

9.8 Definite Integral

Definite integral of a function: $\int_a^b f(x)dx$, $\int_a^b g(x)dx$, ...

Riemann sum: $\sum_{i=1}^n f(\xi_i) \Delta x_i$

Small changes: Δx_i

Antiderivatives: $F(x)$, $G(x)$

Limits of integrations: a , b , c , d

1053. $\int_a^b f(x)dx = \lim_{\substack{n \rightarrow \infty \\ \max \Delta x_i \rightarrow 0}} \sum_{i=1}^n f(\xi_i) \Delta x_i$,

where $\Delta x_i = x_i - x_{i-1}$, $x_{i-1} \leq \xi_i \leq x_i$.

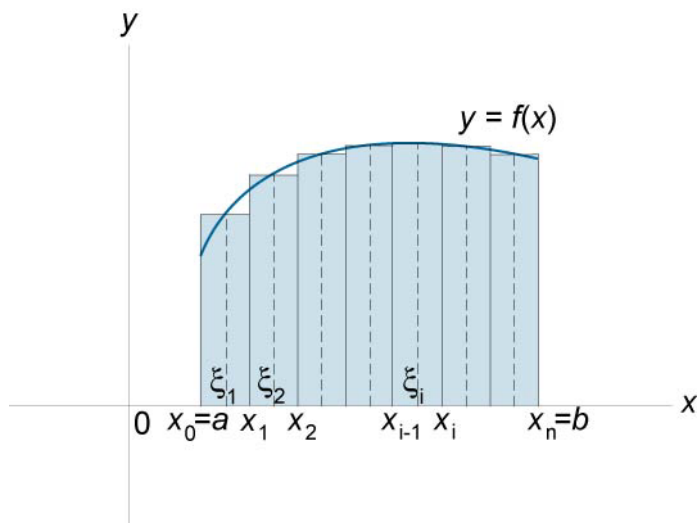


Figure 179.

$$1054. \int_a^b 1 dx = b - a$$

$$1055. \int_a^b k f(x) dx = k \int_a^b f(x) dx$$

$$1056. \int_a^b [f(x) + g(x)] dx = \int_a^b f(x) dx + \int_a^b g(x) dx$$

$$1057. \int_a^b [f(x) - g(x)] dx = \int_a^b f(x) dx - \int_a^b g(x) dx$$

$$1058. \int_a^a f(x) dx = 0$$

$$1059. \int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$1060. \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \text{ for } a < c < b.$$

$$1061. \int_a^b f(x) dx \geq 0 \text{ if } f(x) \geq 0 \text{ on } [a, b].$$

$$1062. \int_a^b f(x) dx \leq 0 \text{ if } f(x) \leq 0 \text{ on } [a, b].$$

1063. Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a) \text{ if } F'(x) = f(x).$$

1064. Method of Substitution

If $x = g(t)$, then

$$\int_a^b f(x)dx = \int_c^d f(g(t))g'(t)dt,$$

where

$$c = g^{-1}(a), \quad d = g^{-1}(b).$$

1065. Integration by Parts

$$\int_a^b u dv = (uv)\Big|_a^b - \int_a^b v du$$

1066. Trapezoidal Rule

$$\int_a^b f(x)dx = \frac{b-a}{2n} \left[f(x_0) + f(x_n) + 2 \sum_{i=1}^{n-1} f(x_i) \right]$$

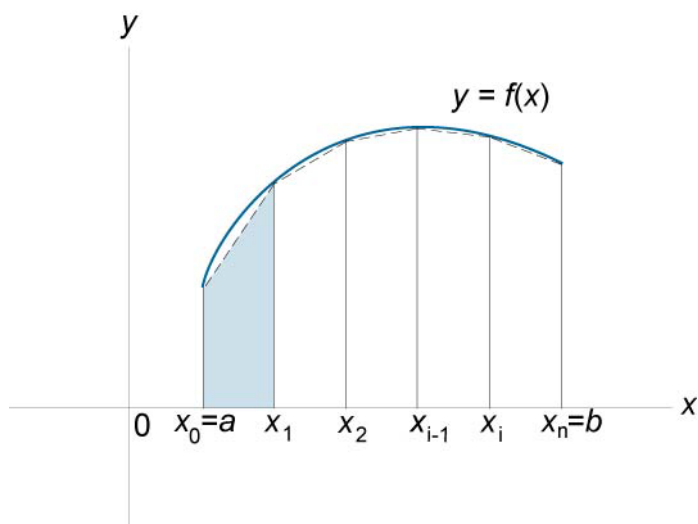


Figure 180.

1067. Simpson's Rule

$$\int_a^b f(x) dx = \frac{b-a}{3n} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 4f(x_{n-1}) + f(x_n)],$$

where

$$x_i = a + \frac{b-a}{n} i, \quad i = 0, 1, 2, \dots, n.$$

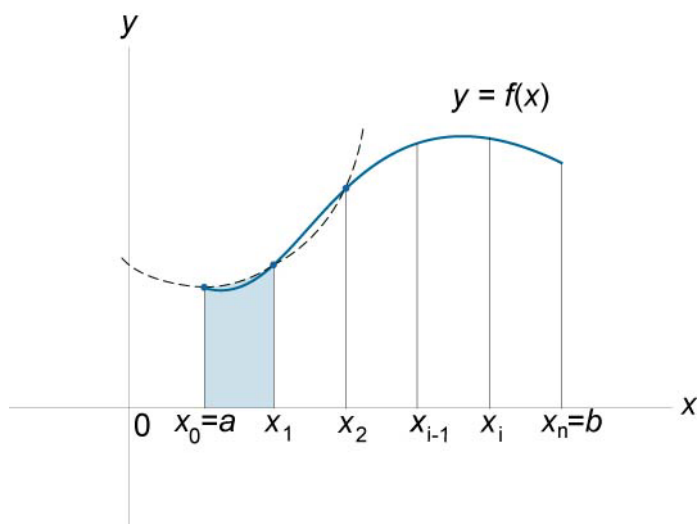


Figure 181.

1068. Area Under a Curve

$$S = \int_a^b f(x) dx = F(b) - F(a),$$

where $F'(x) = f(x)$.

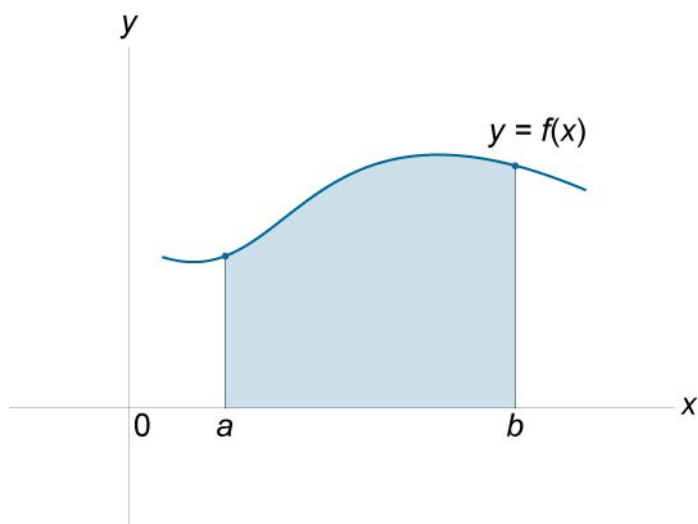


Figure 182.

1069. Area Between Two Curves

$$S = \int_a^b [f(x) - g(x)] dx = F(b) - G(b) - F(a) + G(a),$$

where $F'(x) = f(x)$, $G'(x) = g(x)$.

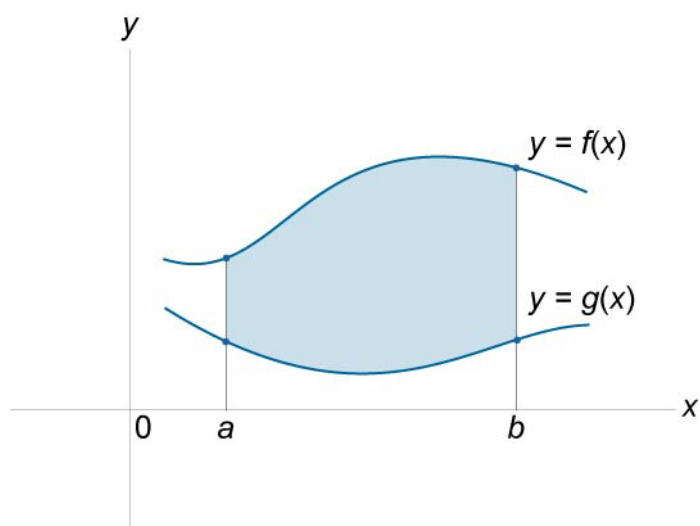


Figure 183.