

8.6 Applications of Derivative

Functions: f, g, y

Position of an object: s

Velocity: v

Acceleration: w

Independent variable: x

Time: t

Natural number: n

825. Velocity and Acceleration

$s = f(t)$ is the position of an object relative to a fixed coordinate system at a time t ,

$v = s' = f'(t)$ is the instantaneous velocity of the object,

$w = v' = s'' = f''(t)$ is the instantaneous acceleration of the object.

826. Tangent Line

$$y - y_0 = f'(x_0)(x - x_0)$$

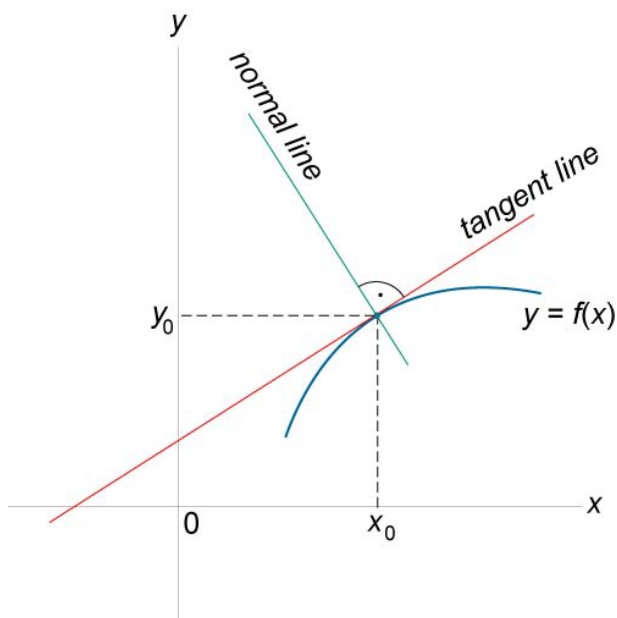


Figure 176.

827. Normal Line

$$y - y_0 = -\frac{1}{f'(x_0)}(x - x_0) \quad (\text{Fig 176})$$

828. Increasing and Decreasing Functions.

If $f'(x_0) > 0$, then $f(x)$ is increasing at x_0 . (Fig 177, $x < x_1$, $x_2 < x$),

If $f'(x_0) < 0$, then $f(x)$ is decreasing at x_0 . (Fig 177, $x_1 < x < x_2$),

If $f'(x_0)$ does not exist or is zero, then the test fails.

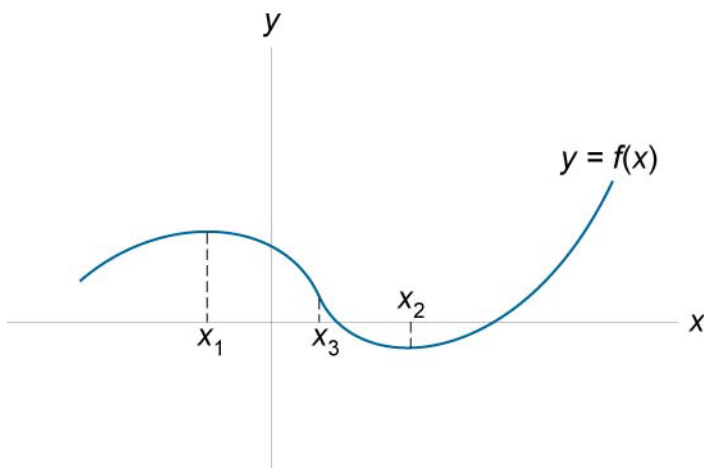


Figure 177.

829. Local extrema

A function $f(x)$ has a **local maximum** at x_1 if and only if there exists some interval containing x_1 such that $f(x_1) \geq f(x)$ for all x in the interval (Fig.177).

A function $f(x)$ has a **local minimum** at x_2 if and only if there exists some interval containing x_2 such that $f(x_2) \leq f(x)$ for all x in the interval (Fig.177).

830. Critical Points

A critical point on $f(x)$ occurs at x_0 if and only if either $f'(x_0)$ is zero or the derivative doesn't exist.

831. First Derivative Test for Local Extrema.

If $f(x)$ is increasing ($f'(x) > 0$) for all x in some interval $(a, x_1]$ and $f(x)$ is decreasing ($f'(x) < 0$) for all x in some interval $[x_1, b)$, then $f(x)$ has a local maximum at x_1 (Fig.177).

832. If $f(x)$ is decreasing ($f'(x) < 0$) for all x in some interval $(a, x_2]$ and $f(x)$ is increasing ($f'(x) > 0$) for all x in some interval $[x_2, b)$, then $f(x)$ has a local minimum at x_2 . (Fig.177).

833. Second Derivative Test for Local Extrema.

If $f'(x_1) = 0$ and $f''(x_1) < 0$, then $f(x)$ has a local maximum at x_1 .

If $f'(x_2) = 0$ and $f''(x_2) > 0$, then $f(x)$ has a local minimum at x_2 . (Fig.177)

834. Concavity.

$f(x)$ is concave upward at x_0 if and only if $f'(x)$ is increasing at x_0 (Fig.177, $x_3 < x$).

$f(x)$ is concave downward at x_0 if and only if $f'(x)$ is decreasing at x_0 . (Fig.177, $x < x_3$).

835. Second Derivative Test for Concavity.

If $f''(x_0) > 0$, then $f(x)$ is concave upward at x_0 .

If $f''(x_0) < 0$, then $f(x)$ is concave downward at x_0 .

If $f''(x)$ does not exist or is zero, then the test fails.

836. Inflection Points

If $f'(x_3)$ exists and $f''(x)$ changes sign at $x = x_3$, then the point $(x_3, f(x_3))$ is an **inflection point** of the graph of $f(x)$. If $f''(x_3)$ exists at the inflection point, then $f''(x_3) = 0$ (Fig.177).

837. L'Hopital's Rule

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)} \text{ if } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} g(x) = \begin{cases} 0 \\ \infty \end{cases}.$$