

8. Transformation Formulae

Exercise 8.1

1. Question

Express each of the following as the sum or difference of sines and cosines:

(i) $2 \sin 3x \cos x$

(ii) $2 \cos 3x \sin 2x$

(iii) $2 \sin 4x \sin 3x$

(iv) $2 \cos 7x \cos 3x$

Answer

(i) We know, $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$$2 \sin 3x \cos x = \sin(3x + x) + \sin(3x - x)$$

$$= \sin(4x) + \sin(2x)$$

$$= \sin 4x + \sin 2x$$

(ii) We know, $2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

$$2 \cos 3x \sin 2x = \sin(3x + 2x) - \sin(3x - 2x)$$

$$= \sin(5x) - \sin(x)$$

$$= \sin 5x - \sin x$$

(iii) We know, $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

$$2 \sin 4x \sin 3x = \cos(4x - 3x) - \cos(4x + 3x)$$

$$= \cos(x) - \cos(7x)$$

$$= \cos x - \cos 7x$$

(iv) We know, $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$

$$2 \sin 3x \cos x = \cos(7x + 3x) + \cos(7x - 3x)$$

$$= \cos(10x) + \cos(4x)$$

$$= \cos 10x + \cos 4x$$

2. Question

Prove that :

i. $2 \sin \frac{5\pi}{12} \sin \frac{\pi}{12} = \frac{1}{2}$

ii. $2 \cos \frac{5\pi}{12} \cos \frac{\pi}{12} = \frac{1}{2}$

iii. $2 \sin \frac{5\pi}{12} \cos \frac{\pi}{12} = \frac{\sqrt{3} + 2}{2}$

Answer

i. We know, $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

$$2 \sin \frac{5\pi}{12} \sin \frac{\pi}{12} = \cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right) - \cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right)$$

$$= \cos\left(\frac{4\pi}{12}\right) - \cos\left(\frac{6\pi}{12}\right)$$

$$= \cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{2}\right)$$

$$= \cos\left(\frac{180^\circ}{3}\right) - \cos\left(\frac{180^\circ}{2}\right)$$

$$= \cos 60^\circ - \cos 90^\circ$$

$$= \frac{1}{2} - 0$$

$$= \frac{1}{2}$$

Hence Proved

ii. We know, $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$

$$\begin{aligned}
2 \cos \frac{5n}{12} \cos \frac{n}{12} &= \cos \left(\frac{5n}{12} + \frac{n}{12} \right) + \cos \left(\frac{5n}{12} - \frac{n}{12} \right) \\
&= \cos \left(\frac{6n}{12} \right) + \cos \left(\frac{4n}{12} \right) \\
&= \cos \left(\frac{n}{2} \right) + \cos \left(\frac{n}{3} \right) \\
&= \cos \left(\frac{180^\circ}{2} \right) + \cos \left(\frac{180^\circ}{3} \right) \\
&= \cos 90^\circ + \cos 60^\circ \\
&= 0 + \frac{1}{2} \\
&= \frac{1}{2}
\end{aligned}$$

Hence Proved

$$\begin{aligned}
\text{iii. We know, } 2 \sin A \cos B &= \sin (A + B) + \sin (A - B) \\
2 \sin \frac{5n}{12} \cos \frac{n}{12} &= \sin \left(\frac{5n}{12} + \frac{n}{12} \right) + \sin \left(\frac{5n}{12} - \frac{n}{12} \right) \\
&= \sin \left(\frac{6n}{12} \right) + \sin \left(\frac{4n}{12} \right) \\
&= \sin \left(\frac{n}{2} \right) + \sin \left(\frac{n}{3} \right) \\
&= \sin \left(\frac{180^\circ}{2} \right) + \sin \left(\frac{180^\circ}{3} \right) \\
&= \sin 90^\circ + \sin 60^\circ \\
&= 1 + \frac{\sqrt{3}}{2} \\
&= \frac{2 + \sqrt{3}}{2}
\end{aligned}$$

Hence Proved

3. Question

Show that:

$$\begin{aligned}
\text{i. } \sin 50^\circ \cos 85^\circ &= \frac{1 - \sqrt{2} \sin 35^\circ}{2\sqrt{2}} \\
\text{ii. } \sin 25^\circ \cos 115^\circ &= \frac{1}{2} \{ \sin (140^\circ) - 1 \}
\end{aligned}$$

Answer

i. We know, $2 \sin A \cos B = \sin (A + B) + \sin (A - B)$

$$\begin{aligned}
\Rightarrow \sin A \cos B &= \frac{\sin (A + B) + \sin (A - B)}{2} \\
\sin 50^\circ \cos 85^\circ &= \frac{\sin (50^\circ + 85^\circ) + \sin (50^\circ - 85^\circ)}{2} \\
&= \frac{\sin (135^\circ) + \sin (-35^\circ)}{2} \\
&= \frac{\sin (135^\circ) - \sin (35^\circ)}{2} \\
\{ \sin (-x) &= -\sin x \} \\
&= \frac{\sin (180^\circ - 45^\circ) - \sin (35^\circ)}{2} \\
\{ \sin (180^\circ - x) &= \sin x \} \\
&= \frac{\sin (45^\circ) - \sin (35^\circ)}{2} \\
&= \frac{\frac{1}{\sqrt{2}} - \sin (35^\circ)}{2} \\
&= \frac{1 - \sin (35^\circ)}{2\sqrt{2}}
\end{aligned}$$

$$= \frac{1 - \sin(35^\circ)}{2\sqrt{2}}$$

Hence Proved

ii. We know, $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$$\Rightarrow \sin A \cos B = \frac{\sin(A + B) + \sin(A - B)}{2}$$

Take L.H.S

$$\sin 25^\circ \cos 115^\circ = \frac{\sin(25^\circ + 115^\circ) + \sin(25^\circ - 115^\circ)}{2}$$

$$= \frac{\sin(140^\circ) + \sin(-90^\circ)}{2}$$

{ $\sin(-x) = -\sin x$ }

$$= \frac{\sin(140^\circ) - \sin(90^\circ)}{2}$$

$$= \frac{1}{2} \{\sin(140^\circ) - 1\}$$

= R.H.S.

Hence Proved

4. Question

Prove that:

$$4 \cos x \cos\left(\frac{\pi}{3} + x\right) \cos\left(\frac{\pi}{3} - x\right) = \cos 3x$$

Answer

Take L.H.S

$$4 \cos x \cos\left(\frac{\pi}{3} + x\right) \cos\left(\frac{\pi}{3} - x\right) = 2 \cos x \left(2 \cos\left(\frac{\pi}{3} + x\right) \cos\left(\frac{\pi}{3} - x\right)\right)$$

{ $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$ }

$$= 2 \cos x \left\{ \cos\left(\frac{\pi}{3} + x + \frac{\pi}{3} - x\right) + \cos\left(\frac{\pi}{3} + x - \frac{\pi}{3} + x\right) \right\}$$

$$= 2 \cos x \left\{ \cos\left(\frac{2\pi}{3}\right) + \cos(2x) \right\}$$

$$= 2 \cos x \{ \cos 120^\circ + \cos 2x \}$$

$$= 2 \cos x \{ \cos(180^\circ - 60^\circ) + \cos 2x \}$$

{ $\cos(180^\circ - A) = -\cos A$ }

$$= 2 \cos x (\cos 2x - \cos 60^\circ)$$

$$= 2 \cos 2x \cos x - 2 \cos x \cos 60^\circ$$

$$= \{ \cos(x + 2x) + \cos(2x - x) \} - \frac{2 \cos x}{2}$$

$$= \cos 3x + \cos x - \cos x$$

$$= \cos 3x = \text{R.H.S.}$$

Hence Proved

5 A. Question

Prove that:

$$\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$$

Answer

Take L.H.S

$$\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ$$

$$= \frac{\sqrt{3} \cos 10^\circ \cos 50^\circ \cos 70^\circ}{2}$$

Multiplying & Dividing by 2:

$$= \frac{\sqrt{3} (2 \cos 10^\circ \cos 50^\circ) \cos 70^\circ}{2 \times 2}$$

$$= \frac{\sqrt{3} (2 \cos 10^\circ \cos 50^\circ) \cos 70^\circ}{2 \times 2}$$

$$\{\because 2 \cos A \cos B = \cos (A + B) + \cos (A - B)\}$$

$$= \frac{\sqrt{3} \{\cos (10^\circ + 50^\circ) + \cos (10^\circ - 40^\circ)\} \cos 70^\circ}{4}$$

$$= \frac{\sqrt{3} \{\cos (60^\circ) + \cos (-40^\circ)\} \cos 70^\circ}{4}$$

$$\{\because \cos (-A) = \cos A\}$$

$$= \frac{\sqrt{3} \left\{ \frac{1}{2} + \cos (40^\circ) \right\} \cos 70^\circ}{4}$$

$$= \frac{\sqrt{3}}{8} \cos 70^\circ + \frac{\sqrt{3}}{4} \cos 70^\circ \cos 40^\circ$$

Multiplying & Dividing by 2:

$$= \frac{\sqrt{3}}{8} \cos 70^\circ + \frac{\sqrt{3}}{4 \times 2} (2 \cos 70^\circ \cos 40^\circ)$$

$$\{\because 2 \cos A \cos B = \cos (A + B) + \cos (A - B)\}$$

$$= \frac{\sqrt{3}}{8} \cos 70^\circ + \frac{\sqrt{3}}{8} \{\cos (70^\circ + 40^\circ) + \cos (70^\circ - 40^\circ)\}$$

$$= \frac{\sqrt{3}}{8} \cos 70^\circ + \frac{\sqrt{3}}{8} \{\cos (110^\circ) + \cos 30^\circ\}$$

$$= \frac{\sqrt{3}}{8} \cos 70^\circ + \frac{\sqrt{3}}{8} \left\{ \cos (180^\circ - 70^\circ) + \frac{\sqrt{3}}{2} \right\}$$

$$\{\because \cos (180^\circ - A) = -\cos A\}$$

$$= \frac{\sqrt{3}}{8} \cos 70^\circ + \frac{\sqrt{3}}{8} \left\{ -\cos 70^\circ + \frac{\sqrt{3}}{2} \right\}$$

$$= \frac{\sqrt{3}}{8} \cos 70^\circ - \frac{\sqrt{3}}{8} \cos 70^\circ + \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2}$$

$$= \frac{3}{16}$$

= R.H.S

Hence Proved

5 B. Question

Prove that:

$$\cos 40^\circ \cos 80^\circ \cos 160^\circ = \frac{-1}{8}$$

Answer

Take L.H.S

$$\cos 40^\circ \cos 80^\circ \cos 160^\circ$$

Multiplying & Dividing by 2:

$$= \frac{(2 \cos 40^\circ \cos 160^\circ) \cos 80^\circ}{2}$$

$$\{\because 2 \cos A \cos B = \cos (A + B) + \cos (A - B)\}$$

$$= \frac{\{\cos (40^\circ + 160^\circ) + \cos (160^\circ - 40^\circ)\} \cos 80^\circ}{2}$$

$$= \frac{\{\cos (200^\circ) + \cos (120^\circ)\} \cos 80^\circ}{2}$$

$$= \frac{\{\cos (180^\circ + 20^\circ) + \cos (180^\circ - 60^\circ)\} \cos 80^\circ}{2}$$

$$\{\because \cos (180^\circ - A) = -\cos A \text{ \& \; } \cos (180^\circ + A) = -\cos A\}$$

$$= \frac{(-\cos 20^\circ - \cos 60^\circ) \cos 80^\circ}{2}$$

$$= \frac{\left(-\cos 20^\circ - \frac{1}{2}\right) \cos 80^\circ}{2}$$

$$= \frac{\left(-\cos 20^\circ \cos 80^\circ - \frac{\cos 80^\circ}{2}\right)}{2}$$

$$= \frac{-\cos 20^\circ \cos 80^\circ}{2} - \frac{\cos 80^\circ}{4}$$

Multiplying & Dividing by 2:

$$= \frac{-2 \cos 20^\circ \cos 80^\circ}{2 \times 2} - \frac{\cos 80^\circ}{4}$$

$$\{\because 2 \cos A \cos B = \cos (A + B) + \cos (A - B)\}$$

$$= \frac{-\{\cos(20^\circ + 80^\circ) + \cos(80^\circ - 20^\circ)\}}{4} - \frac{\cos 80^\circ}{4}$$

$$= \frac{-\{\cos(100^\circ) + \cos(60^\circ)\}}{4} - \frac{\cos 80^\circ}{4}$$

$$= \frac{-\left\{\cos(180^\circ - 80^\circ) + \frac{1}{2}\right\}}{4} - \frac{\cos 80^\circ}{4}$$

$$\{\because \cos(180^\circ - A) = -\cos A\}$$

$$= \frac{-\left\{-\cos(80^\circ) + \frac{1}{2}\right\}}{4} - \frac{\cos 80^\circ}{4}$$

$$= \frac{\cos 80^\circ}{4} + \frac{1}{8} - \frac{\cos 80^\circ}{4}$$

$$= \frac{1}{8}$$

= R.H.S

Hence Proved

5 C. Question

Prove that:

$$\sin 20^\circ \sin 40^\circ \sin 80^\circ = \frac{\sqrt{3}}{8}$$

Answer

Take L.H.S

$$\sin 20^\circ \sin 40^\circ \sin 80^\circ$$

Multiplying & Dividing by 2:

$$= \frac{(2 \sin 20^\circ \sin 40^\circ) \sin 80^\circ}{2}$$

$$\{\because 2 \sin A \sin B = \cos (A - B) - \cos (A + B)\}$$

$$= \frac{\{\cos(20^\circ - 40^\circ) - \cos(20^\circ + 40^\circ)\} \sin 80^\circ}{2}$$

$$= \frac{\{\cos(-20^\circ) - \cos(60^\circ)\} \sin 80^\circ}{2}$$

$$\{\because \cos(-A) = \cos A\}$$

$$= \frac{\left(\cos 20^\circ - \frac{1}{2}\right) \sin 80^\circ}{2}$$

$$= \frac{\cos 20^\circ \sin 80^\circ}{2} - \frac{\sin 80^\circ}{4}$$

Multiplying & Dividing by 2:

$$= \frac{2 \cos 20^\circ \sin 80^\circ}{2 \times 2} - \frac{\sin 80^\circ}{4}$$

$$\{\because 2 \cos A \sin B = \sin (A + B) - \sin (A - B)\}$$

$$= \frac{\sin(20^\circ + 80^\circ) - \sin(20^\circ - 80^\circ)}{4} - \frac{\sin 80^\circ}{4}$$

$$= \frac{\sin(100^\circ) - \sin(-60^\circ)}{4} - \frac{\sin 80^\circ}{4}$$

$$\{\because \sin(-A) = -\sin A\}$$

$$= \frac{\sin(180^\circ - 80^\circ) + \sin(60^\circ)}{4} - \frac{\sin 80^\circ}{4}$$

$$\{\because \sin(180^\circ - A) = \sin A\}$$

$$\begin{aligned}
&= \frac{\sin(80^\circ) + \frac{\sqrt{3}}{2} \sin 80^\circ}{4} \\
&= \frac{\sin(80^\circ)}{4} + \frac{\sqrt{3}}{8} \sin 80^\circ \\
&= \frac{\sqrt{3}}{8} \\
&= \text{R.H.S}
\end{aligned}$$

Hence Proved

5 D. Question

Prove that:

$$\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$$

Answer

Take L.H.S

$$\cos 20^\circ \cos 40^\circ \cos 80^\circ$$

Multiplying & Dividing by 2:

$$\begin{aligned}
&= \frac{(2 \cos 40^\circ \cos 20^\circ) \cos 80^\circ}{2} \\
&\{\because 2 \cos A \cos B = \cos (A + B) + \cos (A - B)\} \\
&= \frac{\{\cos (40^\circ + 20^\circ) + \cos (40^\circ - 20^\circ)\} \cos 80^\circ}{2} \\
&= \frac{\{\cos (60^\circ) + \cos (20^\circ)\} \cos 80^\circ}{2} \\
&= \frac{\left\{\frac{1}{2} + \cos (20^\circ)\right\} \cos 80^\circ}{2} \\
&= \frac{\frac{1}{2} \cos 80^\circ + \cos 20^\circ \cos 80^\circ}{2} \\
&= \frac{\cos 20^\circ \cos 80^\circ}{2} + \frac{\cos 80^\circ}{4}
\end{aligned}$$

Multiplying & Dividing by 2:

$$\begin{aligned}
&= \frac{2 \cos 20^\circ \cos 80^\circ}{2 \times 2} + \frac{\cos 80^\circ}{4} \\
&\{\because 2 \cos A \cos B = \cos (A + B) + \cos (A - B)\} \\
&= \frac{\{\cos (20^\circ + 80^\circ) + \cos (80^\circ - 20^\circ)\}}{4} - \frac{\cos 80^\circ}{4} \\
&= \frac{\{\cos (100^\circ) + \cos (60^\circ)\}}{4} + \frac{\cos 80^\circ}{4} \\
&= \frac{\left\{\cos (180^\circ - 80^\circ) + \frac{1}{2}\right\}}{4} + \frac{\cos 80^\circ}{4}
\end{aligned}$$

$\{\because \cos (180^\circ - A) = -\cos A\}$

$$\begin{aligned}
&= \frac{\left\{-\cos (80^\circ) + \frac{1}{2}\right\}}{4} + \frac{\cos 80^\circ}{4} \\
&= -\frac{\cos 80^\circ}{4} + \frac{1}{8} + \frac{\cos 80^\circ}{4} \\
&= \frac{1}{8} \\
&= \text{R.H.S}
\end{aligned}$$

Hence Proved

5 E. Question

Prove that:

$$\tan 20^\circ \tan 40^\circ \tan 60^\circ \tan 80^\circ = 3$$

Answer

Take L.H.S

$$\tan 20^\circ \tan 40^\circ \tan 60^\circ \tan 80^\circ$$

$$= \frac{\sin 20^\circ \sin 40^\circ \sin 80^\circ}{\cos 20^\circ \cos 40^\circ \cos 80^\circ} (\tan 60^\circ)$$

Multiplying & Dividing by 2:

$$= \frac{(2 \sin 20^\circ \sin 40^\circ) \sin 80^\circ}{(2 \cos 20^\circ \cos 40^\circ) \cos 80^\circ} (\sqrt{3})$$

$$\{\because 2 \sin A \sin B = \cos (A - B) - \cos (A + B) \&$$

$$2 \cos A \cos B = \cos (A + B) + \cos (A - B)\}$$

$$= \frac{\sqrt{3}\{\cos (20^\circ - 40^\circ) - \cos (20^\circ + 40^\circ)\} \sin 80^\circ}{\{\cos (40^\circ + 20^\circ) + \cos (40^\circ - 20^\circ)\} \cos 80^\circ}$$

$$= \frac{\sqrt{3}\{\cos (-20^\circ) - \cos (60^\circ)\} \sin 80^\circ}{\{\cos (60^\circ) + \cos (20^\circ)\} \cos 80^\circ}$$

$$\{\because \cos (-A) = \cos A\}$$

$$= \frac{\sqrt{3}\left\{\cos (20^\circ) - \frac{1}{2}\right\} \sin 80^\circ}{\left\{\frac{1}{2} + \cos (20^\circ)\right\} \cos 80^\circ}$$

$$= \frac{\sqrt{3}\left\{\frac{2 \cos 20^\circ \sin 80^\circ - \sin 80^\circ}{2}\right\}}{\left\{\frac{\cos 80^\circ + 2 \cos 20^\circ \cos 80^\circ}{2}\right\}}$$

$$= \frac{\sqrt{3}\{2 \cos 20^\circ \sin 80^\circ - \sin 80^\circ\}}{\{\cos 80^\circ + 2 \cos 20^\circ \cos 80^\circ\}}$$

$$\{\because 2 \cos A \sin B = \sin (A + B) - \sin (A - B) \&$$

$$2 \cos A \cos B = \cos (A + B) + \cos (A - B)\}$$

$$= \frac{\sqrt{3}\{\sin (20^\circ + 80^\circ) - \sin (20^\circ - 80^\circ) - \sin 80^\circ\}}{\{\cos 80^\circ + \cos (20^\circ + 80^\circ) + \cos (80^\circ - 20^\circ)\}}$$

$$= \frac{\sqrt{3}\{\sin (100^\circ) - \sin (-60^\circ) - \sin 80^\circ\}}{\{\cos 80^\circ + \cos (100^\circ) + \cos (60^\circ)\}}$$

$$\{\because \sin (-A) = -\sin A\}$$

$$= \frac{\sqrt{3}\{\sin (100^\circ) + \sin (60^\circ) - \sin 80^\circ\}}{\{\cos 80^\circ + \cos (100^\circ) + \cos (60^\circ)\}}$$

$$= \frac{\sqrt{3}\{\sin (180^\circ - 80^\circ) + \sin (60^\circ) - \sin 80^\circ\}}{\{\cos 80^\circ + \cos (180^\circ - 80^\circ) + \cos (60^\circ)\}}$$

$$\{\because \sin (180^\circ - A) = \sin A \& \cos (180^\circ - A) = -\cos A\}$$

$$= \frac{\sqrt{3}\left\{\sin (80^\circ) + \frac{\sqrt{3}}{2} - \sin 80^\circ\right\}}{\left\{\cos 80^\circ - \cos (80^\circ) + \frac{1}{2}\right\}}$$

$$= \frac{\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}}$$

$$= 3$$

$$= \text{R.H.S}$$

Hence Proved

5 F. Question

Prove that:

$$\tan 20^\circ \tan 30^\circ \tan 40^\circ \tan 80^\circ = 1$$

Answer

Take L.H.S

$$\tan 20^\circ \tan 40^\circ \tan 30^\circ \tan 80^\circ$$

$$= \frac{\sin 20^\circ \sin 40^\circ \sin 80^\circ}{\cos 20^\circ \cos 40^\circ \cos 80^\circ} (\tan 30^\circ)$$

Multiplying & Dividing by 2:

$$= \frac{(2 \sin 20^\circ \sin 40^\circ) \sin 80^\circ}{(2 \cos 20^\circ \cos 40^\circ) \cos 80^\circ} \left(\frac{1}{\sqrt{3}}\right)$$

$$\{\because 2 \sin A \sin B = \cos (A - B) - \cos (A + B) \&$$

$$2 \cos A \cos B = \cos (A + B) + \cos (A - B)\}$$

$$= \frac{\frac{1}{\sqrt{3}}\{\cos (20^{\circ}-40^{\circ})-\cos (20^{\circ}+40^{\circ})\} \sin 80^{\circ}}{\{\cos (40^{\circ}+20^{\circ})+\cos (40^{\circ}-20^{\circ})\} \cos 80^{\circ}}$$

$$= \frac{\frac{1}{\sqrt{3}}\{\cos (-20^{\circ})-\cos (60^{\circ})\} \sin 80^{\circ}}{\{\cos (60^{\circ})+\cos (20^{\circ})\} \cos 80^{\circ}}$$

$$\{\because \cos (-A)=\cos A\}$$

$$= \frac{\frac{1}{\sqrt{3}}\left\{\cos \left(20^{\circ}\right)-\frac{1}{2}\right\} \sin 80^{\circ}}{\left\{\frac{1}{2}+\cos \left(20^{\circ}\right)\right\} \cos 80^{\circ}}$$

$$= \frac{\frac{1}{\sqrt{3}}\left\{\frac{2 \cos 20^{\circ} \sin 80^{\circ}-\sin 80^{\circ}}{2}\right\}}{\left\{\frac{\cos 80^{\circ}+2 \cos 20^{\circ} \cos 80^{\circ}}{2}\right\}}$$

$$= \frac{\frac{1}{\sqrt{3}}\{2 \cos 20^{\circ} \sin 80^{\circ}-\sin 80^{\circ}\}}{\{\cos 80^{\circ}+2 \cos 20^{\circ} \cos 80^{\circ}\}}$$

$$\{\because 2 \cos A \sin B=\sin (A+B)-\sin (A-B) \&$$

$$2 \cos A \cos B=\cos (A+B)+\cos (A-B)\}$$

$$= \frac{\frac{1}{\sqrt{3}}\{\sin (20^{\circ}+80^{\circ})-\sin (20^{\circ}-80^{\circ})-\sin 80^{\circ}\}}{\{\cos 80^{\circ}+\cos (20^{\circ}+80^{\circ})+\cos (80^{\circ}-20^{\circ})\}}$$

$$= \frac{\frac{1}{\sqrt{3}}\{\sin (100^{\circ})-\sin (-60^{\circ})-\sin 80^{\circ}\}}{\{\cos 80^{\circ}+\cos (100^{\circ})+\cos (60^{\circ})\}}$$

$$\{\because \sin (-A)=-\sin A\}$$

$$= \frac{\frac{1}{\sqrt{3}}\{\sin (100^{\circ})+\sin (60^{\circ})-\sin 80^{\circ}\}}{\{\cos 80^{\circ}+\cos (100^{\circ})+\cos (60^{\circ})\}}$$

$$= \frac{\frac{1}{\sqrt{3}}\{\sin (180^{\circ}-80^{\circ})+\sin (60^{\circ})-\sin 80^{\circ}\}}{\{\cos 80^{\circ}+\cos (180^{\circ}-80^{\circ})+\cos (60^{\circ})\}}$$

$$\{\because \sin (180^{\circ}-A)=\sin A \& \cos (180^{\circ}-A)=-\cos A\}$$

$$= \frac{\frac{1}{\sqrt{3}}\left\{\sin (80^{\circ})+\frac{\sqrt{3}}{2}-\sin 80^{\circ}\right\}}{\left\{\cos 80^{\circ}-\cos (80^{\circ})+\frac{1}{2}\right\}}$$

$$= \frac{\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}}$$

$$= 1$$

$$= \text{R.H.S}$$

Hence Proved

5 G. Question

Prove that:

$$\sin 10^{\circ} \sin 50^{\circ} \sin 60^{\circ} \sin 70^{\circ} = \frac{\sqrt{3}}{16}$$

Answer

Take L.H.S

$$\sin 10^{\circ} \sin 50^{\circ} \sin 60^{\circ} \sin 70^{\circ}$$

Multiplying & Dividing by 2:

$$= \frac{\frac{\sqrt{3}}{2}(2 \sin 10^{\circ} \sin 50^{\circ}) \sin 70^{\circ}}{2}$$

$$\{\because 2 \sin A \sin B=\cos (A-B)-\cos (A+B)\}$$

$$= \frac{\sqrt{3}\{\cos (10^{\circ}-50^{\circ})-\cos (10^{\circ}+50^{\circ})\} \sin 70^{\circ}}{2 \times 2}$$

$$= \frac{\sqrt{3}\{\cos(-40^\circ) - \cos(60^\circ)\} \sin 70^\circ}{4}$$

$$\{\because \cos(-A) = \cos A\}$$

$$\frac{\sqrt{3}\left\{\cos(40^\circ) - \frac{1}{2}\right\} \sin 70^\circ}{4}$$

$$= \frac{\sqrt{3} \cos 40^\circ \sin 70^\circ}{4} - \frac{\sqrt{3}}{8} \sin 70^\circ$$

Multiplying & Dividing by 2:

$$= \frac{\sqrt{3}(2 \cos 40^\circ \sin 70^\circ)}{4 \times 2} - \frac{\sqrt{3}}{8} \sin 70^\circ$$

$$\{\because 2 \cos A \sin B = \sin(A + B) - \sin(A - B)\}$$

$$= \frac{\sqrt{3}}{8} \{\sin(40^\circ + 70^\circ) - \sin(40^\circ - 70^\circ)\} - \frac{\sqrt{3}}{8} \sin 70^\circ$$

$$= \frac{\sqrt{3}}{8} \{\sin(110^\circ) - \sin(-30^\circ)\} - \frac{\sqrt{3}}{8} \sin 70^\circ$$

$$\{\because \sin(-A) = -\sin A\}$$

$$= \frac{\sqrt{3}}{8} \{\sin(180^\circ - 70^\circ) + \sin(30^\circ)\} - \frac{\sqrt{3}}{8} \sin 70^\circ$$

$$\{\because \sin(180^\circ - A) = \sin A\}$$

$$= \frac{\sqrt{3}}{8} \left\{ \sin(70^\circ) + \frac{1}{2} \right\} - \frac{\sqrt{3}}{8} \sin 70^\circ$$

$$= \frac{\sqrt{3}}{8} \sin(70^\circ) + \frac{\sqrt{3}}{16} - \frac{\sqrt{3}}{8} \sin 70^\circ$$

$$= \frac{\sqrt{3}}{16}$$

$$= \text{R.H.S}$$

Hence Proved

5 H. Question

Prove that:

$$\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$$

Answer

Take L.H.S

$$\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$$

Multiplying & Dividing by 2:

$$= \frac{\frac{\sqrt{3}}{2} (2 \sin 20^\circ \sin 40^\circ) \sin 80^\circ}{2}$$

$$\{\because 2 \sin A \sin B = \cos(A - B) - \cos(A + B)\}$$

$$= \frac{\sqrt{3}\{\cos(20^\circ - 40^\circ) - \cos(20^\circ + 40^\circ)\} \sin 80^\circ}{2 \times 2}$$

$$= \frac{\sqrt{3}\{\cos(-20^\circ) - \cos(60^\circ)\} \sin 80^\circ}{4}$$

$$\{\because \cos(-A) = \cos A\}$$

$$\frac{\sqrt{3}\left\{\cos(20^\circ) - \frac{1}{2}\right\} \sin 80^\circ}{4}$$

$$= \frac{\sqrt{3} \cos 20^\circ \sin 80^\circ}{4} - \frac{\sqrt{3}}{8} \sin 80^\circ$$

Multiplying & Dividing by 2:

$$= \frac{\sqrt{3}(2 \cos 20^\circ \sin 80^\circ)}{4 \times 2} - \frac{\sqrt{3}}{8} \sin 80^\circ$$

$$\{\because 2 \cos A \sin B = \sin(A + B) - \sin(A - B)\}$$

$$= \frac{\sqrt{3}}{8} \{\sin(20^\circ + 80^\circ) - \sin(20^\circ - 80^\circ)\} - \frac{\sqrt{3}}{8} \sin 80^\circ$$

$$= \frac{\sqrt{3}}{8} \{\sin(100^\circ) - \sin(-60^\circ)\} - \frac{\sqrt{3}}{8} \sin 80^\circ$$

$$\{\because \sin(-A) = -\sin A\}$$

$$= \frac{\sqrt{3}}{8} \{\sin(180^\circ - 80^\circ) + \sin(60^\circ)\} - \frac{\sqrt{3}}{8} \sin 80^\circ$$

$$\{\because \sin(180^\circ - A) = \sin A\}$$

$$= \frac{\sqrt{3}}{8} \left\{ \sin(80^\circ) + \frac{\sqrt{3}}{2} \right\} - \frac{\sqrt{3}}{8} \sin 80^\circ$$

$$= \frac{\sqrt{3}}{8} \sin(80^\circ) + \frac{3}{16} - \frac{\sqrt{3}}{8} \sin 80^\circ$$

$$= \frac{3}{16}$$

$$= \text{R.H.S}$$

Hence Proved

6. Question

Show that

$$\text{i. } \sin A \sin(B - C) + \sin B \sin(C - A) + \sin C \sin(A - B) = 0$$

$$\text{ii. } \sin(B - C) \cos(A - D) + \sin(C - A) \cos(B - D) + \sin(A - B) \cos(C - D) = 0$$

Answer

i. Take L.H.S

$$\sin A \sin(B - C) + \sin B \sin(C - A) + \sin C \sin(A - B)$$

Multiplying & Dividing by 2:

$$= \frac{1}{2} \{2 \sin A \sin(B - C) + 2 \sin B \sin(C - A) + 2 \sin C \sin(A - B)\}$$

$$\{\because 2 \sin A \sin B = \cos(A - B) - \cos(A + B)\}$$

$$= \frac{1}{2} \{\cos(A - B + C) - \cos(A + B - C) + \cos(B - C + A) - \cos(B + C - A) + \cos(C - A + B) - \cos(C + A - B)\}$$

$$= \frac{1}{2} \{\cos(A - B + C) - \cos(A - B + C) - \cos(A + B - C) + \cos(A + B - C) - \cos(B + C - A) + \cos(B + C - A)\}$$

$$= \frac{1}{2} \{0\}$$

$$= 0$$

$$= \text{R.H.S}$$

Hence Proved

$$\text{ii. } \sin(B - C) \cos(A - D) + \sin(C - A) \cos(B - D) + \sin(A - B) \cos(C - D) = 0$$

Answer:

Take L.H.S

$$\sin(B - C) \cos(A - D) + \sin(C - A) \cos(B - D) + \sin(A - B) \cos(C - D)$$

Multiplying & Dividing by 2:

$$= \frac{1}{2} \{2 \sin(B - C) \cos(A - D) + 2 \sin(C - A) \cos(B - D) + 2 \sin(A - B) \cos(C - D)\}$$

$$\{\because 2 \sin A \cos B = \sin(A + B) + \sin(A - B)\}$$

$$= \frac{1}{2} \{\sin(B - C + A - D) + \sin(B - C - A + D) + \sin(C - A + B - D) + \sin(C - A - B + D) + \sin(A - B + C - D) + \sin(A - B - C + D)\}$$

$$= \frac{1}{2} \{\sin(A + B - C - D) + \sin(-(A + B - C - D)) + \sin(-A + B - C + D) + \sin{-}(-A + B - C + D)\}$$

$$= \frac{1}{2} \{\sin(A + B - C - D) - \sin(A + B - C - D) + \sin(-A + B - C + D) - \sin(-A + B - C + D) + \sin(-A + B + C - D) - \sin(-A + B + C - D)\}$$

$$= \frac{1}{2} \{0\}$$

$$= 0$$

$$= \text{R.H.S}$$

Hence Proved

7. Question

Prove that:

$$\tan x \tan\left(\frac{\pi}{3} - x\right) \tan\left(\frac{\pi}{3} + x\right) = \tan 3x$$

Answer

Take L.H.S.

$$\tan x \tan\left(\frac{\pi}{3} - x\right) \tan\left(\frac{\pi}{3} + x\right)$$

$$= \tan x \tan(60^\circ - x) \tan(60^\circ + x)$$

$$= \frac{\sin x \sin(60^\circ - x) \sin(60^\circ + x)}{\cos x \cos(60^\circ - x) \cos(60^\circ + x)}$$

Multiplying & Dividing by 2:

$$= \frac{\sin x \{2 \sin(60^\circ - x) \sin(60^\circ + x)\}}{\cos x \{2 \cos(60^\circ - x) \cos(60^\circ + x)\}}$$

$$\{\because 2 \sin A \sin B = \cos(A - B) - \cos(A + B) \text{ \&}$$

$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)\}$$

$$= \frac{\sin x \{\cos(60^\circ - x - 60^\circ - x) - \cos(60^\circ - x + 60^\circ + x)\}}{\cos x \{\cos(60^\circ - x + 60^\circ + x) + \cos(60^\circ - x - 60^\circ - x)\}}$$

$$= \frac{\sin x \{\cos(-2x) - \cos(120^\circ)\}}{\cos x \{\cos(120^\circ) + \cos(-2x)\}}$$

$$\{\cos(-A) = \cos A\}$$

$$= \frac{\sin x \{\cos(2x) - \cos(180^\circ - 60^\circ)\}}{\cos x \{\cos(180^\circ - 60^\circ) + \cos(2x)\}}$$

$$\{\cos(180^\circ - A) = -\cos A\}$$

$$= \frac{\sin x \{\cos(2x) + \cos(60^\circ)\}}{\cos x \{-\cos(60^\circ) + \cos(2x)\}}$$

$$= \frac{\cos 2x \sin x + \frac{\sin x}{2}}{-\frac{\cos x}{2} + \cos 2x \cos x}$$

Multiplying & Dividing by 2:

$$= \frac{\frac{1}{2} \{2 \cos 2x \sin x\} + \frac{\sin x}{2}}{-\frac{\cos x}{2} + \frac{1}{2} \{\cos 2x \cos x\}}$$

$$\{\because 2 \cos A \sin B = \sin(A + B) - \sin(A - B) \text{ \&}$$

$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)\}$$

$$= \frac{\frac{1}{2} \{\sin(x + 2x) - \sin(2x - x)\} + \frac{\sin x}{2}}{-\frac{\cos x}{2} + \frac{1}{2} \{\cos(x + 2x) + \cos(2x - x)\}}$$

$$= \frac{\frac{1}{2} \{\sin 3x - \sin x\} + \frac{\sin x}{2}}{-\frac{\cos x}{2} + \frac{1}{2} \{\cos 3x + \cos x\}}$$

$$= \frac{\frac{\sin 3x}{2} - \frac{\sin x}{2} + \frac{\sin x}{2}}{-\frac{\cos x}{2} + \frac{\cos 3x}{2} + \frac{\cos x}{2}}$$

$$= \frac{\frac{\sin 3x}{2}}{\frac{\cos 3x}{2}}$$

$$= \frac{\sin 3x}{\cos 3x}$$

$$= \tan 3x$$

= R.H.S.

Hence Proved

8. Question

If $\alpha + \beta = \frac{\pi}{2}$, show that the maximum value of $\cos \alpha \cos \beta = \frac{1}{2}$

Answer

Take L.H.S:

$$\cos \alpha \cos \beta$$

Multiplying & Dividing by 2:

$$= \frac{1}{2} (2 \cos \alpha \cos \beta)$$

$$\{2 \cos A \cos B = \cos (A + B) + \cos (A - B)\}$$

$$= \frac{1}{2} \{ \cos (\alpha + \beta) + \cos (\alpha - \beta) \}$$

$$= \frac{1}{2} \{ \cos (90^\circ) + \cos (\alpha - \beta) \}$$

$$\left\{ \because \alpha + \beta = \frac{\pi}{2} \right\}$$

$$= \frac{1}{2} \{ 0 + \cos (\alpha - \beta) \}$$

$$= \frac{\cos (\alpha - \beta)}{2}$$

We know,

$$-1 \leq \cos \theta \leq 1$$

$$-1 \leq \cos (\alpha - \beta) \leq 1$$

$$-\frac{1}{2} \leq \frac{\cos (\alpha - \beta)}{2} \leq \frac{1}{2}$$

$$\therefore \text{Maximum value of } \frac{\cos (\alpha - \beta)}{2} = \frac{1}{2}$$

Exercise 8.2

1. Question

Express each of the following as the product of sines and cosines:

i. $\sin 12x + \sin 4x$

ii. $\sin 5x - \sin x$

iii. $\cos 12x + \cos 8x$

iv. $\sin 2x + \cos 4x$

Answer

i. *We know*, $\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$

$$\sin 12x + \sin 4x$$

$$= 2 \sin \frac{12x + 4x}{2} \cos \frac{12x - 4x}{2}$$

$$= 2 \sin \frac{16x}{2} \cos \frac{8x}{2}$$

$$= 2 \sin 8x \cos 4x$$

ii. *We know*, $\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$

$$\sin 5x - \sin x$$

$$= 2 \cos \frac{5x + x}{2} \sin \frac{5x - x}{2}$$

$$= 2 \cos \frac{6x}{2} \sin \frac{4x}{2}$$

$$= 2 \cos 3x \sin 2x$$

iii. *We know*, $\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$

$$\cos 12x + \cos 8x$$

$$= 2 \cos \frac{12x + 8x}{2} \cos \frac{12x - 8x}{2}$$

$$= 2 \cos \frac{20x}{2} \cos \frac{4x}{2}$$

$$= 2 \cos 10x \cos 2x$$

$$\text{iv. } \sin 2x + \cos 4x$$

$$= \sin 2x + \sin (90^\circ - 4x) \left\{ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \sin \frac{2x + 90^\circ - 4x}{2} \cos \frac{2x - 90^\circ + 4x}{2}$$

$$= 2 \sin \frac{90^\circ - 2x}{2} \cos \frac{6x - 90^\circ}{2}$$

$$= 2 \sin (45^\circ - x) \cos (3x - 45^\circ)$$

$$= 2 \sin (45^\circ - x) \cos \{-(45^\circ - 3x)\}$$

$$\{\cos (-x) = \cos x\}$$

$$= 2 \sin (45^\circ - x) \cos (45^\circ - 3x)$$

2 A. Question

Prove that :

$$\sin 38^\circ + \sin 22^\circ = \sin 82^\circ$$

Answer

Take L.H.S:

$$\sin 38^\circ + \sin 22^\circ$$

$$\left\{ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \sin \frac{38^\circ + 22^\circ}{2} \cos \frac{38^\circ - 22^\circ}{2}$$

$$= 2 \sin \frac{60^\circ}{2} \cos \frac{16^\circ}{2}$$

$$= 2 \sin 30^\circ \cos 8^\circ$$

$$= 2 \times \frac{1}{2} \times \cos 8^\circ$$

$$= \cos 8^\circ$$

$$= \cos (90^\circ - 82^\circ)$$

$$\{\cos (90^\circ - A) = \sin A\}$$

$$= \sin 82^\circ$$

$$= \text{R.H.S}$$

Hence Proved

2 B. Question

Prove that :

$$\cos 100^\circ + \cos 20^\circ = \cos 40^\circ$$

Answer

Take L.H.S:

$$\cos 100^\circ + \cos 20^\circ$$

$$\left\{ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \cos \frac{100^\circ + 20^\circ}{2} \cos \frac{100^\circ - 20^\circ}{2}$$

$$= 2 \cos \frac{120^\circ}{2} \cos \frac{80^\circ}{2}$$

$$= 2 \cos 60^\circ \cos 40^\circ$$

$$= 2 \times \frac{1}{2} \times \cos 40^\circ$$

$$= \cos 40^\circ$$

$$= \text{R.H.S}$$

Hence Proved

2 C. Question

Prove that :

$$\sin 50^\circ + \sin 10^\circ = \cos 20^\circ$$

Answer

Take L.H.S:

$$\sin 50^\circ + \sin 10^\circ$$

$$\left\{ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \sin \frac{50^\circ + 10^\circ}{2} \cos \frac{50^\circ - 10^\circ}{2}$$

$$= 2 \sin \frac{60^\circ}{2} \cos \frac{40^\circ}{2}$$

$$= 2 \sin 30^\circ \cos 20^\circ$$

$$= 2 \times \frac{1}{2} \times \cos 20^\circ$$

$$= \cos 20^\circ$$

$$= \text{R.H.S}$$

Hence Proved

2 D. Question

Prove that :

$$\sin 23^\circ + \sin 37^\circ = \cos 7^\circ$$

Answer

Take L.H.S:

$$\sin 23^\circ + \sin 37^\circ$$

$$\left\{ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \sin \frac{23^\circ + 37^\circ}{2} \cos \frac{23^\circ - 37^\circ}{2}$$

$$= 2 \sin \frac{60^\circ}{2} \cos \frac{-14^\circ}{2}$$

$$= 2 \sin 30^\circ \cos -7^\circ$$

$$\{\cos (-A) = \cos A\}$$

$$= 2 \times \frac{1}{2} \times \cos 7^\circ$$

$$= \cos 7^\circ$$

$$= \text{R.H.S}$$

Hence Proved

2 E. Question

Prove that :

$$\sin 105^\circ + \cos 105^\circ = \cos 45^\circ$$

Answer

Take L.H.S:

$$\sin 105^\circ + \cos 105^\circ$$

$$= \sin 105^\circ + \sin (90^\circ - 105^\circ)$$

$$\{\sin (90^\circ - A) = \cos A\}$$

$$= \sin 105^\circ + \sin (-15^\circ)$$

$$\{\sin(-A) = -\sin A\}$$

$$= \sin 105^\circ - \sin (15^\circ)$$

$$\left\{ \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \right\}$$

$$= 2 \cos \frac{105^\circ + 15^\circ}{2} \sin \frac{105^\circ - 15^\circ}{2}$$

$$= 2 \cos \frac{120^\circ}{2} \sin \frac{90^\circ}{2}$$

$$= 2 \cos 60^\circ \sin 45^\circ$$

$$= 2 \times \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}$$

$$= \cos 45^\circ$$

$$= \text{R.H.S}$$

Hence Proved

2 F. Question

Prove that :

$$\sin 40^\circ + \sin 20^\circ = \cos 10^\circ$$

Answer

Take L.H.S:

$$\sin 40^\circ + \sin 20^\circ$$

$$\left\{ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \sin \frac{40^\circ + 20^\circ}{2} \cos \frac{40^\circ - 20^\circ}{2}$$

$$= 2 \sin \frac{60^\circ}{2} \cos \frac{20^\circ}{2}$$

$$= 2 \sin 30^\circ \cos 10^\circ$$

$$= 2 \times \frac{1}{2} \times \cos 10^\circ$$

$$= \cos 10^\circ$$

$$= \text{R.H.S}$$

Hence Proved

3 A. Question

Prove that :

$$\cos 55^\circ + \cos 65^\circ + \cos 175^\circ = 0$$

Answer

Take L.H.S:

$$\cos 55^\circ + \cos 65^\circ + \cos 175^\circ$$

$$\left\{ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \cos \frac{55^\circ + 65^\circ}{2} \cos \frac{55^\circ - 65^\circ}{2} + \cos(180^\circ - 5^\circ)$$

$$= 2 \cos \frac{120^\circ}{2} \cos \frac{-10^\circ}{2} - \cos 5^\circ$$

$$\{\cos(180^\circ - A) = -\cos A\}$$

$$= 2 \cos 60^\circ \cos (-5^\circ) - \cos 5^\circ$$

$$\{\cos(-A) = \cos A\}$$

$$= 2 \times \frac{1}{2} \times \cos 5^\circ - \cos 5^\circ$$

$$= \cos 5^\circ - \cos 5^\circ$$

$$= 0$$

$$= \text{R.H.S}$$

Hence Proved

3 B. Question

Prove that :

$$\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = 0$$

Answer

Take LHS:

$$\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$$

$$\begin{aligned}
& \left\{ \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \right\} \\
& = 2 \cos \frac{50^\circ + 70^\circ}{2} \sin \frac{50^\circ - 70^\circ}{2} + \sin 10^\circ \\
& = 2 \cos \frac{120^\circ}{2} \sin \frac{-20^\circ}{2} + \sin 10^\circ \\
& = 2 \cos 60^\circ \sin (-10^\circ) + \sin 10^\circ \\
& \{ \sin (-A) = -\sin (A) \} \\
& = 2 \times \frac{1}{2} \times -\sin 10^\circ + \sin 10^\circ \\
& = -\sin 10^\circ + \sin 10^\circ \\
& = 0 \\
& = \text{R.H.S}
\end{aligned}$$

Hence Proved

3 C. Question

Prove that :

$$\cos 80^\circ + \cos 40^\circ - \cos 20^\circ = 0$$

Answer

Take L.H.S:

$$\begin{aligned}
& \cos 80^\circ + \cos 40^\circ - \cos 20^\circ \\
& \left\{ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\} \\
& = 2 \cos \frac{80^\circ + 40^\circ}{2} \cos \frac{80^\circ - 40^\circ}{2} - \cos 20^\circ \\
& = 2 \cos \frac{120^\circ}{2} \cos \frac{40^\circ}{2} - \cos 20^\circ \\
& = 2 \cos 60^\circ \cos 20^\circ - \cos 20^\circ \\
& = 2 \times \frac{1}{2} \times \cos 20^\circ - \cos 20^\circ \\
& = \cos 20^\circ - \cos 20^\circ \\
& = 0 \\
& = \text{R.H.S}
\end{aligned}$$

Hence Proved

3 D. Question

Prove that :

$$\cos 20^\circ + \cos 100^\circ + \cos 140^\circ = 0$$

Answer

Take L.H.S:

$$\begin{aligned}
& \cos 20^\circ + \cos 100^\circ + \cos 140^\circ \\
& \left\{ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\} \\
& = 2 \cos \frac{20^\circ + 100^\circ}{2} \cos \frac{20^\circ - 100^\circ}{2} + \cos(180^\circ - 40^\circ) \\
& = 2 \cos \frac{120^\circ}{2} \cos \frac{-80^\circ}{2} - \cos 40^\circ \\
& \{ \cos (180^\circ - A) = -\cos A \} \\
& = 2 \cos 60^\circ \cos (-40^\circ) - \cos 40^\circ \\
& \{ \cos (-A) = \cos A \} \\
& = 2 \times \frac{1}{2} \times \cos 40^\circ - \cos 40^\circ \\
& = \cos 40^\circ - \cos 40^\circ \\
& = 0 \\
& = \text{R.H.S}
\end{aligned}$$

Hence Proved

3 E. Question

$$\sin \frac{5\pi}{18} - \cos \frac{4\pi}{9} = \sqrt{3} \sin \frac{\pi}{9}$$

Answer

Take L.H.S:

$$\sin \frac{5\pi}{18} - \cos \frac{4\pi}{9}$$

$$\{\cos A = \sin (90^\circ - A)\}$$

$$= \sin \frac{5\pi}{18} - \sin \left(\frac{\pi}{2} - \frac{4\pi}{9} \right)$$

$$= \sin \frac{5\pi}{18} - \sin \left(\frac{9\pi - 8\pi}{18} \right)$$

$$= \sin \frac{5\pi}{18} - \sin \frac{\pi}{18}$$

$$\left\{ \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \right\}$$

$$= 2 \cos \left(\frac{\frac{5\pi}{18} + \frac{\pi}{18}}{2} \right) \sin \left(\frac{\frac{5\pi}{18} - \frac{\pi}{18}}{2} \right)$$

$$= 2 \cos \left(\frac{6\pi}{36} \right) \sin \left(\frac{4\pi}{36} \right)$$

$$= 2 \cos \frac{\pi}{6} \sin \frac{\pi}{9}$$

$$= 2 \cos 30^\circ \sin \frac{\pi}{9}$$

$$= 2 \times \frac{\sqrt{3}}{2} \times \sin \frac{\pi}{9}$$

$$= \sqrt{3} \sin \frac{\pi}{9}$$

= R.H.S.

Hence Proved

3 F. Question

Prove that :

$$\cos \frac{\pi}{12} - \sin \frac{\pi}{12} = \frac{1}{\sqrt{2}}$$

Answer

Take L.H.S:

$$\cos \frac{\pi}{12} - \sin \frac{\pi}{12}$$

$$\{\cos A = \sin (90^\circ - A)\}$$

$$= \sin \left(\frac{\pi}{2} - \frac{\pi}{12} \right) - \sin \frac{\pi}{12}$$

$$= \sin \left(\frac{6\pi - 5\pi}{12} \right) - \sin \frac{\pi}{12}$$

$$= \sin \frac{5\pi}{12} - \sin \frac{\pi}{12}$$

$$\left\{ \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \right\}$$

$$= 2 \cos \left(\frac{\frac{5\pi}{12} + \frac{\pi}{12}}{2} \right) \sin \left(\frac{\frac{5\pi}{12} - \frac{\pi}{12}}{2} \right)$$

$$= 2 \cos \left(\frac{6\pi}{24} \right) \sin \left(\frac{4\pi}{24} \right)$$

$$= 2 \cos \frac{\pi}{4} \sin \frac{\pi}{6}$$

$$= 2 \cos 45^\circ \sin 30^\circ$$

$$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{1}{\sqrt{2}}$$

= R.H.S.

Hence Proved

3 G. Question

Prove that :

$$\sin 80^\circ - \cos 70^\circ = \cos 50^\circ$$

Answer

$$\sin 80^\circ - \cos 70^\circ = \cos 50^\circ$$

$$\Rightarrow \sin 80^\circ = \cos 50^\circ + \cos 70^\circ$$

Take RHS:

$$\cos 50^\circ + \cos 70^\circ$$

$$\left\{ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \cos \frac{50^\circ + 70^\circ}{2} \cos \frac{50^\circ - 70^\circ}{2}$$

$$= 2 \cos \frac{120^\circ}{2} \cos \frac{-20^\circ}{2}$$

$$= 2 \cos 60^\circ \cos (-10^\circ)$$

$$\{\cos (-A) = \cos A\}$$

$$= 2 \times \frac{1}{2} \times \cos 10^\circ$$

$$= \cos 10^\circ$$

$$= \cos (90^\circ - 80^\circ)$$

$$\{\cos (90^\circ - A) = \sin A\}$$

$$= \sin 80^\circ$$

$$= \text{L.H.S}$$

Hence Proved

3 H. Question

Prove that :

$$\sin 51^\circ + \cos 81^\circ = \cos 21^\circ$$

Answer

Take L.H.S:

$$\sin 51^\circ + \cos 81^\circ$$

$$= \sin 51^\circ + \sin (90^\circ - 81^\circ)$$

$$\{\sin (90^\circ - A) = \cos A\}$$

$$= \sin 51^\circ + \sin 9^\circ$$

$$\left\{ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \sin \frac{51^\circ + 9^\circ}{2} \cos \frac{51^\circ - 9^\circ}{2}$$

$$= 2 \sin \frac{60^\circ}{2} \cos \frac{42^\circ}{2}$$

$$= 2 \sin 30^\circ \cos 21^\circ$$

$$= 2 \times \frac{1}{2} \times \cos 21^\circ$$

$$= \cos 21^\circ$$

$$= \text{R.H.S}$$

Hence Proved

4. Question

Prove that:

$$\text{i. } \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$$

$$\text{ii. } \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) = \sqrt{2} \cos x$$

Answer

Take L.H.S:

$$\begin{aligned} & \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) \\ & \left\{ \cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \right\} \\ & = -2 \sin \frac{\left(\frac{3\pi}{4} + x\right) + \left(\frac{3\pi}{4} - x\right)}{2} \sin \frac{\left(\frac{3\pi}{4} + x\right) - \left(\frac{3\pi}{4} - x\right)}{2} \\ & = -2 \sin \frac{\left(\frac{6\pi}{4}\right)}{2} \sin \frac{(2x)}{2} \\ & = -2 \sin \frac{6\pi}{8} \sin x \\ & = -2 \sin \frac{3\pi}{4} \sin x \\ & = -2 \sin \left(\pi - \frac{\pi}{4}\right) \sin x \\ & \{\sin(\pi - A) = \sin A\} \\ & = -2 \sin \frac{\pi}{4} \sin x \\ & = -2 \times \frac{1}{\sqrt{2}} \times \sin x \\ & = -\sqrt{2} \sin x \\ & = \text{R.H.S.} \end{aligned}$$

Hence Proved

ii. Take L.H.S:

$$\begin{aligned} & \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) \\ & \left\{ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\} \\ & = 2 \cos \frac{\left(\frac{\pi}{4} + x\right) + \left(\frac{\pi}{4} - x\right)}{2} \cos \frac{\left(\frac{\pi}{4} + x\right) - \left(\frac{\pi}{4} - x\right)}{2} \\ & = 2 \cos \frac{\left(\frac{2\pi}{4}\right)}{2} \cos \frac{(2x)}{2} \\ & = 2 \cos \frac{2\pi}{8} \cos x \\ & = 2 \sin \frac{\pi}{4} \cos x \\ & = 2 \times \frac{1}{\sqrt{2}} \times \cos x \\ & = \sqrt{2} \cos x \\ & = \text{R.H.S.} \end{aligned}$$

Hence Proved

5. Question

Prove that:

$$\text{i. } \sin 65^\circ + \cos 65^\circ = \sqrt{2} \cos 20^\circ$$

$$\text{ii. } \sin 47^\circ + \cos 77^\circ = \cos 17^\circ$$

Answer

Take L.H.S:

$$\begin{aligned} & \sin 65^\circ + \cos 65^\circ \\ & = \sin 65^\circ + \sin (90^\circ - 65^\circ) \\ & \{\sin (90^\circ - A) = \cos A\} \\ & = \sin 65^\circ + \sin 25^\circ \end{aligned}$$

$$\left\{ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \sin \frac{65^\circ + 25^\circ}{2} \cos \frac{65^\circ - 25^\circ}{2}$$

$$= 2 \sin \frac{90^\circ}{2} \cos \frac{40^\circ}{2}$$

$$= 2 \sin 45^\circ \cos 20^\circ$$

$$= 2 \times \frac{1}{\sqrt{2}} \times \cos 20^\circ$$

$$= \sqrt{2} \cos 20^\circ$$

$$= \text{R.H.S}$$

Hence Proved

ii. Take L.H.S:

$$\sin 47^\circ + \cos 77^\circ$$

$$= \sin 47^\circ + \sin (90^\circ - 77^\circ)$$

$$\{ \sin (90^\circ - A) = \cos A \}$$

$$= \sin 47^\circ + \sin 13^\circ$$

$$\left\{ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \sin \frac{47^\circ + 13^\circ}{2} \cos \frac{47^\circ - 13^\circ}{2}$$

$$= 2 \sin \frac{60^\circ}{2} \cos \frac{34^\circ}{2}$$

$$= 2 \sin 30^\circ \cos 17^\circ$$

$$= 2 \times \frac{1}{2} \times \cos 20^\circ$$

$$= \cos 17^\circ$$

$$= \text{R.H.S}$$

Hence Proved

6 A. Question

Prove that :

$$\cos 3A + \cos 5A + \cos 7A + \cos 15A = 4 \cos 4A \cos 5A \cos 6A$$

Answer

Take L.H.S.

$$\cos 3A + \cos 5A + \cos 7A + \cos 15A$$

$$= (\cos 5A + \cos 3A) + (\cos 15A + \cos 7A)$$

$$\left\{ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \cos \frac{(5A+3A)}{2} \cos \frac{(5A-3A)}{2} + 2 \cos \frac{(15A+7A)}{2} \cos \frac{(15A-7A)}{2}$$

$$= 2 \cos \frac{(8A)}{2} \cos \frac{(2A)}{2} + 2 \cos \frac{(22A)}{2} \cos \frac{(8A)}{2}$$

$$= 2 \cos 4A \cos A + 2 \cos 11A \cos 4A$$

$$= 2 \cos 4A (\cos 11A + \cos A)$$

$$\left\{ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \cos 4A \left\{ 2 \cos \frac{(11A+A)}{2} \cos \frac{(11A-A)}{2} \right\}$$

$$= 2 \cos 4A \left(2 \cos \frac{12A}{2} \cos \frac{10A}{2} \right)$$

$$= 4 \cos 4A \cos 5A \cos 6A$$

$$= \text{R.H.S.}$$

Hence Proved

6 B. Question

Prove that :

$$\cos A + \cos 3A + \cos 5A + \cos 7A = 4 \cos A \cos 2A \cos 4A$$

Answer

Take L.H.S.

$$\cos A + \cos 3A + \cos 5A + \cos 7A$$

$$= (\cos 3A + \cos A) + (\cos 7A + \cos 5A)$$

$$\left\{ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \cos \frac{(3A+A)}{2} \cos \frac{(3A-A)}{2} + 2 \cos \frac{(7A+5A)}{2} \cos \frac{(7A-5A)}{2}$$

$$= 2 \cos \frac{(4A)}{2} \cos \frac{(2A)}{2} + 2 \cos \frac{(12A)}{2} \cos \frac{(2A)}{2}$$

$$= 2 \cos 2A \cos A + 2 \cos 6A \cos A$$

$$= 2 \cos A (\cos 6A + \cos 2A)$$

$$\left\{ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \cos A \left\{ 2 \cos \frac{(6A+2A)}{2} \cos \frac{(6A-2A)}{2} \right\}$$

$$= 2 \cos A \left(2 \cos \frac{8A}{2} \cos \frac{4A}{2} \right)$$

$$= 4 \cos A \cos 2A \cos 4A$$

= R.H.S.

Hence Proved

6 C. Question

Prove that :

$$\sin A + \sin 2A + \sin 4A + \sin 5A = 4 \sin 3A \cos \frac{A}{2} \cos \frac{3A}{2}$$

Answer

Take L.H.S.

$$\sin A + \sin 2A + \sin 4A + \sin 5A$$

$$= (\sin 2A + \sin A) + (\sin 5A + \sin 4A)$$

$$\left\{ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \sin \frac{(2A+A)}{2} \cos \frac{(2A-A)}{2} + 2 \sin \frac{(5A+4A)}{2} \cos \frac{(5A-4A)}{2}$$

$$= 2 \sin \frac{(3A)}{2} \cos \frac{(A)}{2} + 2 \sin \frac{(9A)}{2} \cos \frac{(A)}{2}$$

$$\left\{ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \cos \frac{A}{2} \left\{ \sin \frac{(9A)}{2} + \sin \frac{(3A)}{2} \right\}$$

$$= 2 \cos \frac{A}{2} \left\{ 2 \sin \frac{\frac{9A}{2} + \frac{3A}{2}}{2} \cos \frac{\frac{9A}{2} - \frac{3A}{2}}{2} \right\}$$

$$= 2 \cos \frac{A}{2} \left\{ 2 \sin \frac{(9A+3A)}{2} \cos \frac{(9A-3A)}{2} \right\}$$

$$= 2 \cos \frac{A}{2} \left(2 \sin \frac{12A}{4} \cos \frac{6A}{4} \right)$$

$$= 2 \cos \frac{A}{2} \left(2 \sin 3A \cos \frac{3A}{2} \right)$$

$$= 4 \sin 3A \cos \frac{A}{2} \cos \frac{3A}{2}$$

= R.H.S.

Hence Proved

6 D. Question

Prove that :

$$\sin 3A + \sin 2A - \sin A = 4 \sin A \cos \frac{A}{2} \cos \frac{3A}{2}$$

Answer

Take L.H.S.

$$\sin 3A + \sin 2A - \sin A$$

$$= (\sin 3A - \sin A) + \sin 2A$$

$$\left\{ \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \right\}$$

$$= 2 \cos \frac{(3A+A)}{2} \sin \frac{(3A-A)}{2} + \sin 2A$$

$$= 2 \cos \frac{(4A)}{2} \sin \frac{(2A)}{2} + \sin 2A$$

$$\{\sin 2A = 2 \sin A \cos A\}$$

$$= 2 \cos 2A \sin A + 2 \sin A \cos A$$

$$= 2 \sin A (\cos 2A + \cos A)$$

$$\left\{ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= 2 \sin A \left\{ 2 \cos \frac{2A+A}{2} \cos \frac{2A-A}{2} \right\}$$

$$= 2 \sin A \left(2 \cos \frac{3A}{2} \cos \frac{A}{2} \right)$$

$$= 4 \sin A \cos \frac{A}{2} \cos \frac{3A}{2}$$

$$= \text{R.H.S.}$$

Hence Proved

6 E. Question

Prove that :

$$\cos 20^\circ \cos 100^\circ + \cos 100^\circ \cos 140^\circ - \cos 140^\circ \cos 200^\circ = -\frac{3}{4}$$

Answer

Take L.H.S:

$$\cos 20^\circ \cos 100^\circ + \cos 100^\circ \cos 140^\circ - \cos 140^\circ \cos 200^\circ$$

Multiplying & Dividing by 2:

$$= \frac{1}{2} \{ 2 \cos 100^\circ \cos 20^\circ + 2 \cos 140^\circ \cos 100^\circ - 2 \cos 200^\circ \cos 140^\circ \}$$

$$\{\because 2 \cos A \cos B = \cos (A+B) + \cos (A-B)\}$$

$$= \frac{1}{2} \{ \cos (100^\circ + 20^\circ) + \cos (100^\circ - 20^\circ) + \cos (140^\circ + 100^\circ) \\ + \cos (140^\circ - 100^\circ) - \cos (200^\circ + 140^\circ) - \cos (200^\circ - 140^\circ) \}$$

$$= \frac{1}{2} \{ \cos (120^\circ) + \cos 80^\circ + \cos (240^\circ) + \cos (40^\circ) - \cos (340^\circ) - \cos 60^\circ \}$$

$$= \frac{1}{2} \{ \cos (90^\circ + 30^\circ) + \cos 80^\circ + \cos (180^\circ + 60^\circ) \\ + \cos (40^\circ) - \cos (360^\circ - 20^\circ) - \cos 60^\circ \}$$

$$\{ \cos (180^\circ + A) = -\cos A ;$$

$$\cos (90^\circ + A) = -\sin A \&$$

$$\cos (360^\circ - A) = \cos A \}$$

$$= \frac{1}{2} \{ -\sin (30^\circ) + \cos 80^\circ - \cos (60^\circ) + \cos (40^\circ) - \cos (20^\circ) - \cos 60^\circ \}$$

$$= \frac{1}{2} \{ -\sin (30^\circ) + \cos 80^\circ + \cos 40^\circ - \cos (20^\circ) - 2 \cos 60^\circ \}$$

$$\left\{ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= \frac{1}{2} \left\{ -\sin (30^\circ) + 2 \cos \frac{(80^\circ + 40^\circ)}{2} \cos \frac{(80^\circ - 40^\circ)}{2} - \cos (20^\circ) - 2 \times \frac{1}{2} \right\}$$

$$\begin{aligned}
&= \frac{1}{2} \left\{ -\sin(30^\circ) + 2 \cos \frac{(120^\circ)}{2} \cos \frac{(40^\circ)}{2} - \cos(20^\circ) - 1 \right\} \\
&= \frac{1}{2} \{ -\sin(30^\circ) + 2 \cos 60^\circ \cos 20^\circ - \cos(20^\circ) - 1 \} \\
&= \frac{1}{2} \left\{ -\frac{1}{2} + 2 \times \frac{1}{2} \times \cos 20^\circ - \cos(20^\circ) - 1 \right\} \\
&= \frac{1}{2} \left\{ -\frac{1}{2} + \cos 20^\circ - \cos 20^\circ - 1 \right\} \\
&= \frac{1}{2} \left\{ -\frac{3}{2} \right\} \\
&= -\frac{3}{4} \\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved

6 F. Question

Prove that :

$$\sin \frac{x}{2} \sin \frac{7x}{2} + \sin \frac{3x}{2} \sin \frac{11x}{2} = \sin 2x \sin 5x$$

Answer

Take L.H.S.:

$$\sin \frac{x}{2} \sin \frac{7x}{2} + \sin \frac{3x}{2} \sin \frac{11x}{2}$$

Multiplying & Dividing by 2:

$$= \frac{1}{2} \left\{ 2 \sin \frac{7x}{2} \sin \frac{x}{2} + 2 \sin \frac{11x}{2} \sin \frac{3x}{2} \right\}$$

$$\{ \because 2 \sin A \sin B = \cos(A - B) - \cos(A + B) \}$$

$$= \frac{1}{2} \left\{ \cos \left(\frac{7x}{2} - \frac{x}{2} \right) - \cos \left(\frac{7x}{2} + \frac{x}{2} \right) + \cos \left(\frac{11x}{2} - \frac{3x}{2} \right) - \cos \left(\frac{11x}{2} + \frac{3x}{2} \right) \right\}$$

$$= \frac{1}{2} \left\{ \cos \left(\frac{7x - x}{2} \right) - \cos \left(\frac{7x + x}{2} \right) + \cos \left(\frac{11x - 3x}{2} \right) - \cos \left(\frac{11x + 3x}{2} \right) \right\}$$

$$= \frac{1}{2} \left\{ \cos \frac{(6x)}{2} - \cos \frac{(8x)}{2} + \cos \frac{(8x)}{2} - \cos \frac{(14x)}{2} \right\}$$

$$= \frac{1}{2} \{ \cos 3x - \cos 7x \}$$

$$= -\frac{1}{2} \{ -(\cos 3x - \cos 7x) \}$$

$$= -\frac{1}{2} \{ \cos 7x - \cos 3x \}$$

$$\left\{ \cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \right\}$$

$$= -\frac{1}{2} \left\{ -2 \sin \frac{3x + 7x}{2} \sin \frac{7x - 3x}{2} \right\}$$

$$= -\frac{1}{2} \left\{ -2 \sin \frac{10x}{2} \sin \frac{4x}{2} \right\}$$

$$= -\frac{1}{2} \{ -2 \sin 5x \sin 2x \}$$

$$= \sin 5x \sin 2x$$

$$= \text{R.H.S.}$$

Hence Proved

6 G. Question

Prove that :

$$\cos x \cos \frac{x}{2} - \cos 3x \cos \frac{9x}{2} = \sin 4x \sin \frac{7x}{2}$$

Answer

Take L.H.S.:

$$\cos x \cos \frac{x}{2} - \cos 3x \cos \frac{9x}{2}$$

Multiplying & Dividing by 2:

$$= \frac{1}{2} \left\{ 2 \cos x \cos \frac{x}{2} - 2 \cos \frac{9x}{2} \cos 3x \right\}$$

$$\{\because 2 \cos A \cos B = \cos (A + B) + \cos (A - B)\}$$

$$= \frac{1}{2} \left\{ \cos \left(x + \frac{x}{2} \right) + \cos \left(x - \frac{x}{2} \right) - \cos \left(\frac{9x}{2} + 3x \right) - \cos \left(\frac{9x}{2} - 3x \right) \right\}$$

$$= \frac{1}{2} \left\{ \cos \left(\frac{2x + x}{2} \right) + \cos \left(\frac{2x - x}{2} \right) - \cos \left(\frac{9x + 6x}{2} \right) - \cos \left(\frac{9x - 6x}{2} \right) \right\}$$

$$= \frac{1}{2} \left\{ \cos \frac{(3x)}{2} + \cos \frac{(x)}{2} - \cos \frac{(15x)}{2} - \cos \frac{(3x)}{2} \right\}$$

$$= \frac{1}{2} \left\{ \cos \frac{(x)}{2} - \cos \frac{(15x)}{2} \right\}$$

$$= -\frac{1}{2} \left\{ - \left(\cos \frac{(x)}{2} - \cos \frac{(15x)}{2} \right) \right\}$$

$$= -\frac{1}{2} \left\{ \cos \frac{15x}{2} - \cos \frac{x}{2} \right\}$$

$$\left\{ \cos A - \cos B = -2 \sin \frac{A + B}{2} \sin \frac{A - B}{2} \right\}$$

$$= -\frac{1}{2} \left\{ -2 \sin \frac{\frac{15x}{2} + \frac{x}{2}}{2} \sin \frac{\frac{15x}{2} - \frac{x}{2}}{2} \right\}$$

$$= -\frac{1}{2} \left\{ -2 \sin \frac{\frac{16x}{2}}{2} \sin \frac{\frac{14x}{2}}{2} \right\}$$

$$= -\frac{1}{2} \left\{ -2 \sin \frac{16x}{4} \sin \frac{14x}{4} \right\}$$

$$= -\frac{1}{2} \left\{ -2 \sin 4x \sin \frac{7x}{2} \right\}$$

$$= \sin 4x \sin \frac{7x}{2}$$

= R.H.S.

Hence Proved

7 A. Question

Prove that:

$$\frac{\sin A + \sin 3A}{\cos A - \cos 3A} = \cot A$$

Answer

Take L.H.S.:

$$\frac{\sin 3A + \sin A}{\cos A - \cos 3A}$$

$$\left\{ \begin{array}{l} \sin A + \sin B = 2 \sin \frac{A + B}{2} \cos \frac{A - B}{2} \\ \cos A - \cos B = -2 \sin \frac{A + B}{2} \sin \frac{A - B}{2} \end{array} \right\}$$

$$= \frac{2 \left(\sin \frac{A + 3A}{2} \cos \frac{3A - A}{2} \right)}{-2 \left(\sin \frac{A + 3A}{2} \sin \frac{A - 3A}{2} \right)}$$

$$= - \frac{\left(\sin \frac{4A}{2} \cos \frac{2A}{2} \right)}{\left(\sin \frac{4A}{2} \sin \frac{(-2A)}{2} \right)}$$

$$\{\sin (-A) = -\sin A\}$$

$$= - \frac{(\cos A)}{(-\sin A)}$$

$$= \frac{\cos A}{\sin A}$$

= cot A

= R.H.S.

Hence Proved

7 B. Question

Prove that:

$$\frac{\sin 9A - \sin 7A}{\cos 7A - \cos 9A} = \cot 8A$$

Answer

Take L.H.S.:

$$\frac{\sin 9A - \sin 7A}{\cos 7A - \cos 9A}$$

$$\left\{ \begin{array}{l} \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \\ \cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \end{array} \right\}$$

$$= \frac{2 \left(\cos \frac{9A+7A}{2} \sin \frac{9A-7A}{2} \right)}{-2 \left(\sin \frac{7A+9A}{2} \sin \frac{7A-9A}{2} \right)}$$

$$= - \frac{\left(\cos \frac{16A}{2} \sin \frac{2A}{2} \right)}{\left(\sin \frac{16A}{2} \sin \frac{(-2A)}{2} \right)}$$

$$\{ \sin (-A) = - \sin A \}$$

$$= - \frac{(\cos 8A \sin A)}{(-\sin 8A \sin A)}$$

$$= \frac{\cos 8A}{\sin 8A}$$

$$= \cot 8A$$

$$= \text{R.H.S.}$$

Hence Proved

7 C. Question

Prove that:

$$\frac{\sin A - \sin B}{\cos A + \cos B} = \tan \frac{A-B}{2}$$

Answer

Take L.H.S.:

$$\frac{\sin A - \sin B}{\cos A + \cos B}$$

$$\left\{ \begin{array}{l} \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \\ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \end{array} \right\}$$

$$= \frac{2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}}{2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}}$$

$$= \frac{\sin \frac{A-B}{2}}{\cos \frac{A-B}{2}}$$

$$= \tan \left(\frac{A-B}{2} \right)$$

$$= \text{R.H.S.}$$

Hence Proved

7 D. Question

Prove that:

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \tan \left(\frac{A+B}{2} \right) \cot \left(\frac{A-B}{2} \right)$$

Answer

Take L.H.S.:

$$\frac{\sin A + \sin B}{\sin A - \sin B}$$

$$\left\{ \begin{array}{l} \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \\ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \end{array} \right\}$$

$$= \frac{2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}}{2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}}$$

$$= \frac{\sin \frac{A+B}{2} \cos \frac{A-B}{2}}{\cos \frac{A+B}{2} \sin \frac{A-B}{2}}$$

$$= \tan \left(\frac{A+B}{2} \right) \cot \left(\frac{A-B}{2} \right)$$

= R.H.S.

Hence Proved

7 E. Question

Prove that:

$$\frac{\cos A + \cos B}{\cos B - \cos A} = \cot \left(\frac{A+B}{2} \right) \cot \left(\frac{A-B}{2} \right)$$

Answer

Take L.H.S.:

$$\frac{\cos A + \cos B}{\cos A - \cos B}$$

$$\left\{ \begin{array}{l} \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \\ \cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \end{array} \right\}$$

$$= \frac{2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}}{-2 \sin \frac{A+B}{2} \sin \frac{B-A}{2}}$$

$$= - \frac{\cos \frac{A+B}{2} \cos \frac{A-B}{2}}{\sin \frac{A+B}{2} \sin \left(-\frac{A-B}{2} \right)}$$

{sin (-x) = - sin x}

$$= - \frac{\cos \frac{A+B}{2} \cos \frac{A-B}{2}}{-\sin \frac{A+B}{2} \sin \left(\frac{A-B}{2} \right)}$$

$$= \cot \left(\frac{A+B}{2} \right) \cot \left(\frac{A-B}{2} \right)$$

= R.H.S.

Hence Proved

8 A. Question

Prove that:

$$\frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A$$

Answer

Take L.H.S.:

$$\frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A}$$

$$= \frac{(\sin 5A + \sin A) + \sin 3A}{(\cos 5A + \cos A) + \cos 3A}$$

$$\left\{ \begin{array}{l} \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \end{array} \right\}$$

$$\begin{aligned}
&= \frac{\left(2 \sin \frac{5A+A}{2} \cos \frac{5A-A}{2}\right) + \sin 3A}{\left(2 \cos \frac{5A+A}{2} \cos \frac{5A-A}{2}\right) + \cos 3A} \\
&= \frac{\left(2 \sin \frac{6A}{2} \cos \frac{4A}{2}\right) + \sin 3A}{\left(2 \cos \frac{6A}{2} \cos \frac{4A}{2}\right) + \cos 3A} \\
&= \frac{(2 \sin 3A \cos 2A) + \sin 3A}{(2 \cos 3A \cos 2A) + \cos 3A} \\
&= \frac{\sin 3A (2 \cos 2A + 1)}{\cos 3A (2 \cos 2A + 1)} \\
&= \tan 3A \\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved

8 B. Question

Prove that:

$$\frac{\cos 3A + 2 \cos 5A + \cos 7A}{\cos A + 2 \cos 3A + \cos 5A} = \frac{\cos 5A}{\cos 3A}$$

Answer

Take L.H.S.:

$$\begin{aligned}
&\frac{\cos 3A + 2 \cos 5A + \cos 7A}{\cos A + 2 \cos 3A + \cos 5A} \\
&= \frac{(\cos 7A + \cos 3A) + 2 \cos 5A}{(\cos 5A + \cos A) + 2 \cos 3A} \\
&\left\{ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\} \\
&= \frac{\left(2 \cos \frac{7A+3A}{2} \cos \frac{7A-3A}{2}\right) + 2 \cos 5A}{\left(2 \cos \frac{5A+A}{2} \cos \frac{5A-A}{2}\right) + 2 \cos 3A} \\
&= \frac{\left(2 \cos \frac{10A}{2} \cos \frac{4A}{2}\right) + 2 \cos 5A}{\left(2 \cos \frac{6A}{2} \cos \frac{4A}{2}\right) + 2 \cos 3A} \\
&= \frac{(2 \cos 5A \cos 2A) + 2 \cos 5A}{(2 \cos 3A \cos 2A) + 2 \cos 3A} \\
&= \frac{2 \cos 5A (\cos 2A + 1)}{2 \cos 3A (\cos 2A + 1)} \\
&= \frac{\cos 5A}{\cos 3A} \\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved

8 C. Question

Prove that:

$$\frac{\cos 4A + \cos 3A + \cos 2A}{\sin 4A + \sin 3A + \sin 2A} = \cot 3A$$

Answer

Take L.H.S.:

$$\begin{aligned}
&\frac{\cos 4A + \cos 3A + \cos 2A}{\sin 4A + \sin 3A + \sin 2A} \\
&= \frac{(\cos 4A + \cos 2A) + \cos 3A}{(\sin 4A + \sin 2A) + \sin 3A} \\
&\left\{ \begin{aligned} \sin A + \sin B &= 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \cos A + \cos B &= 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \end{aligned} \right\} \\
&= \frac{\left(2 \cos \frac{4A+2A}{2} \cos \frac{4A-2A}{2}\right) + \cos 3A}{\left(2 \sin \frac{4A+2A}{2} \cos \frac{4A-2A}{2}\right) + \sin 3A}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\left(2 \cos \frac{6A}{2} \cos \frac{2A}{2}\right) + \cos 3A}{\left(2 \sin \frac{6A}{2} \cos \frac{2A}{2}\right) + \sin 3A} \\
&= \frac{(2 \cos 3A \cos A) + \cos 3A}{(2 \sin 3A \cos A) + \sin 3A} \\
&= \frac{\cos 3A (2 \cos A + 1)}{\sin 3A (2 \cos A + 1)} \\
&= \cot 3A
\end{aligned}$$

= R.H.S.

Hence Proved

8 D. Question

Prove that:

$$\frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A} = \tan 6A$$

Answer

Take L.H.S.:

$$\begin{aligned}
&\frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A} \\
&= \frac{(\sin 9A + \sin 3A) + (\sin 7A + \sin 5A)}{(\cos 9A + \cos 3A) + (\cos 7A + \cos 5A)} \\
&\left\{ \begin{aligned} \sin A + \sin B &= 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \cos A + \cos B &= 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \end{aligned} \right\} \\
&= \frac{\left(2 \sin \frac{9A+3A}{2} \cos \frac{9A-3A}{2}\right) + \left(2 \sin \frac{7A+5A}{2} \cos \frac{7A-5A}{2}\right)}{\left(2 \cos \frac{9A+3A}{2} \cos \frac{9A-3A}{2}\right) + \left(2 \cos \frac{7A+5A}{2} \cos \frac{7A-5A}{2}\right)} \\
&= \frac{\left(2 \sin \frac{12A}{2} \cos \frac{6A}{2}\right) + \left(2 \sin \frac{12A}{2} \cos \frac{2A}{2}\right)}{\left(2 \cos \frac{12A}{2} \cos \frac{6A}{2}\right) + \left(2 \cos \frac{12A}{2} \cos \frac{2A}{2}\right)} \\
&= \frac{(2 \sin 6A \cos 3A) + (2 \sin 6A \cos A)}{(2 \cos 6A \cos 3A) + (2 \cos 6A \cos A)} \\
&= \frac{2 \sin 6A (\cos 3A + \cos A)}{2 \cos 6A (\cos 3A + \cos A)} \\
&= \frac{\sin 6A}{\cos 6A} \\
&= \tan 6A
\end{aligned}$$

= R.H.S.

Hence Proved

8 E. Question

Prove that:

$$\frac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A} = \cot 6A$$

Answer

Take L.H.S.:

$$\begin{aligned}
&\frac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A} \\
&= \frac{-(\sin 7A - \sin 5A) + (\sin 8A - \sin 4A)}{(\cos 7A - \cos 5A) - (\cos 8A - \cos 4A)} \\
&\left\{ \begin{aligned} \sin A - \sin B &= 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \\ \cos A - \cos B &= -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \end{aligned} \right\} \\
&= \frac{-\left(2 \cos \frac{7A+5A}{2} \sin \frac{7A-5A}{2}\right) + \left(2 \cos \frac{8A+4A}{2} \sin \frac{8A-4A}{2}\right)}{\left(-2 \sin \frac{7A+5A}{2} \sin \frac{7A-5A}{2}\right) - \left(-2 \sin \frac{8A+4A}{2} \sin \frac{8A-4A}{2}\right)}
\end{aligned}$$

$$\begin{aligned}
&= \frac{-\left(2 \cos \frac{12A}{2} \sin \frac{2A}{2}\right) + \left(2 \cos \frac{12A}{2} \sin \frac{4A}{2}\right)}{\left(-2 \sin \frac{12A}{2} \sin \frac{2A}{2}\right) - \left(-2 \sin \frac{12A}{2} \sin \frac{4A}{2}\right)} \\
&= \frac{-(2 \cos 6A \sin A) + (2 \cos 6A \sin 2A)}{-(2 \sin 6A \sin A) + (2 \sin 6A \sin 2A)} \\
&= \frac{2 \cos 6A (-\sin A + \sin 2A)}{2 \sin 6A (-\sin A + \sin 2A)} \\
&= \frac{\cos 6A}{\sin 6A} \\
&= \cot 6A \\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved

8 F. Question

Prove that:

$$\frac{\sin 5A \cos 2A - \sin 6A \cos A}{\sin A \sin 2A - \cos 2A \cos 3A} = \tan A$$

Answer

Take L.H.S.:

$$\frac{\sin 5A \cos 2A - \sin 6A \cos A}{\sin A \sin 2A - \cos 2A \cos 3A}$$

Multiplying & Dividing by 2:

$$= \frac{(2 \sin 5A \cos 2A) - (2 \sin 6A \cos A)}{(2 \sin 2A \sin A) - (2 \cos 3A \cos A)}$$

$$\{\because 2 \sin A \sin B = \cos(A - B) - \cos(A + B)\};$$

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B) \text{ \& }$$

$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)\}$$

$$= \frac{\{\sin(5A + 2A) + \sin(5A - 2A)\} - \{\sin(6A + A) + \sin(6A - A)\}}{\{\cos(2A - A) - \cos(2A + A)\} - \{\cos(3A + 2A) + \cos(3A - 2A)\}}$$

$$= \frac{\{\sin 7A + \sin 3A\} - \{\sin 7A + \sin 5A\}}{\{\cos A - \cos 3A\} - \{\cos 5A + \cos A\}}$$

$$= \frac{\sin 7A + \sin 3A - \sin 7A - \sin 5A}{\cos A - \cos 3A - \cos 5A - \cos A}$$

$$= \frac{\sin 3A - \sin 5A}{-(\cos 5A + \cos 3A)}$$

$$= \frac{-(\sin 5A - \sin 3A)}{-(\cos 5A + \cos 3A)}$$

$$= \frac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A}$$

$$\left\{ \begin{array}{l} \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \\ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \end{array} \right\}$$

$$= \frac{2 \cos \frac{5A+3A}{2} \sin \frac{5A-3A}{2}}{2 \cos \frac{5A+3A}{2} \cos \frac{5A-3A}{2}}$$

$$= \frac{2 \cos \frac{8A}{2} \sin \frac{2A}{2}}{2 \cos \frac{8A}{2} \cos \frac{2A}{2}}$$

$$= \frac{\sin A}{\cos A}$$

$$= \tan A$$

$$= \text{R.H.S.}$$

Hence Proved

8 G. Question

Prove that:

$$\frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A} = \tan 8A$$

Answer

Take L.H.S.:

$$\frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A}$$

Multiplying & Dividing by 2:

$$= \frac{(2 \sin 11A \sin A) + (2 \sin 7A \sin 3A)}{(2 \cos 11A \sin A) + (2 \cos 7A \sin 3A)}$$

$$\{\because 2 \sin A \sin B = \cos(A - B) - \cos(A + B) \text{ \&}$$

$$2 \cos A \sin B = \sin(A + B) - \sin(A - B)\}$$

$$= \frac{\{\cos(11A - A) - \cos(11A + A)\} + \{\cos(7A - 3A) - \cos(7A + 3A)\}}{\{\sin(11A + A) - \sin(11A - A)\} + \{\sin(7A + 3A) - \sin(7A - 3A)\}}$$

$$= \frac{\{\cos 10A - \cos 12A\} + \{\cos 4A - \cos 10A\}}{\{\sin 12A - \sin 10A\} + \{\sin 10A - \sin 4A\}}$$

$$= \frac{\cos 10A - \cos 12A + \cos 4A - \cos 10A}{\sin 12A - \sin 10A + \sin 10A - \sin 4A}$$

$$= \frac{-(\cos 12A - \cos 4A)}{\sin 12A - \sin 4A}$$

$$\left\{ \begin{array}{l} \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \\ \cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \end{array} \right\}$$

$$= - \frac{-2 \sin \frac{12A+4A}{2} \sin \frac{12A-4A}{2}}{2 \cos \frac{12A+4A}{2} \sin \frac{12A-4A}{2}}$$

$$= \frac{\sin \frac{16A}{2} \sin \frac{8A}{2}}{\cos \frac{16A}{2} \sin \frac{8A}{2}}$$

$$= \frac{\sin 8A}{\cos 8A}$$

$$= \tan 8A$$

$$= \text{R.H.S.}$$

Hence Proved

8 H. Question

Prove that:

$$\frac{\sin 3A \cos 4A - \sin A \cos 2A}{\sin 4A \sin A + \cos 6A \cos A} = \tan 2A$$

Answer

Take L.H.S.:

$$\frac{\sin 3A \cos 4A - \sin A \cos 2A}{\sin 4A \sin A + \cos 6A \cos A}$$

Multiplying & Dividing by 2:

$$= \frac{(2 \sin 3A \cos 4A) - (2 \sin A \cos 2A)}{(2 \sin 4A \sin A) + (2 \cos 6A \cos A)}$$

$$\{\because 2 \sin A \sin B = \cos(A - B) - \cos(A + B);\}$$

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B) \text{ \&}$$

$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)\}$$

$$= \frac{\{\sin(3A + 4A) + \sin(3A - 4A)\} - \{\sin(A + 2A) + \sin(A - 2A)\}}{\{\cos(4A - A) - \cos(4A + A)\} + \{\cos(6A + A) + \cos(6A - A)\}}$$

$$= \frac{\{\sin 7A + \sin(-A)\} - \{\sin 3A + \sin(-A)\}}{\{\cos 3A - \cos 5A\} + \{\cos 7A + \cos 5A\}}$$

$$\{\sin(-A) = -\sin A\}$$

$$= \frac{\{\sin 7A - \sin A\} - \{\sin 3A - \sin A\}}{\{\cos 3A - \cos 5A\} + \{\cos 7A + \cos 5A\}}$$

$$\begin{aligned}
&= \frac{\sin 7A - \sin A - \sin 3A + \sin A}{\cos 3A - \cos 5A + \cos 7A + \cos 5A} \\
&= \frac{\sin 7A - \sin 3A}{\cos 7A + \cos 3A} \\
&\left(\begin{aligned} \sin A - \sin B &= 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \\ \cos A + \cos B &= 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \end{aligned} \right) \\
&= \frac{2 \cos \frac{7A+3A}{2} \sin \frac{7A-3A}{2}}{2 \cos \frac{7A+3A}{2} \cos \frac{7A-3A}{2}} \\
&= \frac{\cos \frac{10A}{2} \sin \frac{4A}{2}}{\cos \frac{10A}{2} \cos \frac{4A}{2}} \\
&= \frac{\sin 2A}{\cos 2A} \\
&= \tan 2A \\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved

8 I. Question

Prove that:

$$\frac{\sin A \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A + \sin 3A \cos 6A} = \tan 5A$$

Answer

Take L.H.S.:

$$\frac{\sin A \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A + \sin 3A \cos 6A}$$

Multiplying & Dividing by 2:

$$= \frac{(2 \sin 2A \sin A) + (2 \sin 6A \sin 3A)}{(2 \sin A \cos 2A) + (2 \sin 3A \cos 6A)}$$

$$\{\because 2 \sin A \sin B = \cos(A-B) - \cos(A+B) \text{ \&}$$

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)\}$$

$$= \frac{\{\cos(2A-A) - \cos(2A+A)\} + \{\cos(6A-3A) - \cos(6A+3A)\}}{\{\sin(A+2A) + \sin(A-2A)\} + \{\sin(3A+6A) + \sin(3A-6A)\}}$$

$$= \frac{\{\cos A - \cos 3A\} + \{\cos 3A - \cos 9A\}}{\{\sin 3A + \sin(-A)\} + \{\sin 9A + \sin(-3A)\}}$$

$$\{\sin(-A) = -\sin A\}$$

$$= \frac{\{\cos A - \cos 3A\} + \{\cos 3A - \cos 9A\}}{\{\sin 3A - \sin A\} + \{\sin 9A - \sin 3A\}}$$

$$= \frac{\cos A - \cos 3A + \cos 3A - \cos 9A}{\sin 3A - \sin A + \sin 9A - \sin 3A}$$

$$= \frac{\cos A - \cos 9A}{\sin 9A - \sin A}$$

$$= \frac{-(\cos 9A - \cos A)}{\sin 9A - \sin A}$$

$$\left(\begin{aligned} \sin A - \sin B &= 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \\ \cos A - \cos B &= -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \end{aligned} \right)$$

$$= - \frac{-2 \sin \frac{9A+A}{2} \sin \frac{9A-A}{2}}{2 \cos \frac{9A+A}{2} \sin \frac{9A-A}{2}}$$

$$= \frac{\sin \frac{10A}{2} \sin \frac{8A}{2}}{\cos \frac{10A}{2} \sin \frac{8A}{2}}$$

$$= \frac{\sin 5A}{\cos 5A}$$

$$= \tan 5A$$

= R.H.S.

Hence Proved

8 J. Question

Prove that:

$$\frac{\sin A + 2\sin 3A + \sin 5A}{\sin 3A + 2\sin 5A + \sin 7A} = \frac{\sin 3A}{\sin 5A}$$

Answer

Take L.H.S.:

$$\begin{aligned} & \frac{\sin A + 2\sin 3A + \sin 5A}{\sin 3A + 2\sin 5A + \sin 7A} \\ &= \frac{(\sin 5A + \sin A) + 2\sin 3A}{(\sin 7A + \sin 3A) + 2\sin 5A} \\ & \left\{ \sin A + \sin B = 2\sin \frac{A+B}{2} \cos \frac{A-B}{2} \right\} \\ &= \frac{\left(2\sin \frac{5A+A}{2} \cos \frac{5A-A}{2} \right) + 2\sin 3A}{\left(2\sin \frac{7A+3A}{2} \cos \frac{7A-3A}{2} \right) + 2\sin 5A} \\ &= \frac{\left(2\sin \frac{6A}{2} \cos \frac{4A}{2} \right) + 2\sin 3A}{\left(2\sin \frac{10A}{2} \cos \frac{4A}{2} \right) + 2\sin 5A} \\ &= \frac{(2\sin 3A \cos 2A) + 2\sin 3A}{(2\sin 5A \cos 2A) + 2\sin 5A} \\ &= \frac{2\sin 3A (\cos 2A + 1)}{2\sin 5A (\cos 2A + 1)} \\ &= \frac{\sin 3A}{\sin 5A} \end{aligned}$$

= R.H.S.

Hence Proved

8 K. Question

Prove that:

$$\frac{\sin(\theta + \Phi) - 2\sin \theta + \sin(\theta - \Phi)}{\cos(\theta + \Phi) - 2\cos \theta + \cos(\theta - \Phi)} = \tan \theta$$

Answer

Take R.H.S:

$$\begin{aligned} & \frac{\sin(\theta + \Phi) - 2\sin \theta + \sin(\theta - \Phi)}{\cos(\theta + \Phi) - 2\cos \theta + \cos(\theta - \Phi)} \\ &= \frac{\{\sin(\theta + \Phi) + \sin(\theta - \Phi)\} - 2\sin \theta}{\{\cos(\theta + \Phi) + \cos(\theta - \Phi)\} - 2\cos \theta} \\ & \left\{ \begin{aligned} \sin A + \sin B &= 2\sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \cos A + \cos B &= 2\cos \frac{A+B}{2} \cos \frac{A-B}{2} \end{aligned} \right\} \\ &= \frac{\left(2\sin \frac{\theta + \Phi + \theta - \Phi}{2} \cos \frac{\theta + \Phi - \theta + \Phi}{2} \right) - 2\sin \theta}{\left(2\cos \frac{\theta + \Phi + \theta - \Phi}{2} \cos \frac{\theta + \Phi - \theta + \Phi}{2} \right) - 2\cos \theta} \\ &= \frac{\left(2\sin \frac{2\theta}{2} \cos \frac{2\Phi}{2} \right) - 2\sin \theta}{\left(2\cos \frac{2\theta}{2} \cos \frac{2\Phi}{2} \right) - 2\cos \theta} \\ &= \frac{(2\sin \theta \cos \Phi) - 2\sin \theta}{(2\cos \theta \cos \Phi) - 2\cos \theta} \\ &= \frac{2\sin \theta (\cos \Phi - 1)}{2\cos \theta (\cos \Phi - 1)} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \\ &= \text{R.H.S} \end{aligned}$$

Hence Proved

9. Question

Prove that:

$$i. \sin \alpha + \sin \beta + \sin \gamma - \sin(\alpha + \beta + \gamma) = 4 \sin \frac{\alpha + \beta}{2} \sin \frac{\beta + \gamma}{2} \sin \frac{\alpha + \gamma}{2}$$

$$ii. \cos(A + B + C) + \cos(A - B + C) + \cos(A + B - C) + \cos(-A + B + C) = 4 \cos A \cos B \cos C$$

Answer

Take L.H.S:

$$\sin \alpha + \sin \beta + \sin \gamma - \sin(\alpha + \beta + \gamma)$$

$$\left\{ \begin{array}{l} \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \end{array} \right\}$$

$$= (\sin \alpha + \sin \beta) + \{\sin \gamma - \sin(\alpha + \beta + \gamma)\}$$

$$= \left(2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} \right) + \left(2 \cos \frac{\gamma + \alpha + \beta + \gamma}{2} \sin \frac{\gamma - \alpha - \beta - \gamma}{2} \right)$$

$$= \left(2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} \right) + \left(2 \cos \frac{\alpha + \beta + 2\gamma}{2} \sin \frac{-(\alpha + \beta)}{2} \right)$$

$$\{\sin(-A) = -\sin A\}$$

$$= \left(2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} \right) - \left(2 \cos \frac{\alpha + \beta + 2\gamma}{2} \sin \frac{\alpha + \beta}{2} \right)$$

$$= 2 \sin \frac{\alpha + \beta}{2} \left(\cos \frac{\alpha - \beta}{2} - \cos \frac{\alpha + \beta + 2\gamma}{2} \right)$$

$$\left\{ \cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \right\}$$

$$= 2 \sin \frac{\alpha + \beta}{2} \left(-2 \sin \frac{\frac{\alpha - \beta}{2} + \frac{\alpha + \beta + 2\gamma}{2}}{2} \sin \frac{\frac{\alpha - \beta}{2} - \frac{\alpha + \beta + 2\gamma}{2}}{2} \right)$$

$$= 2 \sin \frac{\alpha + \beta}{2} \left(-2 \sin \frac{\frac{\alpha - \beta + \alpha + \beta + 2\gamma}{2}}{2} \sin \frac{\frac{\alpha - \beta - (\alpha + \beta + 2\gamma)}{2}}{2} \right)$$

$$= 2 \sin \frac{\alpha + \beta}{2} \left(-2 \sin \frac{\frac{2\alpha + 2\gamma}{2}}{2} \sin \frac{\frac{\alpha - \beta - \alpha - \beta - 2\gamma}{2}}{2} \right)$$

$$= 2 \sin \frac{\alpha + \beta}{2} \left(-2 \sin \frac{\frac{2\alpha + 2\gamma}{2}}{2} \sin \frac{\frac{-2\beta - 2\gamma}{2}}{2} \right)$$

$$= 2 \sin \frac{\alpha + \beta}{2} \left(-2 \sin \frac{\frac{2(\alpha + \gamma)}{2}}{2} \sin \frac{\frac{-2(\beta + \gamma)}{2}}{2} \right)$$

$$= 2 \sin \frac{\alpha + \beta}{2} \left(-2 \sin \frac{\alpha + \gamma}{2} \sin \frac{-(\beta + \gamma)}{2} \right)$$

$$\{\sin(-A) = -\sin A\}$$

$$= 2 \sin \frac{\alpha + \beta}{2} \left(2 \sin \frac{\alpha + \gamma}{2} \sin \frac{\beta + \gamma}{2} \right)$$

$$= 4 \sin \frac{\alpha + \beta}{2} \sin \frac{\beta + \gamma}{2} \sin \frac{\alpha + \gamma}{2}$$

= R.H.S.

Hence Proved

ii. Take L.H.S.:

$$\cos(A + B + C) + \cos(A - B + C) + \cos(A + B - C) + \cos(-A + B + C)$$

$$= \{\cos(A + B + C) + \cos(A - B + C)\} + \{\cos(A + B - C) + \cos(-A + B + C)\}$$

$$\left\{ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$= \left\{ 2 \cos \frac{(A + B + C) + (A - B + C)}{2} \cos \frac{(A + B + C) - (A - B + C)}{2} \right\}$$

$$+ \left\{ 2 \cos \frac{(A + B - C) + (-A + B + C)}{2} \cos \frac{(A + B - C) - (-A + B + C)}{2} \right\}$$

$$\begin{aligned}
&= \left\{ 2 \cos \frac{A+B+C+A-B+C}{2} \cos \frac{A+B+C-A+B-C}{2} \right\} \\
&\quad + \left\{ 2 \cos \frac{A+B-C-A+B+C}{2} \cos \frac{A+B-C+A-B-C}{2} \right\} \\
&= \left\{ 2 \cos \frac{2A+2C}{2} \cos \frac{2B}{2} \right\} + \left\{ 2 \cos \frac{2B}{2} \cos \frac{2A-2C}{2} \right\} \\
&= \left\{ 2 \cos \frac{2(A+C)}{2} \cos \frac{2B}{2} \right\} + \left\{ 2 \cos \frac{2B}{2} \cos \frac{2(A-C)}{2} \right\} \\
&= 2 \cos (A+C) \cos B + 2 \cos B \cos (A-C) \\
&= 2 \cos B \{ \cos (A+C) + \cos (A-C) \} \\
&\left\{ \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\} \\
&= 2 \cos B \left\{ 2 \cos \frac{A+C+(A-C)}{2} \cos \frac{A+C-(A-C)}{2} \right\} \\
&= 2 \cos B \left\{ 2 \cos \frac{A+C+A-C}{2} \cos \frac{A+C-A+C}{2} \right\} \\
&= 2 \cos B \left\{ 2 \cos \frac{2A}{2} \cos \frac{2C}{2} \right\} \\
&= 4 \cos A \cos B \cos C \\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved

10. Question

If $\cos A + \cos B = \frac{1}{2}$ and $\sin A + \sin B = \frac{1}{4}$, prove that $\tan\left(\frac{A+B}{2}\right) = \frac{1}{2}$.

Answer

Given,

$$\cos A + \cos B = \frac{1}{2} \text{ and } \sin A + \sin B = \frac{1}{4}$$

$$\therefore \frac{\sin A + \sin B}{\cos A + \cos B} = \frac{\frac{1}{4}}{\frac{1}{2}}$$

$$\Rightarrow \frac{\sin A + \sin B}{\cos A + \cos B} = \frac{1}{4} \times \frac{2}{1}$$

$$\Rightarrow \frac{\sin A + \sin B}{\cos A + \cos B} = \frac{1}{2}$$

$$\left\{ \begin{aligned} \sin A + \sin B &= 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \cos A + \cos B &= 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \end{aligned} \right\}$$

$$\Rightarrow \frac{2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}}{2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}} = \frac{1}{2}$$

$$\Rightarrow \frac{\sin \frac{A+B}{2}}{\cos \frac{A+B}{2}} = \frac{1}{2}$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \frac{1}{2}$$

$$\text{This proves that } \tan\left(\frac{A+B}{2}\right) = \frac{1}{2}$$

11. Question

If $\operatorname{cosec} A + \sec A = \operatorname{cosec} B + \sec B$, prove that: $\tan A \tan B = \cot \frac{A+B}{2}$

Answer

Given,

$$\operatorname{cosec} A + \sec A = \operatorname{cosec} B + \sec B$$

$$\Rightarrow \sec A - \sec B = \operatorname{cosec} B - \operatorname{cosec} A$$

$$\Rightarrow \frac{1}{\cos A} - \frac{1}{\cos B} = \frac{1}{\sin B} - \frac{1}{\sin A}$$

$$\Rightarrow \frac{\cos B - \cos A}{\cos A \cos B} = \frac{\sin A - \sin B}{\sin B \sin A}$$

$$\Rightarrow \frac{\sin B \sin A}{\cos A \cos B} = \frac{\sin A - \sin B}{\cos B - \cos A}$$

$$\Rightarrow \tan A \tan B = \frac{\sin A - \sin B}{\cos B - \cos A}$$

$$\left\{ \begin{array}{l} \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \\ \cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \end{array} \right\}$$

$$\Rightarrow \tan A \tan B = \frac{2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}}{-2 \sin \frac{A+B}{2} \sin \frac{B-A}{2}}$$

$$\Rightarrow \tan A \tan B = \frac{\cos \frac{A+B}{2} \sin \frac{A-B}{2}}{-\sin \frac{A+B}{2} \sin \frac{-(A-B)}{2}}$$

$$\{\sin(-A) = -\sin A\}$$

$$\Rightarrow \tan A \tan B = \frac{\cos \frac{A+B}{2} \sin \frac{A-B}{2}}{\sin \frac{A+B}{2} \sin \frac{A-B}{2}}$$

$$\Rightarrow \tan A \tan B = \frac{\cos \frac{A+B}{2}}{\sin \frac{A+B}{2}}$$

$$\Rightarrow \tan A \tan B = \cot \frac{A+B}{2}$$

12. Question

If $\sin 2A = \lambda \sin 2B$, prove that:

$$\frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda+1}{\lambda-1}$$

Answer

Given,

$$\sin 2A = \lambda \sin 2B$$

$$\Rightarrow \lambda = \frac{\sin 2A}{\sin 2B}$$

Now,

$$\frac{\lambda+1}{\lambda-1} = \frac{\frac{\sin 2A}{\sin 2B} + 1}{\frac{\sin 2A}{\sin 2B} - 1}$$

$$= \frac{\frac{\sin 2A + \sin 2B}{\sin 2B}}{\frac{\sin 2A - \sin 2B}{\sin 2B}}$$

$$= \frac{\sin 2A + \sin 2B}{\sin 2B} \times \frac{\sin 2B}{\sin 2A - \sin 2B}$$

$$= \frac{\sin 2A + \sin 2B}{\sin 2A - \sin 2B}$$

$$\left\{ \begin{array}{l} \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \end{array} \right\}$$

$$= \frac{2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}}{2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}}$$

$$= \frac{\sin \frac{2(A+B)}{2} \cos \frac{2(A-B)}{2}}{\cos \frac{2(A+B)}{2} \sin \frac{2(A-B)}{2}}$$

$$= \frac{\sin(A+B) \cos(A-B)}{\cos(A+B) \sin(A-B)}$$

$$= \frac{\tan(A+B)}{\tan(A-B)}$$

This proves that $\frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda+1}{\lambda-1}$

13. Question

Prove that:

- i. $\frac{\cos(A+B+C) + \cos(-A+B+C) + \cos(A-B+C) + \cos(A+B-C)}{\sin(A+B+C) + \sin(-A+B+C) + \sin(A-B+C) - \sin(A+B-C)} = \cot C$
- ii. $\sin(B-C)\cos(A-D) + \sin(C-A)\cos(B-D) + \sin(A-B)\cos(C-D) = 0$

Answer

Take L.H.S.

$$\begin{aligned} & \frac{\cos(A+B+C) + \cos(-A+B+C) + \cos(A-B+C) + \cos(A+B-C)}{\sin(A+B+C) + \sin(-A+B+C) + \sin(A-B+C) - \sin(A+B-C)} \\ &= \frac{\{\cos(A+B+C) + \cos(-A+B+C)\} + \{\cos(A-B+C) + \cos(A+B-C)\}}{\{\sin(A+B+C) + \sin(-A+B+C)\} + \{\sin(A-B+C) - \sin(A+B-C)\}} \\ &= \frac{\left\{ \begin{aligned} \cos A + \cos B &= 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \\ \sin A + \sin B &= 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \sin A - \sin B &= 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \end{aligned} \right\}}{\left\{ \begin{aligned} 2 \cos \frac{(A+B+C) + (-A+B+C)}{2} \cos \frac{(A+B+C) - (-A+B+C)}{2} &+ 2 \cos \frac{(A-B+C) + (A+B-C)}{2} \cos \frac{(A-B+C) - (A+B-C)}{2} \\ 2 \sin \frac{(A+B+C) + (-A+B+C)}{2} \cos \frac{(A+B+C) - (-A+B+C)}{2} &+ 2 \cos \frac{(A-B+C) + (A+B-C)}{2} \sin \frac{(A-B+C) - (A+B-C)}{2} \end{aligned} \right\}} \\ &= \frac{2 \left\{ \cos \frac{A+B+C-A+B+C}{2} \cos \frac{A+B+C+A-B-C}{2} + \cos \frac{A-B+C+A+B-C}{2} \cos \frac{A-B+C-A-B+C}{2} \right\}}{2 \left\{ \sin \frac{A+B+C-A+B+C}{2} \cos \frac{A+B+C+A-B-C}{2} + \cos \frac{A-B+C+A+B-C}{2} \sin \frac{A-B+C-A-B+C}{2} \right\}} \\ &= \frac{\left\{ \cos \frac{2(B+C)}{2} \cos \frac{2A}{2} + \cos \frac{2A}{2} \cos \frac{-2(B-C)}{2} \right\}}{\left\{ \sin \frac{2(B+C)}{2} \cos \frac{2A}{2} + \cos \frac{2A}{2} \sin \frac{-2(B-C)}{2} \right\}} \\ &= \frac{\{\cos(B+C) \cos A\} + \{\cos A \cos -(B-C)\}}{\{\sin(B+C) \cos A\} + \{\cos A \sin -(B-C)\}} \\ &= \frac{\cos A \{\cos(B+C) + \cos -(B-C)\}}{\cos A \{\sin(B+C) + \sin -(B-C)\}} \end{aligned}$$

{sin (-A) = -sin A & cos (-A) = cos A}

$$\begin{aligned} &= \frac{\cos(B+C) + \cos(B-C)}{\sin(B+C) - \sin(B-C)} \\ &= \frac{\left\{ \begin{aligned} \cos A + \cos B &= 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \\ \sin A - \sin B &= 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \end{aligned} \right\}}{\frac{2 \cos \frac{(B+C) + (B-C)}{2} \cos \frac{(B+C) - (B-C)}{2}}{2 \cos \frac{(B+C) + (B-C)}{2} \sin \frac{(B+C) - (B-C)}{2}}} \\ &= \frac{\cos \frac{B+C+B-C}{2} \cos \frac{B+C-B+C}{2}}{\cos \frac{B+C+B-C}{2} \sin \frac{B+C-B+C}{2}} \\ &= \frac{\cos \frac{2B}{2} \cos \frac{2C}{2}}{\cos \frac{2B}{2} \sin \frac{2C}{2}} \\ &= \frac{\cos B \cos C}{\cos B \sin C} \\ &= \frac{\cos C}{\sin C} \\ &= \cot C \\ &= \text{R.H.S.} \end{aligned}$$

Hence Proved

ii. Take L.H.S.:

$$\sin(B-C)\cos(A-D) + \sin(C-A)\cos(B-D) + \sin(A-B)\cos(C-D)$$

Multiplying & Dividing by 2:

$$= \frac{1}{2} \{ 2 \sin(B-C) \cos(A-D) + 2 \sin(C-A) \cos(B-D) + 2 \sin(A-B) \cos(C-D) \}$$

$$\{ \because 2 \sin A \cos B = \sin(A+B) + \sin(A-B) \}$$

$$= \frac{1}{2} \{ \sin \{ (B-C) + (A-D) \} + \sin \{ (B-C) - (A-D) \} + \sin \{ (C-A) + (B-D) \} + \sin \{ (C-A) - (B-D) \} + \sin \{ (A-B) + (C-D) \} + \sin \{ (A-B) - (C-D) \} \}$$

$$= \frac{1}{2} \{ \sin(B-C+A-D) + \sin(B-C-A+D) + \sin(C-A+B-D) + \sin(C-A-B+D) + \sin(A-B+C-D) + \sin(A-B-C+D) \}$$

$$= \frac{1}{2} \{ \sin(A+B-C-D) + \sin(-A+B-C+D) + \sin(-A+B+C-D) + \sin(-A-B+C+D) + \sin(A-B+C-D) + \sin(A-B-C+D) \}$$

$$= \frac{1}{2} \{ \sin(A+B-C-D) + \sin-(A+B-C-D) + \sin(-A+B-C+D) + \sin-(-A+B-C+D) + \sin(A-B+C-D) + \sin-(A-B+C+D) \}$$

$$\{ \sin(-A) = -\sin A \}$$

$$= \frac{1}{2} \{ \sin(A+B-C-D) - \sin(A+B-C-D) + \sin(-A+B-C+D) - \sin(-A+B-C+D) + \sin(A-B+C-D) - \sin(A-B+C+D) \}$$

$$= \frac{1}{2} \{ 0 + 0 + 0 \}$$

$$= 0$$

$$= \text{R.H.S.}$$

Hence Proved

14. Question

$$\text{If } \frac{\cos(A-B)}{\cos(A+B)} + \frac{\cos(C+D)}{\cos(C-D)} = 0, \text{ prove that } \tan A \tan B \tan C \tan D = -1$$

Answer

Given,

$$\frac{\cos(A-B)}{\cos(A+B)} + \frac{\cos(C+D)}{\cos(C-D)} = 0$$

$$\Rightarrow \frac{\cos(A-B)}{\cos(A+B)} = -\frac{\cos(C+D)}{\cos(C-D)}$$

Adding 1 both sides:

$$\Rightarrow \frac{\cos(A-B)}{\cos(A+B)} + 1 = -\frac{\cos(C+D)}{\cos(C-D)} + 1$$

$$\Rightarrow \frac{\cos(A-B) + \cos(A+B)}{\cos(A+B)} = \frac{\cos(C-D) - \cos(C+D)}{\cos(C-D)}$$

$$\Rightarrow \frac{\cos(A-B) + \cos(A+B)}{\cos(A+B)} = \frac{-\{\cos(C+D) - \cos(C-D)\}}{\cos(C-D)} \quad (i)$$

Now,

$$\frac{\cos(A-B)}{\cos(A+B)} = -\frac{\cos(C+D)}{\cos(C-D)}$$

Subtracting 1 both sides:

$$\Rightarrow \frac{\cos(A-B)}{\cos(A+B)} - 1 = -\frac{\cos(C+D)}{\cos(C-D)} - 1$$

$$\Rightarrow \frac{\cos(A-B) - \cos(A+B)}{\cos(A+B)} = \frac{-\cos(C+D) - \cos(C-D)}{\cos(C-D)}$$

$$\Rightarrow \frac{-\{\cos(A+B) - \cos(A-B)\}}{\cos(A+B)} = \frac{-\{\cos(C+D) + \cos(C-D)\}}{\cos(C-D)}$$

$$\Rightarrow \frac{\cos(A+B) - \cos(A-B)}{\cos(A+B)} = \frac{\cos(C+D) + \cos(C-D)}{\cos(C-D)} \quad (ii)$$

Dividing equation (i) by equation (ii):

$$\begin{aligned} \frac{\frac{\cos(A-B) + \cos(A+B)}{\cos(A+B)}}{\frac{\cos(A+B) - \cos(A-B)}{\cos(A+B)}} &= \frac{\frac{-\{\cos(C+D) - \cos(C-D)\}}{\cos(C-D)}}{\frac{\cos(C+D) + \cos(C-D)}{\cos(C-D)}} \\ \Rightarrow \frac{\cos(A-B) + \cos(A+B)}{\cos(A+B)} \times \frac{\cos(A+B)}{\cos(A+B) - \cos(A-B)} &= \frac{-\{\cos(C+D) - \cos(C-D)\}}{\cos(C-D)} \times \frac{\cos(C-D)}{\cos(C+D) + \cos(C-D)} \\ \Rightarrow \frac{\cos(A+B) + \cos(A-B)}{\cos(A+B) - \cos(A-B)} &= \frac{-\{\cos(C+D) - \cos(C-D)\}}{\cos(C+D) + \cos(C-D)} \\ \left\{ \begin{aligned} \cos A + \cos B &= 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \\ \cos A - \cos B &= -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \end{aligned} \right\} \\ \Rightarrow \frac{2 \cos \frac{(A+B) + (A-B)}{2} \cos \frac{(A+B) - (A-B)}{2}}{-2 \sin \frac{(A+B) + (A-B)}{2} \sin \frac{(A+B) - (A-B)}{2}} &= \frac{-\left\{ -2 \sin \frac{(C+D) + (C-D)}{2} \sin \frac{(C+D) - (C-D)}{2} \right\}}{2 \cos \frac{(C+D) + (C-D)}{2} \cos \frac{(C+D) - (C-D)}{2}} \\ \Rightarrow -\frac{\cos \frac{(A+B+A-B)}{2} \cos \frac{(A+B-A+B)}{2}}{\sin \frac{(A+B+A-B)}{2} \sin \frac{(A+B-A+B)}{2}} &= \frac{\sin \frac{(C+D+C-D)}{2} \sin \frac{(C+D-C+D)}{2}}{\cos \frac{(C+D+C-D)}{2} \cos \frac{(C+D-C+D)}{2}} \\ \Rightarrow -\frac{\cos \frac{2A}{2} \cos \frac{2B}{2}}{\sin \frac{2A}{2} \sin \frac{2B}{2}} &= \frac{\sin \frac{2C}{2} \sin \frac{2D}{2}}{\cos \frac{2C}{2} \cos \frac{2D}{2}} \\ \Rightarrow -\frac{\cos A \cos B}{\sin A \sin B} &= \frac{\sin C \sin D}{\cos C \cos D} \\ \Rightarrow -\frac{1}{\tan A \tan B} &= \tan C \tan D \\ \Rightarrow -1 &= \tan A \tan B \tan C \tan D \end{aligned}$$

This proves that $\tan A \tan B \tan C \tan D = -1$

15. Question

If $\cos(\alpha + \beta) \sin(\gamma + \delta) = \cos(\alpha - \beta) \sin(\gamma - \delta)$, prove that $\cot \alpha \cot \beta \cot \gamma = \cot \delta$

Answer

Given,

$$\cos(\alpha + \beta) \sin(\gamma + \delta) = \cos(\alpha - \beta) \sin(\gamma - \delta)$$

$$\Rightarrow \frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} = \frac{\sin(\gamma - \delta)}{\sin(\gamma + \delta)}$$

Adding 1 both sides:

$$\begin{aligned} \Rightarrow \frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} + 1 &= \frac{\sin(\gamma - \delta)}{\sin(\gamma + \delta)} + 1 \\ \Rightarrow \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{\cos(\alpha - \beta)} &= \frac{\sin(\gamma - \delta) + \sin(\gamma + \delta)}{\sin(\gamma + \delta)} \quad (i) \end{aligned}$$

Now,

$$\frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} = \frac{\sin(\gamma - \delta)}{\sin(\gamma + \delta)}$$

Subtracting 1 both sides:

$$\begin{aligned} \Rightarrow \frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} - 1 &= \frac{\sin(\gamma - \delta)}{\sin(\gamma + \delta)} - 1 \\ \Rightarrow \frac{\cos(\alpha + \beta) - \cos(\alpha - \beta)}{\cos(\alpha - \beta)} &= \frac{\sin(\gamma - \delta) - \sin(\gamma + \delta)}{\sin(\gamma + \delta)} \quad (ii) \end{aligned}$$

Dividing equation (i) by equation (ii):

$$\frac{\frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{\cos(\alpha - \beta)}}{\frac{\cos(\alpha + \beta) - \cos(\alpha - \beta)}{\cos(\alpha - \beta)}} = \frac{\frac{\sin(\gamma - \delta) + \sin(\gamma + \delta)}{\sin(\gamma + \delta)}}{\frac{\sin(\gamma - \delta) - \sin(\gamma + \delta)}{\sin(\gamma + \delta)}}$$

$$\Rightarrow \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{\cos(\alpha - \beta)} \times \frac{\cos(\alpha - \beta)}{\cos(\alpha + \beta) - \cos(\alpha - \beta)} = \frac{\sin(\gamma - \delta) + \sin(\gamma + \delta)}{\sin(\gamma + \delta)} \times \frac{\sin(\gamma + \delta)}{\sin(\gamma - \delta) - \sin(\gamma + \delta)}$$

$$\Rightarrow \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{\cos(\alpha + \beta) - \cos(\alpha - \beta)} = \frac{\sin(\gamma - \delta) + \sin(\gamma + \delta)}{\sin(\gamma - \delta) - \sin(\gamma + \delta)}$$

$$\left\{ \begin{array}{l} \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \\ \cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \\ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \end{array} \right\}$$

$$\Rightarrow \frac{2 \cos \frac{(\alpha + \beta) + (\alpha - \beta)}{2} \cos \frac{(\alpha + \beta) - (\alpha - \beta)}{2}}{-2 \sin \frac{(\alpha + \beta) + (\alpha - \beta)}{2} \sin \frac{(\alpha + \beta) - (\alpha - \beta)}{2}} = \frac{2 \sin \frac{(\gamma - \delta) + (\gamma + \delta)}{2} \cos \frac{(\gamma - \delta) - (\gamma + \delta)}{2}}{2 \cos \frac{(\gamma - \delta) + (\gamma + \delta)}{2} \sin \frac{(\gamma - \delta) - (\gamma + \delta)}{2}}$$

$$\Rightarrow - \frac{\cos \frac{(\alpha + \beta + \alpha - \beta)}{2} \cos \frac{(\alpha + \beta - \alpha + \beta)}{2}}{\sin \frac{(\alpha + \beta + \alpha - \beta)}{2} \sin \frac{(\alpha + \beta - \alpha + \beta)}{2}} = \frac{\sin \frac{(\gamma - \delta + \gamma + \delta)}{2} \cos \frac{(\gamma - \delta - \gamma - \delta)}{2}}{\cos \frac{(\gamma - \delta + \gamma + \delta)}{2} \sin \frac{(\gamma - \delta - \gamma - \delta)}{2}}$$

$$\Rightarrow - \frac{\cos \frac{2\alpha}{2} \cos \frac{2\beta}{2}}{\sin \frac{2\alpha}{2} \sin \frac{2\beta}{2}} = \frac{\sin \frac{2\gamma}{2} \cos \frac{-2\delta}{2}}{\cos \frac{2\gamma}{2} \sin \frac{-2\delta}{2}}$$

$$\Rightarrow - \frac{\cos \alpha \cos \beta}{\sin \alpha \sin \beta} = \frac{\sin \gamma \sin(-\delta)}{\cos \gamma \cos(-\delta)}$$

$$\{\sin(-A) = -\sin A \text{ \& } \cos(-A) = \cos A\}$$

$$\Rightarrow - \frac{\cos \alpha \cos \beta}{\sin \alpha \sin \beta} = - \frac{\sin \gamma \sin \delta}{\cos \gamma \cos \delta}$$

$$\Rightarrow \cot \alpha \cot \beta = \frac{\cot \delta}{\cot \gamma}$$

$$\Rightarrow \cot \alpha \cot \beta \cot \gamma = \cot \delta$$

Hence Proved

16. Question

If $y \sin \phi = x \sin (2\theta + \phi)$, prove that $(x + y) \cot (\theta + \phi) = (y - x) \cot \theta$

Answer

Given, $y \sin \phi = x \sin (2\theta + \phi)$

$$\Rightarrow \frac{\sin \phi}{\sin (2\theta + \phi)} = \frac{x}{y}$$

Adding 1 both sides:

$$\Rightarrow \frac{\sin \phi}{\sin (2\theta + \phi)} + 1 = \frac{x}{y} + 1$$

$$\Rightarrow \frac{\sin \phi + \sin (2\theta + \phi)}{\sin (2\theta + \phi)} = \frac{x + y}{y} \quad (i)$$

Now,

$$\frac{\sin \phi}{\sin (2\theta + \phi)} = \frac{x}{y}$$

Adding 1 both sides:

$$\Rightarrow \frac{\sin \phi}{\sin (2\theta + \phi)} - 1 = \frac{x}{y} - 1$$

$$\Rightarrow \frac{\sin \phi - \sin (2\theta + \phi)}{\sin (2\theta + \phi)} = \frac{x - y}{y} \quad (ii)$$

Dividing equation (i) by equation (ii):

$$\frac{\frac{\sin \phi + \sin (2\theta + \phi)}{\sin (2\theta + \phi)}}{\frac{\sin \phi - \sin (2\theta + \phi)}{\sin (2\theta + \phi)}} = \frac{\frac{x + y}{y}}{\frac{x - y}{y}}$$

$$\Rightarrow \frac{\sin \phi + \sin (2\theta + \phi)}{\sin (2\theta + \phi)} \times \frac{\sin (2\theta + \phi)}{\sin \phi - \sin (2\theta + \phi)} = \frac{x + y}{y} \times \frac{y}{x - y}$$

$$\Rightarrow \frac{\sin \phi + \sin (2\theta + \phi)}{\sin \phi - \sin (2\theta + \phi)} = \frac{x + y}{x - y}$$

$$\left\{ \begin{array}{l} \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \end{array} \right\}$$

$$\Rightarrow \frac{2 \sin \frac{\phi + (2\theta + \phi)}{2} \cos \frac{\phi - (2\theta + \phi)}{2}}{2 \cos \frac{\phi + (2\theta + \phi)}{2} \sin \frac{\phi - (2\theta + \phi)}{2}} = \frac{x + y}{x - y}$$

$$\Rightarrow \frac{\sin \frac{\phi + 2\theta + \phi}{2} \cos \frac{\phi - 2\theta - \phi}{2}}{\cos \frac{\phi + 2\theta + \phi}{2} \sin \frac{\phi - 2\theta - \phi}{2}} = \frac{x + y}{x - y}$$

$$\Rightarrow \frac{\sin \frac{2\phi + 2\theta}{2} \cos \frac{-2\theta}{2}}{\cos \frac{2\phi + 2\theta}{2} \sin \frac{-2\theta}{2}} = \frac{x + y}{x - y}$$

$$\Rightarrow \frac{\sin \frac{2(\phi + \theta)}{2} \cos(-\theta)}{\cos \frac{2(\phi + \theta)}{2} \sin(-\theta)} = \frac{x + y}{-(y - x)}$$

{sin (-A) = -sin A & cos (-A) = cos A}

$$\Rightarrow -\frac{\cot \theta}{\cot(\phi + \theta)} = -\frac{x + y}{y - x}$$

$$\Rightarrow (x + y) \cot(\theta + \phi) = (y - x) \cot \theta$$

Hence Proved

17. Question

If $\cos(A + B) \sin(C - D) = \cos(A - B) \sin(C + D)$, prove that $\tan A \tan B \tan C + \tan D = 0$

Answer

Given,

$$\cos(A + B) \sin(C - D) = \cos(A - B) \sin(C + D)$$

$$\Rightarrow \frac{\cos(A + B)}{\cos(A - B)} = \frac{\sin(C + D)}{\sin(C - D)}$$

Adding 1 both sides:

$$\Rightarrow \frac{\cos(A + B)}{\cos(A - B)} + 1 = \frac{\sin(C + D)}{\sin(C - D)} + 1$$

$$\Rightarrow \frac{\cos(A + B) + \cos(A - B)}{\cos(A - B)} = \frac{\sin(C + D) + \sin(C - D)}{\sin(C - D)} \quad (i)$$

Now,

$$\frac{\cos(A + B)}{\cos(A - B)} = \frac{\sin(C + D)}{\sin(C - D)}$$

Subtracting 1 both sides:

$$\Rightarrow \frac{\cos(A + B)}{\cos(A - B)} - 1 = \frac{\sin(C + D)}{\sin(C - D)} - 1$$

$$\Rightarrow \frac{\cos(A + B) - \cos(A - B)}{\cos(A - B)} = \frac{\sin(C + D) - \sin(C - D)}{\sin(C - D)} \quad (ii)$$

Dividing equation (i) by equation (ii):

$$\frac{\frac{\cos(A + B) + \cos(A - B)}{\cos(A - B)}}{\frac{\cos(A + B) - \cos(A - B)}{\cos(A - B)}} = \frac{\frac{\sin(C + D) + \sin(C - D)}{\sin(C - D)}}{\frac{\sin(C + D) - \sin(C - D)}{\sin(C - D)}}$$

$$\Rightarrow \frac{\cos(A+B) + \cos(A-B)}{\cos(A-B)} \times \frac{\cos(A-B)}{\cos(A+B) - \cos(A-B)}$$

$$= \frac{\sin(C+D) + \sin(C-D)}{\sin(C-D)} \times \frac{\sin(C-D)}{\sin(C+D) - \sin(C-D)}$$

$$\Rightarrow \frac{\cos(A+B) + \cos(A-B)}{\cos(A+B) - \cos(A-B)} = \frac{\sin(C+D) + \sin(C-D)}{\sin(C+D) - \sin(C-D)}$$

$$\left\{ \begin{array}{l} \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \\ \cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \\ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \end{array} \right\}$$

$$\Rightarrow \frac{2 \cos \frac{(A+B) + (A-B)}{2} \cos \frac{(A+B) - (A-B)}{2}}{-2 \sin \frac{(A+B) + (A-B)}{2} \sin \frac{(A+B) - (A-B)}{2}}$$

$$= \frac{2 \sin \frac{(C+D) + (C-D)}{2} \cos \frac{(C+D) - (C-D)}{2}}{2 \cos \frac{(C+D) + (C-D)}{2} \sin \frac{(C+D) - (C-D)}{2}}$$

$$\Rightarrow - \frac{\cos \frac{(A+B) + (A-B)}{2} \cos \frac{(A+B) - (A-B)}{2}}{\sin \frac{(A+B) + (A-B)}{2} \sin \frac{(A+B) - (A-B)}{2}}$$

$$= \frac{\sin \frac{(C+D) + (C-D)}{2} \cos \frac{(C+D) - (C-D)}{2}}{\cos \frac{(C+D) + (C-D)}{2} \sin \frac{(C+D) - (C-D)}{2}}$$

$$\Rightarrow - \frac{\cos \frac{2A}{2} \cos \frac{2B}{2}}{\sin \frac{2A}{2} \sin \frac{2B}{2}} = \frac{\sin \frac{2C}{2} \cos \frac{2D}{2}}{\cos \frac{2C}{2} \sin \frac{2D}{2}}$$

$$\Rightarrow - \frac{\cos A \cos B}{\sin A \sin B} = \frac{\sin C \cos D}{\cos C \sin D}$$

$$\Rightarrow - \frac{1}{\tan A \tan B} = \frac{\tan C}{\tan D}$$

$$\Rightarrow -\tan D = \tan A \tan B \tan C$$

$$\Rightarrow \tan D = -\tan A \tan B \tan C$$

$$\Rightarrow \tan A \tan B \tan C + \tan D = 0$$

Hence Proved

18. Question

If $x \cos \theta = y \cos \left(\theta + \frac{2\pi}{3} \right) = z \cos \left(\theta + \frac{4\pi}{3} \right)$, prove that $xy + yz + zx = 0$.

Answer

Given,

$$x \cos \theta = y \cos \left(\theta + \frac{2\pi}{3} \right) = z \cos \left(\theta + \frac{4\pi}{3} \right) = k$$

Now,

$$x \cos \theta = k$$

$$\Rightarrow x = \frac{k}{\cos \theta}$$

Now,

$$y = \frac{k}{\cos \left(\theta + \frac{2\pi}{3} \right)}$$

Now,

$$z \cos \left(\theta + \frac{4\pi}{3} \right) = k$$

$$z = \frac{k}{\cos \left(\theta + \frac{4\pi}{3} \right)}$$

Now,

$$xy + yz + xz$$

$$\begin{aligned}
&= \frac{k}{\cos \theta} \times \frac{k}{\cos\left(\theta + \frac{2\pi}{3}\right)} + \frac{k}{\cos\left(\theta + \frac{2\pi}{3}\right)} \times \frac{k}{\cos\left(\theta + \frac{4\pi}{3}\right)} \\
&\quad + \frac{k}{\cos \theta} \times \frac{k}{\cos\left(\theta + \frac{4\pi}{3}\right)} \\
&= \frac{k^2}{\cos \theta \cos\left(\theta + \frac{2\pi}{3}\right)} + \frac{k^2}{\cos\left(\theta + \frac{2\pi}{3}\right) \cos\left(\theta + \frac{4\pi}{3}\right)} + \frac{k^2}{\cos \theta \cos\left(\theta + \frac{4\pi}{3}\right)} \\
&= k^2 \left\{ \frac{1}{\cos \theta \cos\left(\theta + \frac{2\pi}{3}\right)} + \frac{1}{\cos\left(\theta + \frac{2\pi}{3}\right) \cos\left(\theta + \frac{4\pi}{3}\right)} \right. \\
&\quad \left. + \frac{1}{\cos \theta \cos\left(\theta + \frac{4\pi}{3}\right)} \right\} \\
&= k^2 \left\{ \frac{\cos \theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right)}{\cos \theta \cos\left(\theta + \frac{2\pi}{3}\right) \cos\left(\theta + \frac{4\pi}{3}\right)} \right\}
\end{aligned}$$

$$\{\because \cos(A + B) = \cos A \cos B - \sin A \sin B\}$$

$$\begin{aligned}
&= k^2 \left\{ \frac{\cos \theta + \cos \theta \cos \frac{2\pi}{3} - \sin \theta \sin \frac{2\pi}{3} + \cos \theta \cos \frac{4\pi}{3} - \sin \theta \sin \frac{4\pi}{3}}{\cos \theta \cos\left(\theta + \frac{2\pi}{3}\right) \cos\left(\theta + \frac{4\pi}{3}\right)} \right\} \\
&= k^2 \left\{ \frac{\cos \theta + \cos \theta \cos \frac{2\pi}{3} - \sin \theta \sin \frac{2\pi}{3} + \cos \theta \cos \frac{4\pi}{3} - \sin \theta \sin \frac{4\pi}{3}}{\cos \theta \cos\left(\theta + \frac{2\pi}{3}\right) \cos\left(\theta + \frac{4\pi}{3}\right)} \right\} \\
&\because \frac{2\pi}{3} = \frac{2\pi}{3} \times \frac{180^\circ}{\pi} = 120^\circ \text{ \& } \frac{4\pi}{3} = \frac{4\pi}{3} \times \frac{180^\circ}{\pi} = 240^\circ \\
&= k^2 \left\{ \frac{\cos \theta + \cos \theta \cos 120^\circ - \sin \theta \sin 120^\circ + \cos \theta \cos 240^\circ - \sin \theta \sin 240^\circ}{\cos \theta \cos\left(\theta + \frac{2\pi}{3}\right) \cos\left(\theta + \frac{4\pi}{3}\right)} \right\} \\
&= k^2 \left\{ \frac{\cos \theta + \cos \theta \cos(180^\circ - 60^\circ) - \sin \theta \sin(180^\circ - 60^\circ) + \cos \theta \cos(180^\circ + 60^\circ) - \sin \theta \sin(180^\circ + 60^\circ)}{\cos \theta \cos\left(\theta + \frac{2\pi}{3}\right) \cos\left(\theta + \frac{4\pi}{3}\right)} \right\}
\end{aligned}$$

$$\{\sin(180^\circ - A) = \sin A ; \sin(180^\circ + A) = -\sin A ;$$

$$\cos(180^\circ - A) = -\cos A \text{ \& } \cos(180^\circ + A) = -\cos A \}$$

$$\begin{aligned}
&= k^2 \left\{ \frac{\cos \theta + \cos \theta (-\cos 60^\circ) - \sin \theta \sin 60^\circ + \cos \theta (-\cos 60^\circ) - \sin \theta (-\sin 60^\circ)}{\cos \theta \cos\left(\theta + \frac{2\pi}{3}\right) \cos\left(\theta + \frac{4\pi}{3}\right)} \right\} \\
&= k^2 \left\{ \frac{\cos \theta - \cos \theta \left(\frac{1}{2}\right) - \sin \theta \left(\frac{\sqrt{3}}{2}\right) - \cos \theta \left(\frac{1}{2}\right) + \sin \theta \left(\frac{\sqrt{3}}{2}\right)}{\cos \theta \cos\left(\theta + \frac{2\pi}{3}\right) \cos\left(\theta + \frac{4\pi}{3}\right)} \right\} \\
&= k^2 \left\{ \frac{\cos \theta - \cos \theta \left(\frac{2}{2}\right) - \frac{\sqrt{3}}{2} \sin \theta + \frac{\sqrt{3}}{2} \sin \theta}{\cos \theta \cos\left(\theta + \frac{2\pi}{3}\right) \cos\left(\theta + \frac{4\pi}{3}\right)} \right\} \\
&= k^2 \left\{ \frac{\cos \theta - \cos \theta - \frac{\sqrt{3}}{2} \sin \theta + \frac{\sqrt{3}}{2} \sin \theta}{\cos \theta \cos\left(\theta + \frac{2\pi}{3}\right) \cos\left(\theta + \frac{4\pi}{3}\right)} \right\} \\
&= k^2 \left\{ \frac{0}{\cos \theta \cos\left(\theta + \frac{2\pi}{3}\right) \cos\left(\theta + \frac{4\pi}{3}\right)} \right\}
\end{aligned}$$

$$= 0$$

$$= \text{R.H.S.}$$

Hence Proved

19. Question

If $m \sin \theta = n \sin(\theta + 2\alpha)$, prove that $\tan(\theta + \alpha) \cot \alpha = \frac{m+n}{m-n}$

Answer

Given, $m \sin \theta = n \sin(\theta + 2\alpha)$

$$\Rightarrow m \sin \theta = n \sin (\theta + 2\alpha)$$

$$\Rightarrow \frac{m}{n} = \frac{\sin (\theta + 2\alpha)}{\sin \theta}$$

Using Componendo - Dividendo:

$$\Rightarrow \frac{m+n}{m-n} = \frac{\sin (\theta + 2\alpha) + \sin \theta}{\sin (\theta + 2\alpha) - \sin \theta}$$

$$\left\{ \begin{array}{l} \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \end{array} \right\}$$

$$\Rightarrow \frac{m+n}{m-n} = \frac{2 \sin \frac{(\theta + 2\alpha) + \theta}{2} \cos \frac{(\theta + 2\alpha) - \theta}{2}}{2 \cos \frac{(\theta + 2\alpha) + \theta}{2} \sin \frac{(\theta + 2\alpha) - \theta}{2}}$$

$$\Rightarrow \frac{m+n}{m-n} = \frac{\sin \frac{(2\theta + 2\alpha)}{2} \cos \frac{\theta + 2\alpha - \theta}{2}}{\cos \frac{(2\theta + 2\alpha)}{2} \sin \frac{\theta + 2\alpha - \theta}{2}}$$

$$\Rightarrow \frac{m+n}{m-n} = \frac{\sin \frac{2(\theta + \alpha)}{2} \cos \frac{2\alpha}{2}}{\cos \frac{2(\theta + \alpha)}{2} \sin \frac{2\alpha}{2}}$$

$$\Rightarrow \frac{m+n}{m-n} = \frac{\sin(\theta + \alpha) \cos \alpha}{\cos(\theta + \alpha) \sin \alpha}$$

$$\Rightarrow \frac{m+n}{m-n} = \tan(\theta + \alpha) \cot \alpha$$

Hence Proved

Very Short Answer

1. Question

If $(\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 = \lambda \cos^2 \left(\frac{\alpha - \beta}{2} \right)$, write the value of λ .

Answer

$$(\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2$$

$$= (\cos^2 \alpha + 2\cos \alpha \cos \beta + \cos^2 \beta) + (\sin^2 \alpha + 2\sin \alpha \sin \beta + \sin^2 \beta)$$

Rearranging the terms,

$$= (\cos^2 \alpha + \sin^2 \alpha) + (\cos^2 \beta + \sin^2 \beta) + 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta)$$

$$= 1 + 1 + 2\cos (\alpha - \beta)$$

$$= 2\{1 + \cos (\alpha - \beta)\}$$

$$= 2\{2 \cos^2 \left(\frac{\alpha - \beta}{2} \right)\}$$

$$= 4\cos^2 \left(\frac{\alpha - \beta}{2} \right)$$

Comparing with question, $\lambda = 4$ (Ans)

2. Question

Write the value of $\sin \frac{\pi}{12} \sin \frac{5\pi}{12}$.

Answer

$$\sin \frac{5\pi}{12} \sin \frac{\pi}{12} = -\frac{1}{2} \left(-2 \sin \frac{5\pi}{12} \sin \frac{\pi}{12} \right)$$

$$= -\frac{1}{2} \left(\cos \left(\frac{5\pi}{12} + \frac{\pi}{12} \right) - \cos \left(\frac{5\pi}{12} - \frac{\pi}{12} \right) \right)$$

$$= -\frac{1}{2} \left(\cos \frac{\pi}{2} - \cos \frac{\pi}{3} \right)$$

$$= -\frac{1}{2} \left(0 - \frac{1}{2} \right)$$

$$= \frac{1}{4} \text{ (Ans)}$$

3. Question

If $\sin A + \sin B = \alpha$ and $\cos A + \cos B = \beta$, then write the value of $\tan\left(\frac{A+B}{2}\right)$.

Answer

$$\sin A + \sin B = \alpha$$

$$\Rightarrow 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) = \alpha \dots (I)$$

$$\cos A + \cos B = \beta$$

$$\Rightarrow 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) = \beta \dots (II)$$

Dividing equation (I) by equation (II), we get -

$$\frac{\sin\left(\frac{A+B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)} = \frac{\alpha}{\beta}$$

$$\text{or, } \tan\left(\frac{A+B}{2}\right) = \frac{\alpha}{\beta} \text{ (Ans)}$$

4. Question

If $\cos A = m \cos B$, then write the value of $\cot \frac{A+B}{2} \cot \frac{A-B}{2}$.

Answer

$$\begin{aligned} & \cot \frac{A+B}{2} \cot \frac{A-B}{2} \\ &= \frac{\cos \frac{A+B}{2} \cos \frac{A-B}{2}}{\sin \frac{A+B}{2} \sin \frac{A-B}{2}} \\ &= \frac{\cos \frac{A+B}{2} \cos \frac{A-B}{2}}{\sin \frac{A+B}{2} \sin \frac{A-B}{2}} \cdot \frac{2}{2} \\ &= \frac{2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}}{2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}} \\ &= \frac{\cos A + \cos B}{-(\cos A - \cos B)} \\ &= \frac{m \cos B + \cos B}{\cos B - m \cos B} \\ &= \frac{\cos B (1 + m)}{\cos B (1 - m)} \\ &= \frac{1 + m}{1 - m} \end{aligned}$$

$$\text{Ans: } \frac{1+m}{1-m}$$

5. Question

Write the value of the expression $\frac{1 - 4 \sin 10^\circ \sin 70^\circ}{2 \sin 10^\circ}$.

Answer

$$\begin{aligned} & \frac{1 - 4 \sin 10^\circ \sin 70^\circ}{2 \sin 10^\circ} \\ &= \frac{1}{2 \sin 10^\circ} - 2 \sin 70^\circ \\ &= \frac{1}{2 \sin 10^\circ} - 2 \cos(90 - 70)^\circ \\ &= \frac{1}{2 \sin 10^\circ} - 2 \cos 20^\circ \\ &= \frac{1}{2 \sin 10^\circ} - 2(1 - 2 \sin^2 10^\circ) \\ &= \frac{1}{2 \sin 10^\circ} - 2 + 4 \sin^2 10^\circ \\ &= \frac{8 \sin^3 10^\circ - 4 \sin 10^\circ + 1}{2 \sin 10^\circ} \end{aligned}$$

$$\begin{aligned}
&= \frac{8 \sin^3 10^\circ - 6 \sin 10^\circ + 2 \sin 10^\circ + 1}{2 \sin 10^\circ} \\
&= \frac{-2(3 \sin 10^\circ - 4 \sin^3 10^\circ) + 2 \sin 10^\circ + 1}{2 \sin 10^\circ} \\
&= \frac{-2(\sin(3 \times 10)^\circ) + 2 \sin 10^\circ + 1}{2 \sin 10^\circ} \\
&= \frac{-2 \sin 30^\circ + 2 \sin 10^\circ + 1}{2 \sin 10^\circ} \\
&= \frac{-2 \cdot \frac{1}{2} + 2 \sin 10^\circ + 1}{2 \sin 10^\circ} \\
&= \frac{-1 + 2 \sin 10^\circ + 1}{2 \sin 10^\circ} \\
&= \frac{2 \sin 10^\circ}{2 \sin 10^\circ} \\
&= 1 \text{ (Ans)}
\end{aligned}$$

6. Question

If $A + B = \frac{\pi}{3}$ and $\cos A + \cos B = 1$, then find the value of $\cos \frac{A-B}{2}$.

Answer

$$\cos A + \cos B = 1$$

$$\Rightarrow 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} = 1$$

$$\Rightarrow 2 \cos \frac{\frac{\pi}{3}}{2} \cos \frac{A-B}{2} = 1$$

$$\Rightarrow 2 \cos \frac{\pi}{6} \cos \frac{A-B}{2} = 1$$

$$\Rightarrow 2 \cdot \frac{\sqrt{3}}{2} \cos \frac{A-B}{2} = 1$$

$$\Rightarrow \sqrt{3} \cos \frac{A-B}{2} = 1$$

$$\Rightarrow \cos \frac{A-B}{2} = \frac{1}{\sqrt{3}} \text{ (Ans)}$$

7. Question

Write the value of $\sin \frac{\pi}{15} \sin \frac{4\pi}{15} \sin \frac{3\pi}{10}$.

Answer

$$\sin \frac{\pi}{15} \sin \frac{4\pi}{15} \sin \frac{3\pi}{10}$$

$$= -\frac{1}{2} \left(-2 \sin \frac{4\pi}{15} \sin \frac{\pi}{15} \right) \sin \frac{3\pi}{10}$$

$$= -\frac{1}{2} \left(\cos \left(\frac{4\pi}{15} + \frac{\pi}{15} \right) - \cos \left(\frac{4\pi}{15} - \frac{\pi}{15} \right) \right) \sin \frac{3\pi}{10}$$

$$= -\frac{1}{2} \left(\cos \frac{5\pi}{15} - \cos \frac{3\pi}{15} \right) \sin \frac{3\pi}{10}$$

$$= -\frac{1}{2} \left(\cos \frac{\pi}{3} - \cos \frac{\pi}{5} \right) \sin \frac{3\pi}{10}$$

$$= -\frac{1}{2} \left(\frac{1}{2} - \cos \frac{\pi}{5} \right) \sin \frac{3\pi}{10}$$

$$= -\frac{1}{4} \sin \frac{3\pi}{10} + \frac{1}{2} \sin \frac{3\pi}{10} \cos \frac{\pi}{5}$$

$$= -\frac{1}{4} \sin \frac{3\pi}{10} + \frac{1}{4} \left(2 \sin \frac{3\pi}{10} \cos \frac{\pi}{5} \right)$$

$$= -\frac{1}{4} \sin \frac{3\pi}{10} + \frac{1}{4} \left(\sin \left(\frac{3\pi}{10} + \frac{\pi}{5} \right) + \sin \left(\frac{3\pi}{10} - \frac{\pi}{5} \right) \right)$$

$$= -\frac{1}{4} \sin \frac{3\pi}{10} + \frac{1}{4} \left(\sin \frac{5\pi}{10} + \sin \frac{\pi}{10} \right)$$

$$= \frac{1}{4} \left(\sin \frac{\pi}{2} + \sin \frac{\pi}{10} - \sin \frac{3\pi}{10} \right)$$

$$= \frac{1}{4} \left(1 + \sin \frac{\pi}{10} - \sin \frac{3\pi}{10} \right) \dots (I)$$

Now,

$$5 \cdot \frac{\pi}{10} = \frac{\pi}{2}$$

$$\Rightarrow 2 \cdot \frac{\pi}{10} + 3 \cdot \frac{\pi}{10} = \frac{\pi}{2}$$

$$\Rightarrow 2 \cdot \frac{\pi}{10} = \frac{\pi}{2} - 3 \cdot \frac{\pi}{10}$$

$$\Rightarrow \sin \left(2 \cdot \frac{\pi}{10} \right) = \sin \left(\frac{\pi}{2} - 3 \cdot \frac{\pi}{10} \right)$$

$$\Rightarrow \sin \frac{2\pi}{10} = \cos \frac{3\pi}{10}$$

$$\Rightarrow 2 \sin \frac{\pi}{10} \cos \frac{\pi}{10} = 4 \cos^3 \frac{\pi}{10} - 3 \cos \frac{\pi}{10}$$

$$\Rightarrow 2 \sin \frac{\pi}{10} \cos \frac{\pi}{10} - 4 \cos^3 \frac{\pi}{10} + 3 \cos \frac{\pi}{10} = 0$$

$$\Rightarrow \cos \frac{\pi}{10} \left(2 \sin \frac{\pi}{10} - 4 \cos^2 \frac{\pi}{10} + 3 \right) = 0$$

$$\Rightarrow 2 \sin \frac{\pi}{10} - 4 \left(1 - \sin^2 \frac{\pi}{10} \right) + 3 = 0$$

$$\Rightarrow 2 \sin \frac{\pi}{10} - 4 + 4 \sin^2 \frac{\pi}{10} + 3 = 0$$

$$\Rightarrow 4 \sin^2 \frac{\pi}{10} + 2 \sin \frac{\pi}{10} - 1 = 0$$

$$\Rightarrow \sin \frac{\pi}{10} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 4 \cdot (-1)}}{2 \cdot 4}$$

$$\Rightarrow \sin \frac{\pi}{10} = \frac{-2 \pm \sqrt{4 + 16}}{8}$$

$$\Rightarrow \sin \frac{\pi}{10} = \frac{-2 \pm \sqrt{20}}{8}$$

$$\Rightarrow \sin \frac{\pi}{10} = \frac{-2 \pm 2\sqrt{5}}{8}$$

$$\Rightarrow \sin \frac{\pi}{10} = \frac{-1 \pm \sqrt{5}}{4}$$

Since, $\sin \frac{\pi}{10}$ is positive,

$$\Rightarrow \sin \frac{\pi}{10} = \frac{\sqrt{5} - 1}{4}$$

$$\text{Now, } \sin \frac{3\pi}{10} = \cos \left(\frac{\pi}{2} - \frac{3\pi}{10} \right) = \cos \frac{2\pi}{10}$$

$$\text{And, } \cos \frac{2\pi}{10} = 1 - 2 \sin^2 \frac{\pi}{10}$$

$$= 1 - 2 \left(\frac{\sqrt{5} - 1}{4} \right)^2$$

$$= 1 - 2 \left(\frac{5 - 2\sqrt{5} + 1}{16} \right)$$

$$= 1 - \frac{6 - 2\sqrt{5}}{8}$$

$$= \frac{8 - 6 + 2\sqrt{5}}{8}$$

$$= \frac{2 + 2\sqrt{5}}{8}$$

$$\text{or, } \sin \frac{3\pi}{10} = \cos \frac{2\pi}{10} = \frac{\sqrt{5} + 1}{4}$$

Putting these values in (I),

$$\frac{1}{4} \left(1 + \sin \frac{\pi}{10} - \sin \frac{3\pi}{10} \right)$$

$$\begin{aligned}
&= \frac{1}{4} \left(1 + \frac{\sqrt{5}-1}{4} - \frac{\sqrt{5}+1}{4} \right) \\
&= \frac{1}{4} \left(1 - \frac{2}{4} \right) \\
&= \frac{1}{4} \left(\frac{1}{2} \right) \\
&= \frac{1}{8} \text{ (Ans)}
\end{aligned}$$

8. Question

If $\sin 2A = \lambda \sin 2B$, then write the value of $\frac{\lambda+1}{\lambda-1}$.

Answer

$$\begin{aligned}
\frac{\lambda+1}{\lambda-1} &= \frac{\lambda+1}{\lambda-1} \cdot \frac{\sin 2B}{\sin 2B} \\
&= \frac{\lambda \sin 2B + \sin 2B}{\lambda \sin 2B - \sin 2B} \\
&= \frac{\sin 2A + \sin 2B}{\sin 2A - \sin 2B} \\
&= \frac{2 \sin \frac{2A+2B}{2} \cos \frac{2A-2B}{2}}{2 \cos \frac{2A+2B}{2} \sin \frac{2A-2B}{2}} \\
&= \frac{2 \sin(A+B) \cos(A-B)}{2 \cos(A+B) \sin(A-B)} \\
&= \frac{\sin(A+B)/\cos(A+B)}{\sin(A-B)/\cos(A-B)} \\
&= \frac{\tan(A+B)}{\tan(A-B)} \text{ (Ans)}
\end{aligned}$$

9. Question

Write the value of $\frac{\sin A + \sin 3A}{\cos A + \cos 3A}$.

Answer

$$\begin{aligned}
&\frac{\sin A + \sin 3A}{\cos A + \cos 3A} \\
&= \frac{2 \sin \frac{A+3A}{2} \cos \frac{A-3A}{2}}{2 \cos \frac{A+3A}{2} \cos \frac{A-3A}{2}} \\
&= \frac{\sin 2A \cos(-A)}{\cos 2A \cos(-A)} \\
&= \tan 2A \text{ (Ans)}
\end{aligned}$$

10. Question

If $\cos(A+B) \sin(C-D) = \cos(A-B) \sin(C+D)$, then write the value $\tan A \tan B \tan C$.

Answer

$$\cos(A+B) \sin(C-D) = \cos(A-B) \sin(C+D)$$

(To make the math easier, while expanding this out I'll use c A instead of cos A and s A instead of sin A)

$$\Rightarrow (c A c B - s A s B)(s C c D - c C s D) = (c A c B + s A s B)(s C c D + c C s D)$$

$$\Rightarrow c A c B s C c D - c A c B c C s D - s A s B s C c D + s A s B c C s D = c A c B s C c D + c A c B c C s D + s A s B s C c D + s A s B c C s D$$

$$c A c B c C s D + s A s B s C c D + s A s B c C s D$$

$$\Rightarrow 2s A s B s C c D = -2c A c B c C s D$$

$$\Rightarrow \sin A \sin B \sin C \cos D = -\cos A \cos B \cos C \sin D$$

$$\Rightarrow \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B} \cdot \frac{\sin C}{\cos C} = -\frac{\sin D}{\cos D}$$

$$\Rightarrow \tan A \tan B \tan C = -\tan D \text{ (Ans)}$$

MCQ

1. Question

Mark the Correct alternative in the following:

$$\cos 40^\circ + \cos 80^\circ + \cos 160^\circ + \cos 240^\circ =$$

- A. 0
- B. 1
- C. 1/2
- D. -1/2

Answer

$$\cos 40^\circ + \cos 80^\circ + \cos 160^\circ + \cos 240^\circ$$

$$= (\cos 40^\circ + \cos 80^\circ) + (\cos 160^\circ + \cos 240^\circ)$$

$$= 2\cos\frac{80^\circ+40^\circ}{2}\cos\frac{80^\circ-40^\circ}{2} + 2\cos\frac{240^\circ+160^\circ}{2}\cos\frac{240^\circ-160^\circ}{2}$$

$$= 2\cos 60^\circ \cos 20^\circ + 2\cos 200^\circ \cos 40^\circ$$

$$= 2 \cdot \frac{1}{2} \cdot \cos 20^\circ + 2\cos (180 + 20)^\circ \cos 40^\circ$$

$$= \cos 20^\circ - 2\cos 20^\circ \cos 40^\circ$$

$$= \cos 20^\circ - (\cos (40^\circ + 20^\circ) + \cos (40^\circ - 20^\circ))$$

$$= \cos 20^\circ - (\cos 60^\circ + \cos 20^\circ)$$

$$= -\cos 60^\circ$$

$$= -\frac{1}{2} \text{ (Ans)}$$

2. Question

Mark the Correct alternative in the following:

$$\sin 163^\circ \cos 347^\circ + \sin 73^\circ \sin 167^\circ =$$

- A. 0
- B. 1/2
- C. 1
- D. None of these

Answer

$$\sin 163^\circ \cos 347^\circ + \sin 73^\circ \sin 167^\circ$$

$$= \sin (180^\circ - 17^\circ) \cos (360^\circ - 13^\circ) + \sin (90^\circ - 17^\circ) \sin (180^\circ - 13^\circ)$$

$$= \sin 17^\circ \cos 13^\circ + \cos 17^\circ \sin 13^\circ$$

$$= \sin (17^\circ + 13^\circ)$$

$$= \sin 30^\circ$$

$$= \frac{1}{2} \text{ (Ans)}$$

3. Question

Mark the Correct alternative in the following:

$$\text{If } \sin 2\theta + \sin 2\phi = \frac{1}{2} \text{ and } \cos 2\theta + \cos 2\phi = \frac{3}{2}, \text{ then } \cos^2(\theta - \phi) =$$

- A. 3/8
- B. 5/8
- C. 3/4
- D. 5/4

Answer

$$\sin 2\theta + \sin 2\phi = \frac{1}{2}$$

$$\Rightarrow 2\sin(\theta + \phi) \cos(\theta - \phi) = \frac{1}{2} \dots \text{(I)}$$

$$\cos 2\theta + \cos 2\phi = \frac{3}{2}$$

$$\Rightarrow 2\cos(\theta + \phi) \cos(\theta - \phi) = \frac{3}{2} \dots \text{(II)}$$

Dividing equation (I) by equation (II) -

$$\tan(\theta + \phi) = \frac{1}{3}$$

$$\Rightarrow \tan^2(\theta + \phi) = \frac{1}{9}$$

$$\Rightarrow 1 + \tan^2(\theta + \phi) = 1 + \frac{1}{9} = \frac{10}{9}$$

$$\Rightarrow \sec^2(\theta + \phi) = \frac{10}{9}$$

$$\Rightarrow \cos^2(\theta + \phi) = \frac{9}{10}$$

$$\Rightarrow \cos(\theta + \phi) = \frac{3}{\sqrt{10}}$$

Substituting this value of $\cos(\theta + \phi)$ in (II), we get -

$$2 \cdot \frac{3}{\sqrt{10}} \cdot \cos(\theta - \phi) = \frac{3}{2}$$

$$\Rightarrow \cos(\theta - \phi) = \frac{\sqrt{10}}{4}$$

$$\Rightarrow \cos^2(\theta - \phi) = \frac{10}{16}$$

$$= \frac{5}{8} \text{ (Ans)}$$

4. Question

Mark the Correct alternative in the following:

The value of $\cos 52^\circ + \cos 68^\circ + \cos 172^\circ$ is

- A. 0
- B. 1
- C. 2
- D. $3/2$

Answer

$$\cos 52^\circ + \cos 68^\circ + \cos 172^\circ$$

$$= (\cos 52^\circ + \cos 68^\circ) + \cos 172^\circ$$

$$= 2\cos 60^\circ \cos 8^\circ + \cos (180^\circ - 8^\circ)$$

$$= 2 \cdot \frac{1}{2} \cdot \cos 8^\circ - \cos 8^\circ$$

$$= \cos 8^\circ - \cos 8^\circ$$

$$= 0 \text{ (Ans)}$$

5. Question

Mark the Correct alternative in the following:

The value of $\sin 78^\circ - \sin 66^\circ - \sin 42^\circ + \sin 6^\circ$ is

- A. $1/2$
- B. $-1/2$
- C. -1
- D. None of these

Answer

$$\sin 78^\circ - \sin 66^\circ - \sin 42^\circ + \sin 6^\circ$$

$$= (\sin 78^\circ - \sin 42^\circ) - (\sin 66^\circ - \sin 6^\circ)$$

$$= 2\cos 60^\circ \sin 18^\circ - 2\cos 36^\circ \sin 30^\circ$$

$$= 2 \cdot \frac{1}{2} \cdot \sin 18^\circ - 2 \cdot \frac{1}{2} \cdot \cos 36^\circ$$

$$= \sin 18^\circ - \cos 36^\circ$$

$$= \frac{\sqrt{5}-1}{4} - \frac{\sqrt{5}+1}{4}$$

$$= -1/2 \text{ (Ans)}$$

6. Question

Mark the Correct alternative in the following:

If $\sin \alpha + \sin \beta = a$ and $\cos \alpha - \cos \beta = b$, then $\tan \frac{\alpha - \beta}{2} =$

- A. $-\frac{a}{b}$
- B. $-\frac{b}{a}$
- C. $\sqrt{a^2 + b^2}$

D. None of these

Answer

$$\sin \alpha + \sin \beta = a$$

$$\Rightarrow 2 \sin \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2} = a \dots(I)$$

$$\cos \alpha - \cos \beta = b$$

$$\Rightarrow -2 \sin \frac{\alpha+\beta}{2} \sin \frac{\alpha-\beta}{2} = b \dots(II)$$

Dividing equation (II) by equation (I) -

$$-\tan \frac{\alpha-\beta}{2} = \frac{b}{a}$$

or,

$$\tan \frac{\alpha-\beta}{2} = -\frac{b}{a} \text{ (Ans)}$$

7. Question

Mark the Correct alternative in the following:

$$\cos 35^\circ + \cos 85^\circ + \cos 155^\circ =$$

A. 0

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{2}}$

D. $\cos 275^\circ$

Answer

$$\cos 35^\circ + \cos 85^\circ + \cos 155^\circ$$

$$= 2 \cos \frac{85^\circ+35^\circ}{2} \cos \frac{85^\circ-35^\circ}{2} + \cos 155^\circ$$

$$= 2 \cos 60^\circ \cos 25^\circ + \cos (180^\circ-25^\circ)$$

$$= 2 \cdot \frac{1}{2} \cdot \cos 25^\circ - \cos 25^\circ$$

$$= \cos 25^\circ - \cos 25^\circ$$

$$= 0 \text{ (Ans)}$$

8. Question

Mark the Correct alternative in the following:

The value of $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$ is equal to

A. 1

B. 0

C. $\frac{1}{2}$

D. 2

Answer

$$\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$$

$$= (\sin 50^\circ + \sin 10^\circ) - \sin 70^\circ$$

$$= 2 \sin 30^\circ \cos 20^\circ - \sin (90^\circ - 20^\circ)$$

$$= 2 \cdot \frac{1}{2} \cdot \cos 20^\circ - \cos 20^\circ$$

$$= \cos 20^\circ - \cos 20^\circ$$

$$= 0$$

9. Question

Mark the Correct alternative in the following:

$\sin 47^\circ + \sin 61^\circ - \sin 11^\circ - \sin 25^\circ$ is equal to

A. $\sin 36^\circ$

B. $\cos 36^\circ$

C. $\sin 7^\circ$

D. $\cos 7^\circ$

Answer

$$\sin 47^\circ + \sin 61^\circ - \sin 11^\circ - \sin 25^\circ$$

$$= (\sin 47^\circ - \sin 25^\circ) + (\sin 61^\circ - \sin 11^\circ)$$

$$= 2\cos 36^\circ \sin 11^\circ + 2\cos 36^\circ \sin 25^\circ$$

$$= 2\cos 36^\circ (\sin 11^\circ + \sin 25^\circ)$$

$$= 2\cos 36^\circ (2\sin 18^\circ \cos 7^\circ)$$

$$= 4\cos 36^\circ \sin 18^\circ \cos 7^\circ$$

$$= 4 \cdot \frac{\sqrt{5}+1}{4} \cdot \frac{\sqrt{5}-1}{4} \cos 7^\circ$$

$$= \frac{5-1}{4} \cos 7^\circ$$

$$= \cos 7^\circ$$

10. Question

Mark the Correct alternative in the following:

If $\cos A = m \cos B$, then $\cot \frac{A+B}{2} \cot \frac{B-A}{2} =$

A. $\frac{m-1}{m+1}$

B. $\frac{m+2}{m-2}$

C. $\frac{m+1}{m-1}$

D. None of these

Answer

$$\cot \frac{A+B}{2} \cot \frac{B-A}{2}$$

$$= \frac{\cos \frac{A+B}{2} \cos \frac{B-A}{2}}{\sin \frac{A+B}{2} \sin \frac{B-A}{2}}$$

$$= \frac{\cos \frac{A+B}{2} \cos \frac{B-A}{2}}{\sin \frac{A+B}{2} \sin \frac{B-A}{2}} \cdot \frac{2}{2}$$

$$= \frac{2 \cos \frac{A+B}{2} \cos \frac{B-A}{2}}{2 \sin \frac{A+B}{2} \sin \frac{B-A}{2}}$$

$$= \frac{\cos B + \cos A}{-(\cos B - \cos A)}$$

$$= \frac{\cos A + \cos B}{\cos A - \cos B}$$

$$= \frac{m \cos B + \cos B}{m \cos B - \cos B}$$

$$= \frac{\cos B (m+1)}{\cos B (m-1)}$$

$$= \frac{m+1}{m-1}$$

11. Question

Mark the Correct alternative in the following:

If A, B, C are in A.P., then $\frac{\sin A - \sin C}{\cos C - \cos A} =$

A. $\tan B$

B. $\cot B$

C. $\tan 2B$

D. None of these

Answer

Let common difference be d.

Then $A = B - d$ and $C = B + d$

So, $\frac{\sin A - \sin C}{\cos C - \cos A}$

$$= \frac{\sin(B-d) - \sin(B+d)}{\cos(B+d) - \cos(B-d)}$$

$$= \frac{2 \cos B \sin(-d)}{-2 \sin B \sin d}$$

$$= \frac{-2 \cos B \sin d}{-2 \sin B \sin d}$$

$$= \cot B \text{ (Ans)}$$

12. Question

Mark the Correct alternative in the following:

If $\sin(B + C - A)$, $\sin(C + A - B)$, $\sin(A + B - C)$ are in A.P., then $\cot A$, $\cot B$, $\cot C$ are in

- A. GP
- B. HP
- C. AP
- D. None of these

Answer

$\sin(B + C - A)$, $\sin(C + A - B)$, $\sin(A + B - C)$ are in A.P.

$$\Rightarrow 2\sin(C + A - B) = \sin(B + C - A) + \sin(A + B - C)$$

$$\Rightarrow 2\sin(C + A - B) = 2\sin\{(B + C - A + A + B - C)/2\} \cos\{(B + C - A - A - B + C)/2\}$$

$$\Rightarrow \sin\{(C + A) - B\} = \sin B \cos(C - A)$$

$$\Rightarrow \sin(C + A) \cos B - \cos(C + A) \sin B = \sin B \cos(C - A)$$

$$\Rightarrow \{\sin C \cos A + \cos C \sin A\} \cos B - \{\cos C \cos A - \sin C \sin A\} \sin B = \sin B \{\cos C \cos A + \sin C \sin A\}$$

$$\Rightarrow \cos A \cos B \sin C + \sin A \cos B \cos C - \cos A \sin B \cos C + \sin A \sin B \sin C = \cos A \sin B \cos C + \sin A \sin B \sin C$$

$$\Rightarrow \cos A \cos B \sin C + \sin A \cos B \cos C = 2\cos A \sin B \cos C$$

$$\Rightarrow \cos B (\cos A \sin C + \sin A \cos C) = 2\cos A \sin B \cos C$$

$$\Rightarrow \cot B = \frac{2 \cos A \cos C}{\cos A \sin C + \sin A \cos C}$$

$$\Rightarrow \cot B = \frac{\frac{2 \cos A \cos C}{\sin A \sin C}}{\frac{\cos A \sin C + \sin A \cos C}{\sin A \sin C}}$$

$$\Rightarrow \cot B = \frac{2 \cot A \cot C}{\cot A + \cot C}$$

$\therefore \cot A$, $\cot B$, $\cot C$ are in H.P. (Ans)

13. Question

Mark the Correct alternative in the following:

If $\sin x + \sin y = \sqrt{3}(\cos y - \cos x)$, then $\sin 3x + \sin 3y =$

- A. $2 \sin 3x$
- B. 0
- C. 1
- D. None of these

Answer

$$\sin x + \sin y = \sqrt{3}(\cos y - \cos x)$$

$$\Rightarrow 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2} = \sqrt{3} \left(-2 \sin \frac{y+x}{2} \sin \frac{y-x}{2} \right)$$

$$\Rightarrow \sin \frac{x+y}{2} \left\{ \cos \frac{x-y}{2} - \sqrt{3} \sin \frac{x-y}{2} \right\} = 0$$

$$\Rightarrow \sin \frac{x+y}{2} = 0 \text{ or } \cos \frac{x-y}{2} = \sqrt{3} \sin \frac{x-y}{2}$$

$$\Rightarrow \frac{x+y}{2} = 0 \text{ or } \tan \frac{x-y}{2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x + y = 0 \text{ or } \frac{x-y}{2} = \frac{\pi}{6}$$

$$\Rightarrow x = -y$$

$$\text{or } x - y = \frac{\pi}{3}$$

$$\Rightarrow x = -y$$

$$\text{or } x = y + \frac{\pi}{3}$$

Putting these values of x in $\sin 3x + \sin 3y$,

For $x = -y$

$$\sin 3x + \sin 3y$$

$$= \sin 3(-y) + \sin 3y$$

$$= -\sin 3y + \sin 3y$$

$$= 0$$

$$\text{For } x = y + \frac{\pi}{3}$$

$$\sin 3x + \sin 3y$$

$$= \sin 3\left(y + \frac{\pi}{3}\right) + \sin 3y$$

$$= \sin (3y + \pi) + \sin 3y$$

$$= -\sin 3y + \sin 3y$$

$$= 0$$

So, $\sin 3x + \sin 3y = 0$ (Ans)

14. Question

Mark the Correct alternative in the following:

If $\tan \alpha = \frac{x}{x+1}$ and $\tan \beta = \frac{1}{2x+1}$, then $\alpha + \beta$ is equal to

A. $\pi/2$

B. $\pi/3$

C. $\pi/6$

D. $\pi/4$

Answer

$$\tan \alpha = \frac{x}{x+1} \text{ and } \tan \beta = \frac{1}{2x+1}$$

$$\tan (\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\Rightarrow \tan (\alpha + \beta) = \frac{\frac{x}{x+1} + \frac{1}{2x+1}}{1 - \frac{x}{x+1} \cdot \frac{1}{2x+1}}$$

$$\Rightarrow \tan (\alpha + \beta) = \frac{2x^2 + x + x + 1 / (x+1)(2x+1)}{2x^2 + 3x + 1 - x / (x+1)(2x+1)}$$

$$\Rightarrow \tan (\alpha + \beta) = \frac{2x^2 + 2x + 1}{2x^2 + 2x + 1}$$

$$\Rightarrow \tan (\alpha + \beta) = 1$$

$$= \tan \pi/4$$

$$\Rightarrow \alpha + \beta = \frac{\pi}{4} \text{ (Ans)}$$