

12.2 Probability Formulas

Events: A, B

Probability: P

Random variables: X, Y, Z

Values of random variables: x, y, z

Expected value of X: μ

Any positive real number: ε

Standard deviation: σ

Variance: σ^2

Density functions: $f(x)$, $f(t)$

1259. Probability of an Event

$$P(A) = \frac{m}{n},$$

where

m is the number of possible positive outcomes,

n is the total number of possible outcomes.

1260. Range of Probability Values

$$0 \leq P(A) \leq 1$$

1261. Certain Event

$$P(A) = 1$$

1262. Impossible Event

$$P(A) = 0$$

1263. Complement

$$P(\bar{A}) = 1 - P(A)$$

1264. Independent Events

$$P(A/B) = P(A),$$

$$P(B/A) = P(B)$$

1265. Addition Rule for Independent Events

$$P(A \cup B) = P(A) + P(B)$$

1266. Multiplication Rule for Independent Events

$$P(A \cap B) = P(A) \cdot P(B)$$

1267. General Addition Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B),$$

where

$A \cup B$ is the union of events A and B,

$A \cap B$ is the intersection of events A and B.

1268. Conditional Probability

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

1269. $P(A \cap B) = P(B) \cdot P(A/B) = P(A) \cdot P(B/A)$

1270. Law of Total Probability

$$P(A) = \sum_{i=1}^m P(B_i)P(A/B_i),$$

where B_i is a sequence of mutually exclusive events.

1271. Bayes' Theorem

$$P(B/A) = \frac{P(A/B) \cdot P(B)}{P(A)}$$

1272. Bayes' Formula

$$P(B_i/A) = \frac{P(B_i) \cdot P(A/B_i)}{\sum_{k=1}^m P(B_k) \cdot P(A/B_k)},$$

where

B_i is a set of mutually exclusive events (hypotheses),

A is the final event,

$P(B_i)$ are the prior probabilities,

$P(B_i/A)$ are the posterior probabilities.

1273. Law of Large Numbers

$$P\left(\left|\frac{S_n}{n} - \mu\right| \geq \varepsilon\right) \rightarrow 0 \text{ as } n \rightarrow \infty,$$

$$P\left(\left|\frac{S_n}{n} - \mu\right| < \varepsilon\right) \rightarrow 1 \text{ as } n \rightarrow \infty,$$

where

S_n is the sum of random variables,

n is the number of possible outcomes.

1274. Chebyshev Inequality

$$P(|X - \mu| \geq \varepsilon) \leq \frac{V(X)}{\varepsilon^2},$$

where $V(X)$ is the variance of X .

1275. Normal Density Function

$$\varphi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where x is a particular outcome.

1276. Standard Normal Density Function

$$\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

Average value $\mu = 0$, deviation $\sigma = 1$.

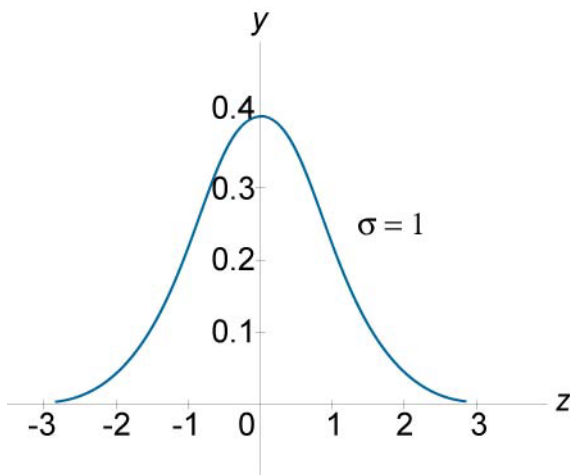


Figure 210.

1277. Standard Z Value

$$Z = \frac{X - \mu}{\sigma}$$

1278. Cumulative Normal Distribution Function

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt,$$

where

x is a particular outcome,

t is a variable of integration.

1279. $P(\alpha < X < \beta) = F\left(\frac{\alpha - \mu}{\sigma}\right) - F\left(\frac{\beta - \mu}{\sigma}\right),$

where

X is normally distributed random variable,

F is cumulative normal distribution function,

$P(\alpha < X < \beta)$ is interval probability.

1280. $P(|X - \mu| < \varepsilon) = 2F\left(\frac{\varepsilon}{\sigma}\right),$

where

X is normally distributed random variable,

F is cumulative normal distribution function.

1281. Cumulative Distribution Function

$$F(x) = P(X < x) = \int_{-\infty}^x f(t) dt,$$

where t is a variable of integration.

1282. Bernoulli Trials Process

$$\mu = np, \quad \sigma^2 = npq,$$

where

n is a sequence of experiments,

p is the probability of success of each experiments,

q is the probability of failure, $q = 1 - p$.

1283. Binomial Distribution Function

$$b(n, p, q) = \binom{n}{k} p^k q^{n-k},$$

$$\mu = np, \sigma^2 = npq,$$

$$f(x) = (q + pe^x)^n,$$

where

n is the number of trials of selections,

p is the probability of success,

q is the probability of failure, $q = 1 - p$.

1284. Geometric Distribution

$$P(T = j) = q^{j-1} p,$$

$$\mu = \frac{1}{p}, \sigma^2 = \frac{q}{p^2},$$

where

T is the first successful event in the series,

j is the event number,

p is the probability that any one event is successful,

q is the probability of failure, $q = 1 - p$.

1285. Poisson Distribution

$$P(X = k) \approx \frac{\lambda^k}{k!} e^{-\lambda}, \lambda = np,$$

$$\mu = \lambda, \sigma^2 = \lambda,$$

where

λ is the rate of occurrence,

k is the number of positive outcomes.

1286. Density Function

$$P(a \leq X \leq b) = \int_a^b f(x) dx$$

1287. Continuous Uniform Density

$$f = \frac{1}{b-a}, \mu = \frac{a+b}{2},$$

where f is the density function.

1288. Exponential Density Function

$$f(t) = \lambda e^{-\lambda t}, \mu = \lambda, \sigma^2 = \lambda^2$$

where t is time, λ is the failure rate.

1289. Exponential Distribution Function

$$F(t) = 1 - e^{-\lambda t},$$

where t is time, λ is the failure rate.

1290. Expected Value of Discrete Random Variables

$$\mu = E(X) = \sum_{i=1}^n x_i p_i,$$

where x_i is a particular outcome, p_i is its probability.

1291. Expected Value of Continuous Random Variables

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

1292. Properties of Expectations

$$E(X + Y) = E(X) + E(Y),$$

$$E(X - Y) = E(X) - E(Y),$$

$$E(cX) = cE(X),$$

$$E(XY) = E(X) \cdot E(Y),$$

where c is a constant.

1293. $E(X^2) = V(X) + \mu^2$,

where

$\mu = E(X)$ is the expected value,

$V(X)$ is the variance.

1294. Markov Inequality

$$P(X > k) \leq \frac{E(X)}{k},$$

where k is some constant.

1295. Variance of Discrete Random Variables

$$\sigma^2 = V(X) = E[(X - \mu)^2] = \sum_{i=1}^n (x_i - \mu)^2 p_i,$$

where

x_i is a particular outcome,

p_i is its probability.

1296. Variance of Continuous Random Variables

$$\sigma^2 = V(X) = E[(X - \mu)^2] = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$$

1297. Properties of Variance

$$V(X + Y) = V(X) + V(Y),$$

$$V(X - Y) = V(X) + V(Y),$$

$$V(X + c) = V(X),$$

$$V(cX) = c^2 V(X),$$

where c is a constant.

1298. Standard Deviation

$$D(X) = \sqrt{V(X)} = \sqrt{E[(X - \mu)^2]}$$

1299. Covariance

$$\text{cov}(X, Y) = E[(X - \mu(X))(Y - \mu(Y))] = E(XY) - \mu(X)\mu(Y),$$

where

X is random variable,

$V(X)$ is the variance of X ,

μ is the expected value of X or Y .

1300. Correlation

$$\rho(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{V(X)V(Y)}},$$

where

$V(X)$ is the variance of X ,

$V(Y)$ is the variance of Y .