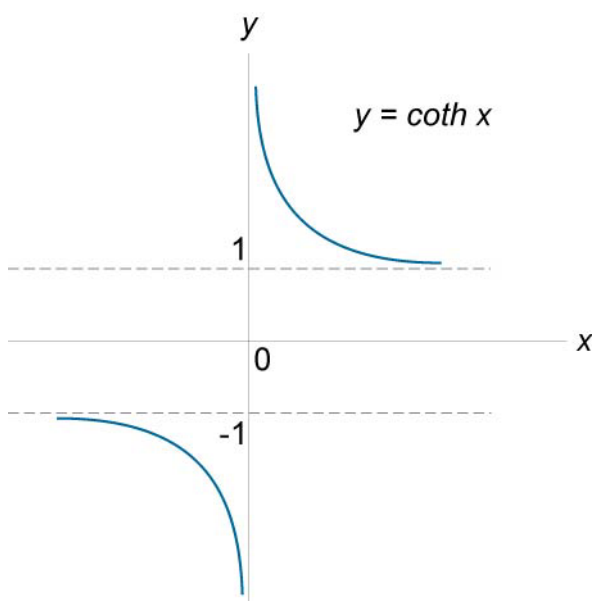


Figure 166.

741. Hyperbolic Secant Function

$$y = \operatorname{sech} x, \quad y = \operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}, \quad x \in \mathbb{R}.$$

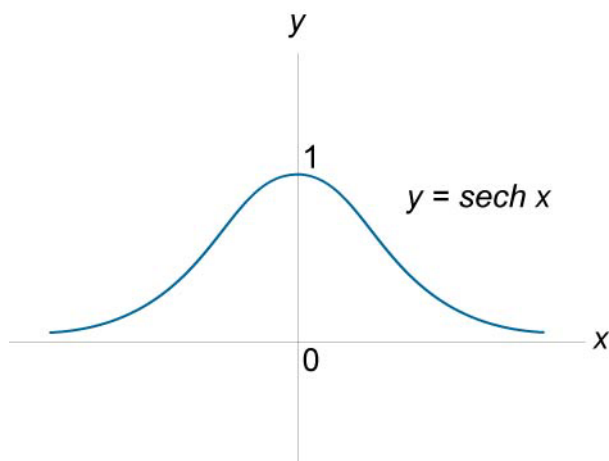


Figure 167.

742. Hyperbolic Cosecant Function

$$y = \operatorname{csch} x, \quad y = \operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}, \quad x \in \mathbb{R}, \quad x \neq 0.$$

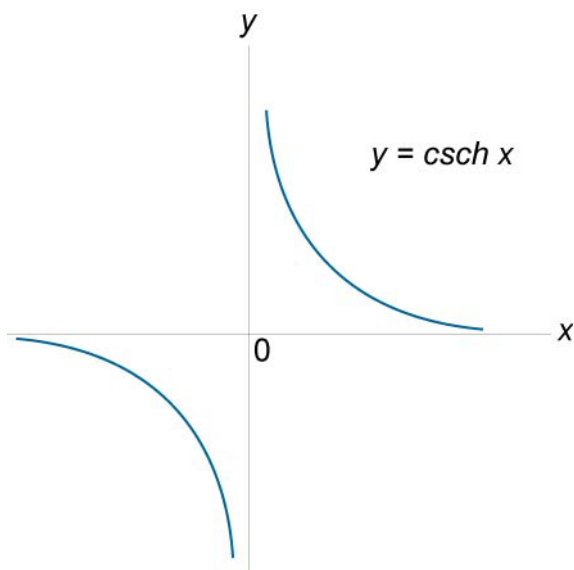


Figure 168.

743. Inverse Hyperbolic Sine Function

$$y = \operatorname{arcsinh} x, \quad x \in \mathbb{R}.$$

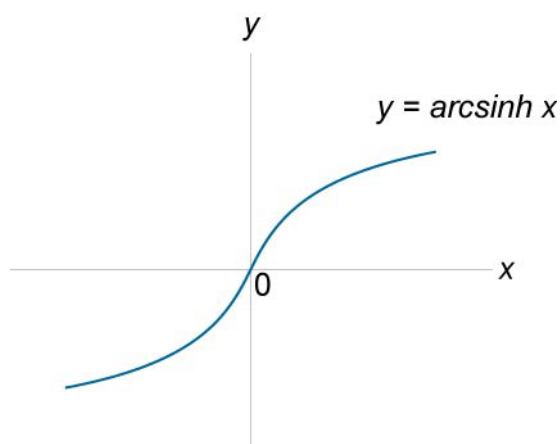


Figure 169.

- 744.** Inverse Hyperbolic Cosine Function
 $y = \operatorname{arccosh} x$, $x \in [1, \infty)$.

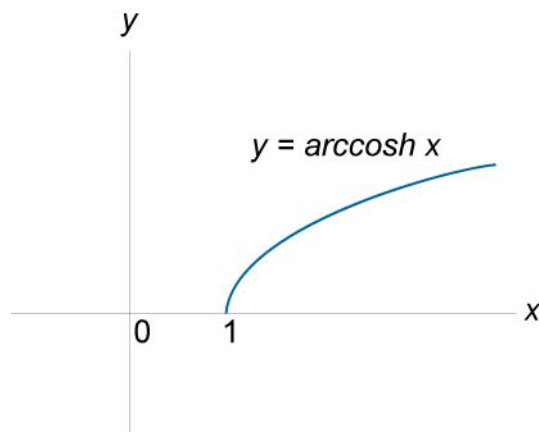


Figure 170.

- 745.** Inverse Hyperbolic Tangent Function
 $y = \operatorname{arctanh} x$, $x \in (-1, 1)$.

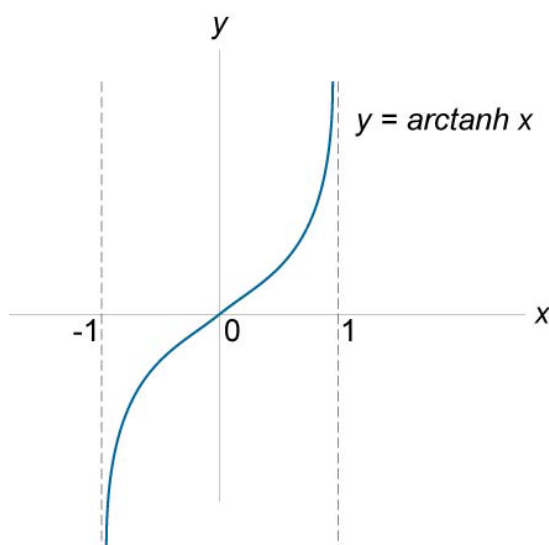


Figure 171.

746. Inverse Hyperbolic Cotangent Function
 $y = \operatorname{arccoth} x$, $x \in (-\infty, -1) \cup (1, \infty)$.

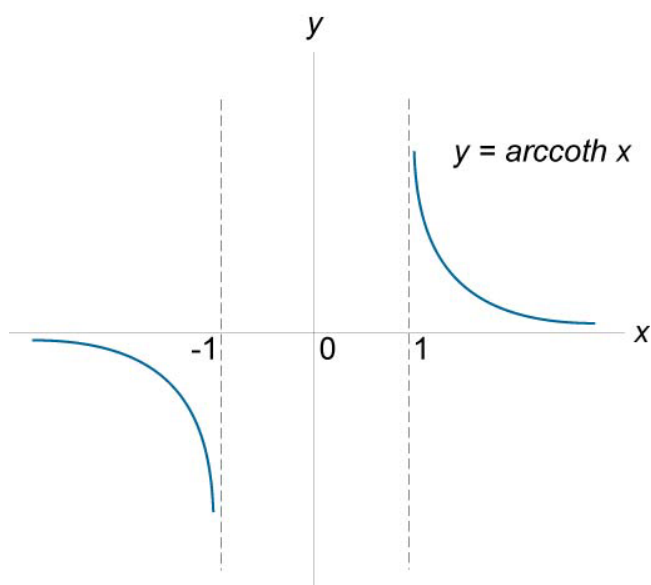


Figure 172.

747. Inverse Hyperbolic Secant Function
 $y = \operatorname{arcsech} x$, $x \in (0, 1]$.

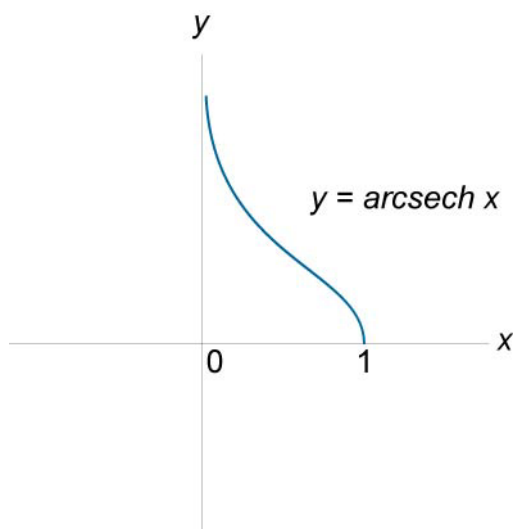


Figure 173.

- 748.** Inverse Hyperbolic Cosecant Function
 $y = \operatorname{arccsch} x$, $x \in \mathbb{R}$, $x \neq 0$.

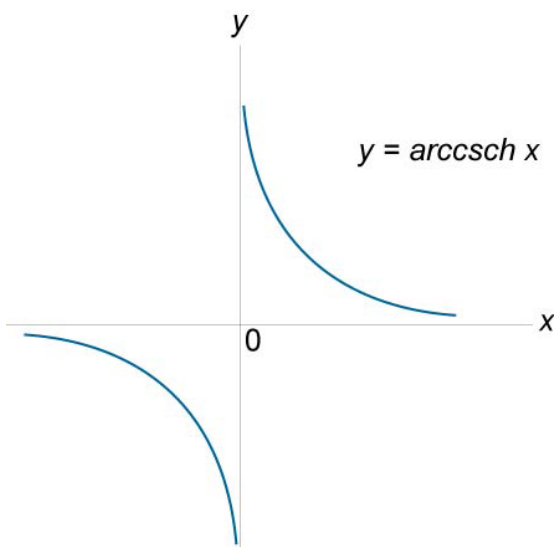


Figure 174.