

## 8.3 Definition and Properties of the Derivative

Functions:  $f, g, y, u, v$

Independent variable:  $x$

Real constant:  $k$

Angle:  $\alpha$

$$764. \quad y'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

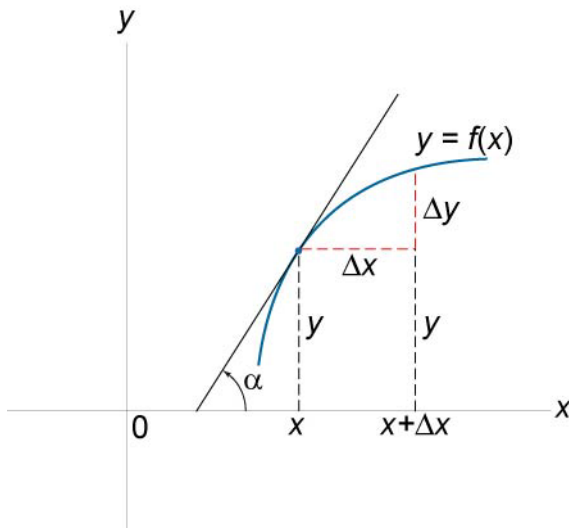


Figure 175.

$$765. \quad \frac{dy}{dx} = \tan \alpha$$

$$766. \quad \frac{d(u + v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$767. \frac{d(u - v)}{dx} = \frac{du}{dx} - \frac{dv}{dx}$$

$$768. \frac{d(ku)}{dx} = k \frac{du}{dx}$$

769. Product Rule

$$\frac{d(u \cdot v)}{dx} = \frac{du}{dx} \cdot v + u \cdot \frac{dv}{dx}$$

770. Quotient Rule

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{\frac{du}{dx} \cdot v - u \cdot \frac{dv}{dx}}{v^2}$$

771. Chain Rule

$$y = f(g(x)), \quad u = g(x),$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

772. Derivative of Inverse Function

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}, \quad y = f(x).$$

where  $x(y)$  is the inverse function of  $f(x)$

773. Reciprocal Rule

$$\frac{d\left(\frac{1}{y}\right)}{dx} = -\frac{\frac{dy}{dx}}{y^2}$$

774. Logarithmic Differentiation

$$y = f(x), \quad \ln y = \ln f(x),$$

$$\frac{dy}{dx} = f(x) \cdot \frac{d}{dx} [\ln f(x)].$$