4. Cubes and Cube-Roots

(Including use of tables for natural numbers)

EXERCISE 4(A)

Question 1.

Find the cube of:

- (i) 7
- (ii) 11
- (iii) 16
- (iv) 23
- (v) 31
- (vi) 42
- (vii) 54

Solution:

- (i) $(7)^3 = 7 \times 7 \times 7 = 343$
- (ii) $(11)^3 = 11 \times 11 \times 11 = 1331$
- (iii) $(16)^3 = 16 \times 16 \times 16 = 4096$
- (iv) $(23)^3 = 23 \times 23 \times 23 = 12167$
- (v) $(31)^3 = 31 \times 31 \times 31 = 29791$
- (vi) $(42)^3 = 42 \times 42 \times 42 = 74088$
- (vii) $(54)^3 = 54 \times 54 \times 54 = 157464$

Question 2.

Find which of the following are perfect cubes:

- (i) 243
- (ii) 588
- (iii) 1331
- (iv) 24000
- (v) 1728
- (vi) 1938

$$(i)$$
 243

$$\therefore 243 = 3 \times 3 \times 3 \times 3$$

$$= (3 \times 3 \times 3) \times 3$$

$$= 3^3 \times 3$$

:. 297 is not a perfect cube.

(ii) 588

$$588 = 2 \times 2 \times 7 \times 7 \times 3$$

.. 588 is not a perfect cube.

(iii) 1331

$$1331 = 11 \times 11 \times 11 = (11)^3$$

:. 1331 is a perfect cube.

$$24000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 5$$

$$= (2)^3 \times (2)^3 \times (5)^3 \times 3$$

(v) 1728

$$: 1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$$
$$= (2)^{3} \times (2)^{3} \times (3)^{3}$$

:. 1728 is a perfect cube.

(vi) 1938

| 2 | 1938 |
|----|------|
| 3 | 936 |
| 17 | 323 |
| 19 | 19 |
| | 1 |

 $1938 = 2 \times 3 \times 17 \times 19$ 1938 is not a perfect cube.

Question 3.

Find the cubes of:

- (i) 2.1
- (ii) 0.4
- (iii) 1.6
- (iv) 2.5
- (v) 0.12
- (vi) 0.02
- (vii) 0.8

(i)
$$2.1 = (2.1)^3 = \left(\frac{21}{10}\right)^3 = \frac{21 \times 21 \times 21}{10 \times 10 \times 10}$$
$$= \frac{9261}{1000} = 9.261$$

(ii)
$$0.4 = (0.4)^3 = \left(\frac{4}{10}\right)^3 = \frac{4 \times 4 \times 4}{10 \times 10 \times 10}$$
$$= \frac{64}{1000} = 0.064$$

(iii)
$$1.6 = (1.6)^3 = \left(\frac{16}{10}\right)^3 = \frac{16 \times 16 \times 16}{10 \times 10 \times 10}$$
$$= \frac{4096}{1000} = 4.096$$

(iv)
$$2.5 = (2.5)^3 = \left(\frac{25}{10}\right)^3 = \frac{25 \times 25 \times 25}{10 \times 10 \times 10}$$
$$= \frac{15625}{1000} = 15.625$$

(v)
$$0.12 = (0.12)^3 = \left(\frac{12}{100}\right)^3 = \frac{12 \times 12 \times 12}{100 \times 100 \times 100}$$
$$= \frac{1728}{1000000} = 0.001728$$

(vi)
$$0.02 = (0.02)^3 = \left(\frac{2}{100}\right)^3 = \frac{2 \times 2 \times 2}{100 \times 100 \times 100}$$
$$= \frac{8}{1000000} = 0.000008$$

(vii)
$$0.8 = (0.8)^3 = \left(\frac{8}{10}\right)^3 = \frac{8 \times 8 \times 8}{10 \times 10 \times 10}$$
$$= \frac{512}{1000} = 0.512$$

Question 4.

Find the cubes of:

- (i) $\frac{3}{7}$
- (ii) $\frac{8}{9}$
- (iii) $\frac{10}{13}$
- (iv) $1\frac{2}{7}$ (v) $2\frac{1}{2}$ Solution:

(i)
$$\frac{3}{7} = \left(\frac{3}{7}\right)^3 = \frac{3 \times 3 \times 3}{7 \times 7 \times 7} = \frac{27}{343}$$

(ii)
$$\frac{8}{9} = \left(\frac{8}{9}\right)^3 = \frac{8 \times 8 \times 8}{9 \times 9 \times 9} = \frac{512}{729}$$

(iii)
$$\frac{10}{13} = \left(\frac{10}{13}\right)^3 = \frac{10 \times 10 \times 10}{13 \times 13 \times 13} = \frac{1000}{2197}$$

(iv)
$$1\frac{2}{7} = \left(1\frac{2}{7}\right)^3 = \left(\frac{1\times7+2}{7}\right)^3 = \left(\frac{9}{7}\right)^3$$
$$= \frac{9\times9\times9}{7\times7\times7} = \frac{729}{343} = 2\frac{43}{343}$$

$$(v) \ 2\frac{1}{2} = \left(2\frac{1}{2}\right)^3 = \left(\frac{5}{2}\right)^3$$
$$= \frac{5 \times 5 \times 5}{2 \times 2 \times 2} = \frac{125}{8} = 15\frac{5}{8}.$$

Question 5.

Find the cubes of:

(i) -3

$$(v) -25$$

Solution:

(i)
$$-3 = (-3)^3 = -3 \times -3 \times -3$$

= $-(3 \times 3 \times 3) = -27$

(ii)
$$-7 = (-7)^3 = -7 \times -7 \times -7$$

= $-(7 \times 7 \times 7) = -343$

(iii)
$$-12 = (-12)^3 = -12 \times -12 \times -12$$

= $-(12 \times 12 \times 12) = -1728$

$$(iv)$$
 -18 = $(-18)^3$ = -18 × -18 × -18
= -(18 × 18 × 18) = -5832

$$(v)$$
 $-25 = (-25)^3 = -25 \times -25 \times -25$
= $-(25 \times 25 \times 25) = -15625$

$$(vi)$$
 $-30 = (-30)^3 = -30 \times -30 \times -30$
= $-(30 \times 30 \times 30) = -27000$

$$(vii)$$
 -50 = $(-50)^3$ = -50 × -50 × -50
= -(50 × 50 × 50) = -125000

Question 6.

Which of the following are cubes of:

- (i) an even number
- (ii) an odd number

216, 729, 3375, 8000, 125, 343, 4096 and 9261.

$$\therefore 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

$$= (2)^3 \times (3)^3 = (6)^3$$

$$\because 729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$= (3)^3 \times (3)^3 = (9)^3$$

$$\therefore 3375 = 5 \times 5 \times 5 \times 3 \times 3 \times 3$$

$$= (5)^{3} \times (3)^{3} = (15)^{3}$$

$$\therefore 8000 = 20 \times 20 \times 20 = (20)^{3}$$

$$\frac{20 \mid 8000}{20 \mid 400}$$

$$\frac{20 \mid 20}{11}$$

$$125 = 5 \times 5 \times 5 = (5)^{3}$$

$$\frac{5 \mid 125}{5 \mid 25}$$

$$\frac{5 \mid 5}{11}$$

$$\therefore 343 = 7 \times 7 \times 7 = (7)^{3}$$

7 343

$$= (2)^3 \times (2)^3 \times (2)^3 \times (2)^3 = (16)^3$$

- (i) Cubes of an even number are 216, 8000, 4096.
- (ii) Cubes of an odd number are 729, 3375, 125, 343, 9261.

Question 7.

Find the least number by which 1323 must be multiplied so that the product is a perfect

cube.

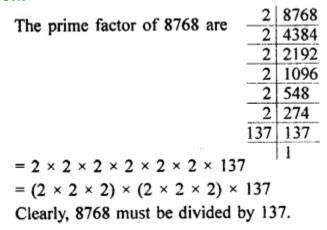
Solution:

The prime factor of 1323 are =3 x 3 x 3 x 7 x 7 = $(3 \times 3 \times 3) \times 7 \times 7$ Clearly, 1323 must be multiplied by 7.

Question 8.

Find the smallest number by which 8768 must be divided so that the quotient is a perfect cube.

Solution:



Question 9.

Find the smallest number by which 27783 be multiplied to get a perfect square number. **Solution:**

3 27783
3 9261
3 3087
3 1029
7 343
7 49
7 7
1
=
$$3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$$

= $(3 \times 3 \times 3) \times (7 \times 7 \times 7) \times 3$
Clearly, 27783 must be multiplied by 3×3
= 9

Question 10.

With what least number must 8640 be divided so that the quotient is a perfect cube? **Solution:**

The prime factors of 8640 are

Clearly, 8640 must be divided by 5.

Question 11.

Which is the smallest number that must be multiplied to 77175 to make it a perfect cube?

Solution:

× 5

The prime factors of 77175 are

| 5 |
|-----|
| 5 |
| |
| |
| |
| |
| |
| 10. |
| |

$$= 3 \times 3 \times 5 \times 5 \times 7 \times 7 \times 7$$
$$= (7 \times 7 \times 7) \times 3 \times 3 \times 5 \times 5$$

Clearly, 77175 must be multiplied by 3 \times 5

= 15

EXERCISE 4(B)

Question 1.

Find the cube-roots of:

- (i) 64
- (ii) 343
- (iii) 729
- (iv) 1728
- (v) 9261
- (vi) 4096
- (vii) 8000
- (viii) 3375

(i)
$$64 = \sqrt[3]{64} = (2 \times 2 \times 2) \times (2 \times 2 \times 2)$$

= $2 \times 2 = 4$

(ii)
$$343 = \sqrt[3]{343} = 7 \times 7 \times 7 = 7$$

(iii)
$$729 = \sqrt[3]{729} = (3 \times 3 \times 3) \times (3 \times 3 \times 3)$$

= $3 \times 3 = 9$

(iv)
$$1728 = \sqrt[3]{1728} = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3) = 2 \times 2 \times 3 = 12$$

| 2 | 1728 |
|---|------|
| 2 | 864 |
| 2 | 432 |
| 2 | 216 |
| 2 | 108 |
| 2 | 54 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

(v)
$$9261 = \sqrt[3]{9261} = (3 \times 3 \times 3) \times (7 \times 7 \times 7)$$

= $3 \times 7 = 21$

(vi)
$$4096 = \sqrt[3]{4096} = (2 \times 2 \times 2) \times (2$$

(vii)
$$8000 = \sqrt[3]{8000} = (4 \times 4 \times 4) \times (5 \times 5 \times 5)$$

= $4 \times 5 = 20$

| 4 | 8000 |
|---|------|
| 4 | 2000 |
| 4 | 500 |
| 5 | 125 |
| 5 | 25 |
| 5 | 5 |
| | 1 |

(viii)
$$3375 = \sqrt[3]{3375} = (5 \times 5 \times 5) \times (3 \times 3 \times 3)$$

= $5 \times 3 = 15$

Question 2.

Find the cube-roots of:

- (i) $\frac{27}{64}$ (ii) $\frac{125}{216}$ (iii) $\frac{343}{512}$
- (iv) 64 x 729
- (v) 64 x 27
- (vi) 729 x 8000
- (vii) 3375 x 512

Solution:

(i)
$$\frac{27}{64} = \sqrt[3]{\frac{27}{64}} = \frac{\sqrt{3 \times 3 \times 3}}{\sqrt{4 \times 4 \times 4}} = \frac{3}{4}$$

(ii)
$$\frac{125}{216} = \sqrt[3]{\frac{125}{216}} = \frac{\sqrt{5 \times 5 \times 5}}{\sqrt{6 \times 6 \times 6}} = \frac{5}{6}$$

(iii)
$$\frac{343}{512} = \sqrt[3]{\frac{343}{512}} = \frac{\sqrt{7 \times 7 \times 7}}{\sqrt{8 \times 8 \times 8}} = \frac{7}{8}$$

(iv)
$$64 \times 729 = \sqrt[3]{64 \times 729}$$

= $\sqrt{4 \times 4 \times 4 \times 9 \times 9 \times 9} = 4 \times 9 = 36$

(v)
$$64 \times 27 = \sqrt[3]{64 \times 27}$$

= $\sqrt{4 \times 4 \times 4 \times 3 \times 3 \times 3} = 4 \times 3 = 12$

(vi)
$$729 \times 8000 = \sqrt[3]{729 \times 8000}$$

= $\sqrt{9 \times 9 \times 9 \times 20 \times 20 \times 20}$
= $9 \times 20 = 180$

(vii)
$$3375 \times 512 = \sqrt[3]{3375 \times 512}$$

= $\sqrt{15 \times 15 \times 15 \times 8 \times 8 \times 8}$
= $15 \times 8 = 120$

Question 3.

Find the cube-roots of:

- (i) -216
- (ii) -512
- (iii) -1331
- $(iv)^{\frac{-27}{125}}$
- $(v)^{\frac{-64}{343}}$
- (vi) $\frac{-512}{343}$
- (vii) -2197
- (vii) -2137
- (viii) -5832
- (ix) -2744000

(i)
$$-216 = \sqrt[3]{-216} = \sqrt{-6 \times -6 \times -6} = -6$$

(ii)
$$-512 = \sqrt[3]{-512} = \sqrt{-8 \times -8 \times -8} = -8$$

(iii)
$$-1331 = \sqrt[3]{-1331}$$

= $\sqrt{-11 \times -11 \times -11} = -11$

(iv)
$$-\frac{27}{125} = -\frac{\sqrt{27}}{\sqrt{125}} = -\sqrt{\frac{3\times3\times3}{5\times5\times5}} = -\frac{3}{5}$$

$$(v) \frac{-64}{343} = \frac{\sqrt[3]{-64}}{\sqrt[3]{343}} = \frac{\sqrt[3]{-4 \times -4 \times -4}}{\sqrt[3]{7 \times 7 \times 7}} = \frac{-4}{7}$$

$$(vi) \quad -\frac{512}{343} = -\sqrt[3]{\frac{512}{343}} = -\sqrt[3]{\frac{8 \times 8 \times 8}{7 \times 7 \times 7}} = -\frac{8}{7}$$

$$(vii) -2197 = \sqrt[3]{-2197}$$

$$= \sqrt[3]{-13 \times -13 \times -13} = -13$$

$$(viii) - 5832 = \sqrt[3]{-5832}$$

$$(ix) -2744000 = \sqrt[3]{-2744000}$$

$$= \sqrt{-2 \times -2 \times -7 \times -7 \times -7} \\ \times -10 \times -10 \times -10$$
$$= -2 \times -7 \times -10 = -140$$

Question 4.

Find the cube-roots of:

- (i) 2.744
- (ii) 9.261
- (iii) 0.000027
- (iv) -0.512
- (v) -15.625
- (vi) -125 x 1000

(i)
$$2.744 = \sqrt[3]{\frac{2744}{1000}}$$

$$= \sqrt[3]{\frac{2 \times 2 \times 2 \times 7 \times 7 \times 7}{10 \times 10 \times 10}}$$
$$= \frac{2 \times 7}{10} = \frac{14}{10} = 1.4$$

(ii) 9.261 =
$$\sqrt[3]{\frac{9261}{1000}}$$
 = $\sqrt{\frac{3 \times 3 \times 3 \times 7 \times 7 \times 7}{10 \times 10 \times 10}}$

$$=\frac{3\times7}{10}=\frac{21}{10}=2.1$$

(iii)
$$0.000027 = \sqrt[3]{\frac{27}{1000000}}$$

$$= \sqrt[3]{\frac{3 \times 3 \times 3}{100 \times 100 \times 100}} = \frac{3}{100} = 0.03$$

$$(iv) -0.512 = \sqrt[3]{\frac{-512}{1000}} = \sqrt{\frac{-8 \times -8 \times -8}{10 \times 10 \times 10}}$$
$$= \frac{-8}{10} = -0.8$$

$$(v) -15.625 = \sqrt[3]{\frac{-15625}{1000}}$$

$$\sqrt{\frac{-(5\times5\times5)\times(5\times5\times5)}{10\times10\times10}}$$

$$=\frac{-5\times5}{10}=\frac{-25}{10}=-2.5$$

$$(vi) -125 \times 1000 = \sqrt{-125 \times 100}$$
$$= \sqrt{-(5 \times 5 \times 5) \times (10 \times 10 \times 10)}$$
$$= -5 \times 10 = -50$$

Question 5.

Find the smallest number by which 26244 may be divided so that the quotient is a perfect cube.

Solution:

The prime factors of 26244 are

Clearly, 26244 must be divided by $3 \times 3 \times 2 \times 2 = 36$

Question 6.

What is the least number by which 30375 should be multiplied to get a perfect cube? **Solution:**

The prime factors of 30375 are

$$= 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$$
$$= (3 \times 3 \times 3) \times (5 \times 5 \times 5) \times 3 \times 3$$

Clearly, 30375 must be multiplied with 3

Question 7.

Find the cube-roots of:

- (i) 700 x 2 x 49 x 5
- (ii) -216 x 1728
- (iii) -64 x -125
- (iv) $\frac{-27}{343}$
- $(v) \frac{729}{-1331}$
- (vi) 250.047

(vii) -175616

(i)
$$700 \times 2 \times 49 \times 5$$

$$=2\times2\times5\times5\times7\times2\times7\times7\times5$$

$$= (2 \times 2 \times 2) \times (5 \times 5 \times 5) \times (7 \times 7 \times 7)$$

$$= 2 \times 5 \times 10 = 70$$

$$(ii)$$
 -216 × 1728

$$2 \times 2 \times 2 \times 3 \times 3 \times 3$$

$$= -2 \times 3 \times 2 \times 2 \times 3 = -72$$

(iii)
$$-64 \times -125$$

$$= -(4 \times 4 \times 4) \times -(5 \times 5 \times 5)$$

$$= -4 \times -5 = 20$$

$$(iv)$$
 $-\frac{27}{343} = \frac{3 \times 3 \times 3}{7 \times 7 \times 7} = -\frac{3}{7}$

$$(v) \ \frac{729}{-1331} = \frac{(9 \times 9 \times 9)}{-(11 \times 11 \times 11)} = -\frac{9}{11}$$

$$(vi) 250.047 = \frac{250047}{1000}$$

$$= \frac{(3 \times 3 \times 3) \times (3 \times 3 \times 3) \times (7 \times 7 \times 7)}{(10 \times 10 \times 10)}$$
$$= \frac{3 \times 3 \times 7}{10} = \frac{63}{10} = 6.3$$

$$= -[(2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (7 \times 7 \times 7)]$$

$$= (2 \times 2 \times 2 \times 7) = 56$$

$$= -[2 \times 2 \times 2 \times 7] = -56$$