12.2 Probability Formulas

Events: A, B
Probability: P
Random variables: X, Y, Z
Values of random variables: x, y, z
Expected value of X: μ
Any positive real number: ε
Standard deviation: σ
Variance: σ^2
Density functions: $f(x)$, $f(t)$

1259. Probability of an Event

$$P(A)=\frac{m}{n},$$

where
m is the number of possible positive outcomes,
n is the total number of possible outcomes.

1260. Range of Probability Values

$$0 \le P(A) \le 1$$

1261. Certain Event

$$P(A)=1$$

1262. Impossible Event

$$P(A)=0$$

1263. Complement

$$P(\overline{A})=1-P(A)$$

1264. Independent Events

$$P(A/B)=P(A),$$
$$P(B/A)=P(B)$$

1265. Addition Rule for Independent Events

$$P(A \cup B)=P(A)+P(B)$$

where f is the density function.

1288. Exponential Density Function

$f(t) = \lambda e^{-\lambda t}$, $\mu = \lambda$, $\sigma^2 = \lambda^2$

where t is time, λ is the failure rate.

1289. Exponential Distribution Function

$F(t) = 1 - e^{-\lambda t}$,

where t is time, λ is the failure rate.

1290. Expected Value of Discrete Random Variables

$$\mu = E(X) = \sum_{i=1}^{n} x_i p_i ,$$

where x_i is a particular outcome, p_i is its probability.

1291. Expected Value of Continuous Random Variables

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

1292. Properties of Expectations

$E(X + Y) = E(X) + E(Y)$,

$E(X - Y) = E(X) - E(Y)$,

$E(cX) = cE(X)$,

$E(XY) = E(X) \cdot E(Y)$,

where c is a constant.

1293. $E(X^2) = V(X) + \mu^2$,

where

$\mu = E(X)$ is the expected value,

$V(X)$ is the variance.

1294. Markov Inequality

$$P(X > k) \leq \frac{E(X)}{k},$$

where k is some constant.

1295. Variance of Discrete Random Variables

$$\sigma^2 = V(X) = E\left[(X-\mu)^2\right] = \sum_{i=1}^{n} (x_i - \mu)^2 p_i,$$

where

x_i is a particular outcome,

p_i is its probability.

1296. Variance of Continuous Random Variables

$$\sigma^2 = V(X) = E\left[(X-\mu)^2\right] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$$

1297. Properties of Variance

$$V(X+Y) = V(X) + V(Y),$$
$$V(X-Y) = V(X) + V(Y),$$
$$V(X+c) = V(X),$$
$$V(cX) = c^2 V(X),$$

where c is a constant.

1298. Standard Deviation

$$D(X) = \sqrt{V(X)} = \sqrt{E\left[(X-\mu)^2\right]}$$

1299. Covariance

$$\operatorname{cov}(X,Y) = E[(X-\mu(X))(Y-\mu(Y))] = E(XY) - \mu(X)\mu(Y),$$

where

X is random variable,

$V(X)$ is the variance of X,

μ is the expected value of X or Y.

1300. Correlation

$$\rho(X,Y)=\frac{\operatorname{cov}(X,Y)}{\sqrt{V(X)V(Y)}},$$

where

$V(X)$ is the variance of X,

$V(Y)$ is the variance of Y.