

11. Algebraic Expressions

EXERCISE 11(A)

Question 1.

Separate the constants and variables from the following :

$$-7, 7+x, 7x+yz, \sqrt{5}, \sqrt{xy}, \frac{3yz}{8}, 4.5y-3x,$$

$$8-5, 8-5x, 8x-5y \times p \text{ and } 3y^2z \div 4x$$

Solution:

Clearly constants are : $-7, \sqrt{5}, 8-5$

Variable are : $7+x, 7x+yz, \sqrt{xy}, \frac{3yz}{8}, 4.5y$

$$-3x$$

$$8-5x, 8x-5y \times p \text{ and } 3y^2z \div 4x$$

Question 2.

Write the number of terms in each of the following polynomials.

(i) $5x^2 + 3 \times ax$

(ii) $ax \div 4 - 7$

(iii) $ax - by + y \times z$

(iv) $23 + a \times b \div 2$.

Solution:

(i) $5x^2 + 3 \times ax = 5x^2 + 3ax$

$\therefore$ The number of terms in this polynomial
= 2

(ii) $ax \div 4 - 7 = \frac{ax}{4} - 7$

$\therefore$ The number of terms in this polynomial
= 2

(iii) $ax - by + y \times z = ax - by + yz$

$\therefore$ The number of terms in this polynomial
= 3

(iv) $23 + a \times b \div 2 = 23 + \frac{ab}{2}$

$\therefore$ The number of terms in this Polynomial
= 2

Question 3.

Separate monomials, binomials, trinomials and polynomials from the following algebraic expressions :

$$8 - 3x, xy^2, 3y^2 - 5y + 8, 9x - 3x^2 + 15x^3 - 7, \\ 3x \times 5y, 3x \div 5y, 2y \div 7 + 3x - 7 \text{ and } 4 - ax^2 + bx + y$$

Solution:

Monomials are : $xy^2, 3x \times 5y, 3x \div 5y$;

Binomials are : $8 - 3x$

Trinomials are : $3y^2 - 5y + 8, 2y \div 7 + 3x - 7$

Polynomials are : $8 - 3x, 3y^2 - 5y + 8, 9x - 3x^2 + 15x^3 - 7, 2y \div 7 + 3x - 7, 4 - ax^2 + bx + y$

Question 4.

Write the degree of each polynomial given below :

$$\begin{array}{ll} \text{(i) } xy + 7z & \text{(ii) } x^2 - 6x^3 + 8 \\ \text{(iii) } y - 6y^2 + 5y^8 & \text{(iv) } xyz - 3 \\ \text{(v) } xy + yz^2 - zx^3 & \text{(vi) } x^5y^7 - 8x^3y^8 + 10x^4y^4z^4 \end{array}$$

Solution:

- (i) degree = 2 (Polynomial is $xy + 7z$)
- (ii) degree = 3 (Polynomial is $x^2 - 6x^3 + 8y$)
- (iii) degree = 8 (Polynomial is $y - 6y^2 + 5y^8$)
- (iv) degree = 3 (Polynomial is $xyz - 3$)
- (v) degree = 4 (Polynomial is $xy + yz^2 - xz^3$)
- (vi) degree = 12 (Polynomial is $x^5y^7 - 8x^3y^8 + 10x^4y^4z^4$)

Question 5.

Write the coefficient of :

- (i) ab in $7abx$,
- (ii) $7a$ in $7abx$;
- (iii) $5x^2$ in $5x^2 - 5x$;
- (iv) 8 in $a^2 - 8ax + a$;
- (v) $4xy$ in $x^2 - 4xy + y^2$.

Solution:

- (i) The coefficient of ab in $7abx = 7x$
- (ii) The coefficient of $7a$ in $7abx = bx$
- (iii) The coefficient of $5x^2$ in $5x^2 - 5x = 1$
- (iv) The coefficient of 8 in $a^2 - 8ax + a = -ax$
- (v) The coefficient of $4xy$ in $x^2 - 4xy + y^2 = -1$

Question 6.

In $\frac{5}{7}xy^2z^3$, write the coefficient of

- | | | |
|---------------|------------------------|-------------|
| (i) 5 | (ii) $\frac{5}{7}$ | (iii) $5x$ |
| (iv) xy^2 | (v) z^3 | (vi) xz^3 |
| (vii) $5xy^2$ | (viii) $\frac{1}{7}yz$ | (ix) z |
| (x) yz^2 | (xi) $5xyz$ | |

Solution:

In $\frac{5}{7}xy^2z^3$, Co-efficient of

- | | |
|-----------------------------------|------------------------------------|
| (i) 5 is $\frac{1}{7}xy^2z^3$ | (ii) $\frac{5}{7}$ is xy^2z^3 |
| (iii) $5x$ is $\frac{1}{7}y^2z^3$ | (iv) xy^2 is $\frac{5}{7}z^3$ |
| (v) z^3 is $\frac{5}{7}xy^2$ | (vi) xz^3 is $\frac{5}{7}y^2$ |
| (vii) $5xy^2$ is $\frac{1}{7}z^3$ | (viii) $\frac{1}{7}yz$ is $5xyz^2$ |
| (ix) z is $\frac{5}{7}xy^2z^2$ | (x) yz^2 is $\frac{5}{7}xy - z$ |
| (xi) $5xyz$ is $\frac{1}{7}yz^2$ | |

Question 7.

In each polynomial, given below, separate the like terms :

- (i) $3xy, -4yx^2, 2xy^2, 2.5x^2y, -8yx, -3.2y^2x$ and x^2y
- (ii) $y^2z^3, xy^2z^3, -5x^2yz, -4y^2z^3, -8xz^3y^2, 3x^2yz$ and $2z^3y^2$

Solution:

- (i) Like terms are
 $3xy, -8yx : -4yx^2, 2.5x^2y$ and $x^2y ; 2xy^2$ and $-3.2y^2x$
(ii) $y^2z^3, -x^2z^3$ and $2z^3y^2 ; xy^2z^3$ and $-8xz^3y^2 ; -5x^2yz$ and $3x^2yz$

EXERCISE 11(B)

Question 1.

Evaluate :

- (i) $-7x^2 + 18x^2 + 3x^2 - 5x^2$
(ii) $b^2y - 9b^2y + 2b^2y - 5b^2y$
(iii) $abx - 15abx - 10abx + 32abx$
(iv) $7x - 9y + 3 - 3x - 5y + 8$
(v) $3x^2 + 5xy - 4y^2 + x^2 - 8xy - 5y^2$

Solution:

- (i) $-7x^2 + 18x^2 + 3x^2 - 5x^2$
 $= 21x^2 - 12x^2$
 $= 9x^2$
(ii) $b^2y - 9b^2y + 2b^2y - 5b^2y$
 $= 3b^2y - 14b^2y$
 $= -11b^2y$
(iii) $abx - 15abx - 10abx + 32abx$
 $= 33abx - 25abx$
 $= 8abx$
(iv) $7x - 9y + 3 - 3x - 5y + 8$
 $= 7x - 3x - 9y - 5y + 3 + 8$
 $= 4x - 14y + 11$
(v) $3x^2 + 5xy - 4y^2 - 8xy - 5y^2$
 $= 3x^2 + 5xy - 8xy - 4y^2 - 5y^2$
 $= 3x^2 - 3xy - 9y^2$

Question 2.

Add :

- (i) $5a + 3b, a - 2b, 3a + 5b$
- (ii) $8x - 3y + 7z, -4x + 5y - 4z, -x - y - 2z$
- (iii) $3b - 7c + 10, 5c - 2b - 15, 15 + 12c + b$
- (iv) $a - 3b + 3 ; 2a + 5 - 3c ; 6c - 15 + 6b$
- (v) $13ab - 9cd - xy ; 5xy ; 15cd - 7ab ; 6xy - 3cd$
- (vi) $x^3 - x^2y + 5xy^2 + y^3 ; -x^3 - 9xy^2 + y^3 ; 3x^2y + 9xy^2$
- (vii) $a^6 - 4a^4 + 6a ; 5a^6 + 5a^4 + 6a ; 12a^6 - 10a$
- (viii) $2ax - 6by + 4cz, 4by - 14ax, 9cz - 4ax - 6by$

Solution:

$$\begin{array}{r}
 (i) \quad 5a + 3b \\
 a - 2b \\
 3a + 5b \\
 \hline
 9a + 6b
 \end{array}$$

$$\begin{array}{r}
 (ii) \quad 8x - 3y + 7z \\
 -4x + 5y - 4z \\
 -x - y - 2z \\
 \hline
 3x + y + z
 \end{array}$$

$$\begin{array}{r}
 (iii) \quad 3b - 7c + 10 \\
 -2b + 5c - 15 \\
 +b + 12c + 15 \\
 \hline
 2b + 10c + 10
 \end{array}$$

$$\begin{array}{r}
 (iv) \quad a - 3b \qquad + 3 \\
 2a \qquad - 3c + 5 \\
 \qquad + 6b + 6c - 15 \\
 \hline
 3a + 3b + 3c - 7
 \end{array}$$

$$\begin{array}{r}
 (v) \quad 13ab - 9cd + xy \\
 \qquad \qquad + 5xy \\
 -7ab + 15cd \\
 \qquad - 3cd + 6xy \\
 \hline
 6ab + 3cd + 10xy
 \end{array}$$

$$\begin{array}{r}
 (vi) \quad x^3 - x^2y + 5xy^2 + y^3 \\
 -x^3 \qquad \qquad - 9xy^2 + y^3 \\
 \qquad + 3x^2y + 9xy^2 \\
 \hline
 2x^2y + 5xy^2 + 2y^3
 \end{array}$$

Question 3.

Find the total savings of a boy who saves ₹ $(4x - 6y)$; ₹ $(6x + 2y)$; ₹ $(4y - x)$ and ₹ $(y - 2x)$ for four consecutive weeks.

Solution:

$$\begin{array}{r}
 4x - 6y \\
 6x + 2y \\
 -x + 4y \\
 -2x + y \\
 \hline
 7x + y
 \end{array}$$

∴ Total savings = ₹ $(7x + y)$

Question 4.

- (i) $4xy^2$ from $3xy^2$;
 (ii) $-2x^2y + 3xy^2$ from $8x^2y$;
 (iii) $3a-5b+c+2d$ from $7a-3b+c-2d$
 (iv) $x^3 - 4x - 1$ from $3x^3 - x^2 + 6$
 (v) $6a+3$ from a^3-3a^2+4a+1
 (vi) $cab - 4cad - cbd$ from $3abc + 5bcd - cda$
 (vii) $a^2 + ab + b^2$ from $4a^2 - 3ab + 2b^2$.

Solution:

(i) $3xy^2 - 4xy^2 = -xy^2$

(ii)

$$\begin{array}{r} 8x^2y \\ -2x^2y + 3xy^2 \\ + \quad - \\ \hline 10x^2y - 3xy^2 \end{array}$$

(iii)

$$\begin{array}{r} 7a - 3b + c - 2d \\ 3a - 5b + c + 2d \\ - + \quad - - \\ \hline 4a + 2b \quad - 4d \end{array}$$

(iv)

$$\begin{array}{r} 3x^3 - x^2 \quad + 6 \\ x^3 \quad - 4x - 1 \\ - \quad + \quad + \\ \hline 2x^3 - x^2 + 4x + 7 \end{array}$$

(v)

$$\begin{array}{r} a^3 - 3a^2 + 4a + 1 \\ \quad + 6a + 3 \\ \quad - \quad - \\ \hline a^3 - 3a^2 - 2a - 2 \end{array}$$

(vi)

$$\begin{array}{r} 3abc + 5bcd - cda \\ + cab - cbd - 4cad \\ - + \quad + \\ \hline 2abc + 6bcd + 3cad \end{array}$$

(vii)

$$\begin{array}{r} 4a^2 - 3ab + 2b^2 \\ + a^2 + ab + b^2 \\ - \quad - \quad - \\ \hline 3a^2 - 4ab + b^2 \end{array}$$

Question 5.

- (i) Take away $-3x^3 + 4x^2 - 5x + 6$ from $3x^3 - 4x^2 + 5x - 6$
 (ii) Take $m^2 + m + 4$ from $-m^2 + 3m + 6$ and the result from $m^2 + m + 1$.

Solution:

$$\begin{array}{r}
 (i) \quad 3x^3 - 4x^2 + 5x - 6 \\
 -3x^3 + 4x^2 - 5x + 6 \\
 \hline
 + \quad - \quad + \quad - \\
 \hline
 6x^3 - 8x^2 + 10x - 12
 \end{array}$$

$$\begin{array}{r}
 (ii) \quad -m^2 + 3m + 6 \\
 \pm m^2 \pm m \pm 4 \\
 \hline
 -2m^2 + 2m + 2
 \end{array}$$

$$\begin{array}{r}
 \text{A.T.Q.} \quad m^2 + m + 1 \\
 -2m^2 + 2m + 2 \\
 \hline
 + \quad - \quad - \\
 \hline
 3m^2 - m - 1
 \end{array}$$

Question 6.

Subtract the sum of $5y^2 + y - 3$ and $y^2 - 3y + 7$ from $6y^2 + y - 2$.

Solution:

$$\begin{array}{r}
 5y^2 + y - 3 \\
 y^2 - 3y + 7 \\
 \hline
 6y^2 - 2y + 4 \\
 \hline
 6y^2 + y - 2 \\
 6y^2 - 2y + 4 \\
 \hline
 - \quad + \quad - \\
 \hline
 3y - 6
 \end{array}$$

Question 7.

What must be added to $x^4 - x^3 + x^2 + x + 3$ to obtain $x^4 + x^2 - 1$?

Solution:

$$\begin{array}{r}
 x^4 \quad + x^2 \quad - 1 \\
 +x^4 - x^3 + x^2 + x + 3 \\
 \hline
 - \quad + \quad - \quad - \quad - \\
 \hline
 x^3 \quad - x \quad - 4
 \end{array}$$

Question 8.

(i) How much more than $2x^2 + 4xy + 2y^2$ is $5x^2 + 10xy - y^2$?

(ii) How much less $2a^2 + 1$ is than $3a^2 - 6$?

Solution:

$$\begin{array}{r}
 (i) \quad \begin{array}{rrrr}
 5x^2 & + & 10xy & - & y^2 \\
 +2x^2 & + & 4xy & + & 2y^2 \\
 \hline
 3x^2 & + & 6xy & - & 3y^2
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 (ii) \quad \begin{array}{rr}
 3a^2 & - & 6 \\
 + & 2a^2 & + & 1 \\
 \hline
 a^2 & - & 7
 \end{array}
 \end{array}$$

Question 9.

If $x = 6a + 8b + 9c$; $y = 2b - 3a - 6c$ and $z = c - b + 3a$; find

(i) $x + y + z$

(ii) $x - y + z$

(iii) $2x - y - 3z$

(iv) $3y - 2z - 5x$

Solution:

$$\begin{array}{rcl}
 (i) \quad x & = & 6a + 8b + 9c \\
 y & = & -3a + 2b - 6c \\
 z & = & +3a - b + c \\
 \hline
 \end{array}$$

Adding $x+y+z = 6a + 9b + 4c$

$$\begin{aligned}
 (ii) \quad x-y+z &= (6a + 8b + 9c) - (2b - 3a - 6c) \\
 &\quad + (c - b + 3a) \\
 &= 6a + 8b + 9c - 2b + 3a + 6c + c - b + 3a \\
 &= 6a + 3a + 3a + 8b - 2b - b + 9c + 6c + c \\
 &= 12a + 5b + 16c
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad 2x - y - 3z &= 2(6a + 8b + 9c) - (2b - 3a - 6c) \\
 &\quad - 3(c - b + 3a) \\
 &= 12a + 16b + 18c - 2b + 3a + 6c - 3c + 3b - 9a \\
 &= 12a + 3a - 9a + 16b + 3b - 2b + 18c + 6c - 3c \\
 &= 6a + 17b + 21c
 \end{aligned}$$

$$\begin{aligned}
 (iv) \quad 3y - 2z - 5x &= 3(2b - 3a - 6c) - 2(c - b + 3a) - \\
 &\quad 5(6a + 8b + 9c) \\
 &= 6b - 9a - 18c - 2c + 2b - 6a - 30a - 40b - 45c \\
 &= -9a - 6a - 30a + 6b + 2b - 40b - 18c - 2c - 45c \\
 &= -45a - 32b - 65c
 \end{aligned}$$

Question 10.

The sides of a triangle are $x^2 - 3xy + 8$, $4x^2 + 5xy - 3$ and $6 - 3x^2 + 4xy$. Find its perimeter.

Solution:

$$\begin{aligned}
 \text{Required perimeter} &= \text{Sum of three sides} \\
 &= x^2 - 3xy + 8 + 4x^2 + 5xy - 3 + 6 - 3x^2 + 4xy \\
 &= x^2 + 4x^2 - 3x^2 - 3xy + 5xy + 4xy + 8 - 3 + 6 \\
 &= 2x^2 + 6xy + 11
 \end{aligned}$$

Question 11.

The perimeter of a triangle is $8y^2 - 9y + 4$ and its two sides are $3y^2 - 5y$ and $4y^2 + 12$. Find its third side.

Solution:

$$\begin{aligned}
 \text{Perimeter of the triangle} &= \text{Sum of three sides} \\
 &= 8y^2 - 9y + 4 \\
 \text{Sum of two sides} &= 3y^2 - 5y + 4y^2 + 12 \\
 &= 7y^2 - 5y + 12 \\
 \therefore (8y^2 - 9y + 4) - (7y^2 - 5y + 12) \\
 &= 8y^2 - 9y + 4 - 7y^2 + 5y - 12 \\
 &= y^2 - 4y - 8 \\
 \text{Hence third side} &= y^2 - 4y - 8
 \end{aligned}$$

Question 12.

The two adjacent sides of a rectangle are $2x^2 - 5xy + 3z^2$ and $4xy - x^2 - z^2$. Find its perimeter.

Solution:

$$\begin{aligned}
 \text{Adjacent sides of a rectangle are} \\
 2x^2 - 5xy + 3z^2 \text{ and } 4xy - x^2 - z^2 \\
 \therefore \text{Perimeter} &= 2(2x^2 - 5xy + 3z^2 + 4xy - x^2 - z^2) \\
 &= 4x^2 - 10xy + 6z^2 + 8xy - 2x^2 - 2z^2 \\
 &= 2x^2 - 2xy + 4z^2
 \end{aligned}$$

Question 13.

What must be subtracted from $19x^4 + 2x^3 + 30x - 37$ to get $8x^4 + 22x^3 - 7x - 60$?

Solution:

The required result will be

$$\begin{aligned}& (19x^4 + 2x^3 + 30x - 37) - (8x^4 + 22x^3 - 7x - 60) \\&= 19x^4 + 2x^3 + 30x - 37 - 8x^4 - 22x^3 + 7x + 60 \\&= 11x^4 - 20x^3 + 37x + 23\end{aligned}$$

Question 14.

How much smaller is $15x - 18y + 19z$ than $22x - 20y - 13z + 26$?

Solution:

The required result is

$$\begin{aligned}& (22x - 20y - 13z + 26) - (15x - 18y + 19z) \\&= 22x - 20y - 13z + 26 - 15x + 18y - 19z \\&= 7x - 2y - 32z + 26\end{aligned}$$

Question 15.

How much bigger is $15x^2y^2 - 18xy^2 - 10x^2y$ than $-5x^2 + 6x^2y - 7xy$?

Solution:

The required result,

$$\begin{aligned}& (5x^2y^2 - 18xy^2 - 10x^2y) - (-5x^2 + 6x^2y - 7xy) \\&= 5x^2y^2 - 18xy^2 - 10x^2y + 5x^2 - 6x^2y + 7xy \\&= 5x^2y^2 - 18xy^2 - 16x^2y + 5x^2 + 7xy\end{aligned}$$

EXERCISE 11(C)

Question 1.

Multiply :

(i) $8ab^2$ by $-4a^3b^4$

(ii) $\frac{2}{3}ab$ by $-\frac{1}{4}a^2b$

(iii) $-5cd^2$ by $-5cd^2$.

(iv) $4a$ and $(6a + 7)$

(v) $-8x$ and $(4 - 2x - x^2)$

(vi) $2a^2 - 5a - 4$ and $-3a$.

(vii) $x + 4$ by $x - 5$

(viii) $5a - 1$ by $7a - 3$

(ix) $12a + 5b$ by $7a - b$

(x) $x^2 + x + 1$ by $1 - x$

(xi) $2m^2 - 3m - 1$ and $4m^2 - m - 1$

(xii) a^2 , ab and b^2

(xiii) abx , $-3a^2x$ and $7b^2x^3$

(xiv) $-3bx$, $-5xy$ and $-7b^3y^2$

(xv) $\left(-\frac{3}{2}x^5y^3\right)$ and $\left(\frac{4}{9}a^2x^3y\right)$

(xvi) $\left(-\frac{2}{3}a^7b^2\right)$ and $\left(-\frac{9}{4}ab^5\right)$

(xvii) $(2a^3 - 3a^2b)$ and $\left(-\frac{1}{2}ab^2\right)$

(xviii) $\left(2x + \frac{1}{2}y\right)$ and $\left(2x - \frac{1}{2}y\right)$

Solution:

$$\begin{aligned} \text{(i)} \quad 8ab^2 \times -4a^3b^4 &= (8 \times -4)(ab^2 \times a^3b^4) \\ &= -32a^{1+3} \cdot b^{2+4} \\ &= -32a^4b^6 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \frac{2}{3}ab \times -\frac{1}{4}a^2b &= \left(\frac{2}{3} \times \frac{-1}{4}\right) (ab \times a^2b) \\ &= -\frac{1}{6}a^{1+2} \cdot b^{1+1} \\ &= -\frac{1}{6}a^3b^2 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad -5cd^2 \times -5cd^2 &= (-5 \times -5) (cd^2 \times cd^2) \\ &= 25c^{1+1}d^{2+2} \\ &= 25c^2d^4 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad 4a(6a + 7) \\ &= 4a \times 6a + 4a \times 7 \\ &= 24a^2 + 28a \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad -8x(4 - 2x - x^2) \\ &= -8x \times 4 - 8x \times -2x - 8x \times -x^2 \\ &= -32x + 16x^2 + 8x^3 \end{aligned}$$

$$\begin{aligned} \text{(vi)} \quad -3a(2a^2 - 5a - 4) \\ &= -3a \times 2a^2 - 5a \times -3a - 4 \\ &\quad \times -3a \\ &= -6a^3 + 15a^2 + 12a \end{aligned}$$

$$\begin{aligned} \text{(vii)} \quad (x+4)(x-5) &= x(x-5) + 4(x-5) \\ &= x^2 - 5x + 4x - 20 \\ &= x^2 - x - 20 \end{aligned}$$

$$\begin{aligned} \text{(viii)} \quad (5a - 1)(7a - 3) \\ &= 5a(7a - 3) - 1(7a - 3) \\ &= 35a^2 - 15a - 7a + 3 \\ &= 35a^2 - 22a + 3 \end{aligned}$$

$$\begin{aligned} \text{(ix)} \quad (12a + 5b)(7a - b) &= 12a(7a - b) + 5b \\ &\quad (7a - b) \\ &= 84a^2 - 12ab + 35ab - 5b^2 \\ &= 84a^2 + 23ab - 5b^2 \end{aligned}$$

$$\begin{aligned} \text{(x)} \quad (x^2+x+1)(1-x) &= 1(x^2+x+1) - x(x^2+x+1) \\ &= x^2+x+1 - x^3-x^2-x \\ &= 1-x^3 \end{aligned}$$

$$\begin{aligned}
 (xi) \quad & (2m^2 - 3m - 1)(4m^2 - m - 1) \\
 &= 2m^2(4m^2 - m - 1) - 3m(4m^2 - m - 1) - 1(4m^2 - m - 1) \\
 &= 8m^4 - 2m^3 - 2m^2 - 12m^3 + 3m^2 + 3m - 4m^2 + m + 1 \\
 &= 8m^4 - 14m^3 - 6m^2 + 3m^2 + 4m + 1 \\
 &= 8m^4 - 14m^3 - 3m^2 + 4m + 1
 \end{aligned}$$

$$\begin{aligned}
 (xii) \quad a^2 \times ab \times b^2 &= a^{2+1} \cdot b^{1+2} \\
 &= a^3 b^3
 \end{aligned}$$

$$\begin{aligned}
 (xiii) \quad abx \times -3a^2x \times 7b^2x^3 \\
 &= (-3 \times 7)(a \times a^2)(b \times b^2)(x \times x \times x^3) \\
 &= -21a^3b^3x^5
 \end{aligned}$$

$$\begin{aligned}
 (xiv) \quad -3bx \times -5xy \times -7b^3y^2 \\
 &= (-3 \times -5 \times -7)(b \times b^3)(x \times x)(y \times y^2) \\
 &= -105 b^4x^2y^3
 \end{aligned}$$

$$\begin{aligned}
 (xv) \quad \left(-\frac{3}{2}x^5y^3\right)\left(\frac{4}{9}a^2x^3y\right) \\
 &= \left(-\frac{3}{2} \times \frac{4}{9}\right)(a^2)(x^5 \times x^3)(y^3 \times y)
 \end{aligned}$$

$$\begin{aligned}
 (xvi) \quad \left(-\frac{2}{3}a^7b^2\right)\left(-\frac{9}{4}ab^5\right) \\
 &= \left(-\frac{2}{3} \times -\frac{9}{4}\right)(a^7 \times a)(b^2 \times b^5) \\
 &= \frac{3}{2}a^8b^7
 \end{aligned}$$

$$\begin{aligned}
 (xvii) \quad (2a^3 - 3a^2b) \left(-\frac{1}{2}ab^2\right) \\
 &= -\frac{1}{2}ab^2(2a^3 - 3a^2b) \\
 &= 2a^3 \times -\frac{1}{2}ab^2 - 3a^2b \times -\frac{1}{2}ab^2 \\
 &= -a^4b^2 + \frac{3}{2}a^3b^3
 \end{aligned}$$

$$\begin{aligned}
 (xviii) \quad \left(2x + \frac{1}{2}y\right)\left(2x - \frac{1}{2}y\right) \\
 &= 2x\left(2x - \frac{1}{2}y\right) + \frac{1}{2}y\left(2x - \frac{1}{2}y\right) \\
 &= 4x^2 - xy + xy - \frac{1}{4}y^2 \\
 &= 4x^2 - \frac{1}{4}y^2
 \end{aligned}$$

Question 2.

Multiply :

(i) $5x^2 - 8xy + 6y^2 - 3$ by $-3xy$

(ii) $3 - \frac{2}{3}xy + \frac{5}{7}xy^2 - \frac{16}{21}x^2y$ by $-21x^2y^2$

(iii) $6x^3 - 5x + 10$ by $4 - 3x^2$

(iv) $2y - 4y^3 + 6y^5$ by $y^2 + y - 3$

(v) $5p^2 + 25pq + 4q^2$ by $2p^2 - 2pq + 3q^2$

Solution:

(i) $5x^2 - 8xy + 6y^2 - 3 \times -3xy$
 $= 15x^3y^3 + 24x^2y^2 - 18xy^3 + 9xy$

(ii) $3 - \frac{2}{3}xy + \frac{5}{7}xy^2 - \frac{16}{21}x^2y$
 $\times -21x^2y^2$
 $-63x^2y^2 + 14x^3y^3 - 15x^3y^4 + 16x^4y^3$

(iii) $6x^3 - 5x + 10$
 $\times 4 - 3x^2$

 $24x^3 - 20x + 40$
 $- 18x^5 + 15x^3 - 30x^2$

 $- 18x^5 + 39x^3 - 30x^2 - 20x + 40$

(iv) $2y - 4y^3 + 6y^5$
 $\times y^2 + y - 3$

 $2y^3 - 4y^5 + 6y^7$
 $+ 2y^2 - 4y^4 + 6y^6$

 $- 6y + 12y^3 - 18y^5$
 $6y^7 + 6y^6 - (4 + 18)y^5 - 4y^4 + (2 + 12)y^3 + 2y^2 - 6y$
 $= 6y^7 + 6y^6 - 22y^5 - 4y^4 + 14y^3 + 2y^2 - 6y$

(v) $5p^2 + 25pq + 4q^2$
 $\times 2p^2 - 2pq + 3q^2$

 $10p^4 + 50p^3q + 8p^2q^2$
 $- 10p^3q - 50p^2q^2 - 8pq^3$
 $+ 15p^2q^2 + 75pq^3 + 12q^4$

 $10p^4 + 40p^3q - 27p^2q^2 + 67pq^3 + 12q^4$

Question 3.

Simplify :

(i) $(7x - 8)(3x + 2)$

(ii) $(px - q)(px + q)$

(iii) $(5a + 5b - c)(2b - 3c)$

(iv) $(4x - 5y)(5x - 4y)$

(v) $(3y + 4z)(3y - 4z) + (2y + 7z)(y + z)$

Solution:

$$\begin{aligned} \text{(i)} \quad (7x - 8)(3x + 2) &= 7x(3x + 2) - 8(3x + 2) \\ &= 21x^2 + 14x - 24x - 16 = 21x^2 - 10x - 16 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (px - q)(px + q) &= px(px + q) - q(px + q) \\ &= p^2x^2 + pxq - pqx - q^2 = p^2x^2 - q^2 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad (5a + 5b - c)(2b - 3c) \\ &= 5a(2b - 3c) + 5b(2b - 3c) - c(2b - 3c) \\ &= 10ab - 15ac + 10b^2 - 15bc - 2bc + 3c^2 \\ &= 10ab + 10b^2 - 17bc - 15ac + 3c^2 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad (4x - 5y)(5x - 4y) \\ &= 4x(5x - 4y) - 5y(5x - 4y) \\ &= 20x^2 - 16xy - 25xy + 20y^2 \\ &= 20x^2 - 41xy + 20y^2 \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad (3y + 4z)(3y - 4z) + (2y + 7z)(y + z) \\ &= 3y(3y - 4z) + 4z(3y - 4z) + 2y(y + z) + 7z(y + z) \\ &= 9y^2 - 12yz + 12yz - 16z^2 + 2y^2 + 2yz + 7yz + 7z^2 \\ &= (9 + 2)y^2 + (-12 + 12 + 2 + 7)yz + (-16 + 7)z^2 \\ &= 11y^2 + 9yz - 9z^2 \end{aligned}$$

Question 4.

The adjacent sides of a rectangle are $x^2 - 4xy + 7y^2$ and $x^3 - 5xy^2$. Find its area.

Solution:

$$\begin{aligned}
 \text{Reqd. area} &= (x^2 - 4xy + 7y^2) (x^3 - 5xy^2) \\
 &= x^2 (x^3 - 5xy^2) - 4xy (x^3 - 5xy^2) + 7y^2 (x^3 - 5xy^2) \\
 &= x^5 - 5x^3y^2 - 4x^4y + 20x^2y^3 + 7x^3y^2 - 35xy^4 \\
 &= x^5 + (7 - 5)x^3y^2 - 4x^4y + 20x^2y^3 - 35xy^4 \\
 &= x^5 + 2x^3y^2 - 4x^4y + 20x^2y^3 - 35xy^4 \\
 &= (x^5 - 4x^4y + 2x^3y^2 + 20x^2y^3 - 35xy^4) \text{ sq. unit.}
 \end{aligned}$$

Question 5.

The base and the altitude of a triangle are $(3x - 4y)$ and $(6x + 5y)$ respectively. Find its area.

Solution:

$$\begin{aligned}
 \text{Reqd. Area} &= \frac{1}{2} (\text{base}) \times (\text{altitude}) \\
 &= \frac{1}{2} (3x - 4y) (6x + 5y) \\
 &= \frac{1}{2} (18x^2 + 15xy - 24xy - 20y^2) \\
 &= \frac{1}{2} (18x^2 - 9xy - 20y^2) \text{ sq. unit.}
 \end{aligned}$$

Question 6.

Multiply $-4xy^3$ and $6x^2y$ and verify your result for $x = 2$ and $y = 1$.

Solution:

$$\begin{aligned}
 (-4xy^3) \times (6x^2y) &= (-4 \times 6) (x \times x^2) (y^3 \times y) \\
 &= -24x^3y^4
 \end{aligned}$$

For $x = 2$ and $y = 1$

$$\begin{aligned}
 (-4xy^3) \times (6x^2y) &= (-4 \times 2 \times 1^3) \times (6 \times 2^2 \times 1) \\
 &= (-8) \times 24 = -192
 \end{aligned}$$

$$\begin{aligned}
 \text{And, } -24x^3y^4 &= -24 \times 2^3 \times 1^4 \\
 &= -24 \times 8 \times 1 = -192
 \end{aligned}$$

$\therefore$ For $x = 2$ and $y = 1$, it is verified that

$$(-4xy^3) \times (6x^2y) = -24x^3y^4$$

Question 7.

Find the value of $(3x^3) \times (-5xy^2) \times (2x^2yz^3)$ for $x = 1$, $y = 2$ and $z = 3$.

Solution:

For $x = 1$, $y = 2$ and $z = 3$

$$\begin{aligned} & (3x^3) \times (-5xy^2) \times (2x^2yz^3) \\ & (3 \times 1^3) \times (-5 \times 1 \times 2^2) \times (2 \times 1^2 \times 2 \times 3^3) \\ & 3 \times (-5 \times 4) \times (2 \times 1 \times 2 \times 27) \\ & 3 \times (-20) \times 108 = -6480 \end{aligned}$$

Question 8.

Evaluate $(3x^4y^2) (2x^2y^3)$ for $x = 1$ and $y = 2$.

Solution:

$$\begin{aligned} & (3x^4y^2) (2x^2y^3) \\ & (3 \times 1^4 \times 2^2) \times (2 \times 1^2 \times 2^3) \\ & (3 \times 1 \times 4) \times (2 \times 1 \times 8) \\ & = 12 \times 16 = 192 \end{aligned}$$

Question 9.

Evaluate $(x^5) \times (3x^2) \times (-2x)$ for $x = 1$.

Solution:

$$\begin{aligned} & \text{For } x = 1 \\ & (x^5) \times (3x^2) \times (-2x) \\ & (1^5) \times (3 \times 1^2) \times (-2 \times 1) \\ & 1 \times 3 \times (-2) = -6 \end{aligned}$$

Question 10.

If $x = 2$ and $y = 1$; find the value of $(-4x^2y^3) \times (-5x^2y^5)$.

Solution:

$$\begin{aligned} & \text{For } x = 2 \text{ and } y = 1 \\ & (-4x^2y^3) \times (-5x^2y^5) \\ & (-4 \times 2^2 \times 1^3) \times (-5 \times 2^2 \times 1^5) \\ & (-4 \times 4 \times 1) \times (-5 \times 4 \times 1) \\ & -16 \times -20 = 320 \end{aligned}$$

Question 11.

Evaluate:

- (i) $(3x - 2)(x + 5)$ for $x = 2$.
- (ii) $(2x - 5y)(2x + 3y)$ for $x = 2$ and $y = 3$.
- (iii) $xz (x^2 + y^2)$ for $x = 2$, $y = 1$ and $z = 1$.

Solution:

(i) For $x = 2$

$$(3x - 2)(x + 5)$$

$$(3 \times 2 - 2)(2 + 5)$$

$$(6 - 2) \times 7$$

$$4 \times 7 = 28$$

(ii) For $x = 2$ and $y = 1$

$$xy^2(x - 5y) + 1$$

$$2 \times 1^2(2 - 5 \times 1) + 1$$

$$2 \times (2 - 5) + 1$$

$$2 \times (-3) + 1$$

$$-6 + 1 = -5$$

(iii) For $x = 2$, $y = 1$ and $z = 1$

$$xz(x^2 + y^2)$$

$$2 \times 1(2^2 + 1^2)$$

$$2(2 + 1)$$

$$= 2 \times 3 = 6$$

Question 12.

Evaluate:

(i) $x(x - 5) + 2$ for $x = 1$.

(ii) $xy^2(x - 5y) + 1$ for $x = 2$ and $y = 1$.

(iii) $2x(3x - 5) - 5(x - 2) - 18$ for $x = 2$.

Solution:

(i) For $x = 1$

$$x(x - 5) + 2$$

$$1(1 - 5) + 2$$

$$-4 + 2 = -2$$

(ii) For $x = 2$ and $y = 1$

$$xy^2(x - 5y)$$

$$2 \times 1^2(2 - 5 \times 1)$$

$$2 \times (2 - 5)$$

$$2 \times (-3) = -6$$

(iii) For $x = 2$

$$2x(3x - 5) - 5(x - 2) - 18$$

$$2 \times 2(3 \times 2 - 5) - 5(2 - 2) - 18$$

$$4(6 - 5) - 5 \times 0 - 18$$

$$4 - 18 = -14$$

Question 13.

Multiply and then verify :

$-3x^2y^2$ and $(x - 2y)$ for $x = 1$ and $y = 2$.

Solution:

$$\begin{aligned} & (-3x^2y^2) \times (x - 2y) \\ &= (-3x^2y^2) \times (x) - (-3x^2y^2)(2y) \\ &= -3x^3y^2 + 6x^2y^3 \\ &= 6x^2y^3 - 3x^3y^2 \end{aligned}$$

For $x = 1$ and $y = 2$

$$\begin{aligned} & (-3x^2y^2) \times (x - 2y) \\ &= (-3 \times 1^2 \times 2^2) \times (1 - 2 \times 2) \\ &= (6 \times 1 \times 8) - (3 \times 1 \times 4) \\ &= 48 - 12 = 36 \end{aligned}$$

$\therefore$ For $x = 1$ and $y = 2$, it is verified that,

$$(-3x^2y^2) \times (x - 2y) = 6x^2y^3 - 3x^3y^2$$

Question 14.

Multiply:

(i) $2x^2 - 4x + 5$ by $x^2 + 3x - 7$

(ii) $(ab - 1)(3 - 2ab)$

$$\begin{aligned} & (i) \ 2x^2 - 4x + 5 \text{ by } x^2 + 3x - 7 \\ & (2x^2 - 4x + 5) \times (x^2 + 3x - 7) \\ & 2x^2(x^2 + 3x - 7) - 4x(x^2 + 3x - 7) + 5(x^2 + 3x - 7) \\ & 2x^4 + 6x^3 - 14x^2 - 4x^3 - 12x^2 + 28x + 5x^2 + 15x - 35 \\ & 2x^4 + 6x^3 - 4x^3 - 14x^2 - 12x^2 + 5x^2 + 28x + 15x - 35 \\ & 2x^4 + 2x^3 - 21x^2 + 43x - 35 \end{aligned}$$

(ii) $(ab - 1)(3 - 2ab)$

$$\begin{aligned} & ab(3 - 2ab) - 1(3 - 2ab) \\ & 3ab - 2a^2b^2 - 3 + 2ab \\ & -2a^2b^2 + 5ab - 3 \\ & 2a^2b^2 - 5ab + 3 \end{aligned}$$

Question 15.

Simplify : $(5 - x)(6 - 5x)(2 - x)$.

Solution:

$$(5 - x)(6 - 5x)(2 - x)$$

$$[(5 - x)(6 - 5x)](2 - x)$$

$$[5(6 - 5x) - x(6 - 5x)](2 - x)$$

$$[30 - 25x - 6x + 5x^2](2 - x)$$

$$(5x^2 - 31x + 30)(2 - x)$$

$$2(5x^2 - 31x + 30) - x(5x^2 - 31x + 30)$$

$$10x^2 - 62x + 60 - 5x^3 + 31x^2 - 30x$$

$$-5x^3 + 10x^2 + 31x^2 - 62x - 30x + 60$$

$$-5x^3 + 41x^2 - 92x + 60$$

EXERCISE 11(D)

Question 1.

Divide :

(i) $-70a^3$ by $14a^2$

(ii) $24x^3y^3$ by $-8y^2$

(iii) $15a^4b$ by $-5a^3b$

(iv) $-24x^4d^3$ by $-2x^2d^5$

(v) $63a^4b^5c^6$ by $-9a^2b^4c^3$

(vi) $8x - 10y + 6c$ by 2.

(vii) $15a^3b^4 - 10a^4b^3 - 25a^3b^6$ by $-5a^3b^2$

(viii) $-14x^6y^3 - 21x^4y^5 + 7x^5y^4$ by $7x^2y^2$

(ix) $a^2 + 7a + 12$ by $a + 4$

(x) $x^2 + 3x - 54$ by $x - 6$

(xi) $12x^2 + 7xy - 12y^2$ by $3x + 4y$

(xii) $x^6 - 8$ by $x^2 - 2$

(xiii) $6x^3 - 13x^2 - 13x + 30$ by $2x^2 - x - 6$

(xiv) $4a^2 + 12ab + 9b^2 - 25c^2$ by $2a + 3b + 5c$.

(xv) $16 + 8x + x^6 - 8x^3 - 2x^4 + x^2$ by $x + 4 - x^3$

Solution:

$$\begin{aligned}(i) \quad \frac{-70a^3}{14a^2} &= \left(\frac{-70}{14}\right)\left(\frac{a^3}{a^2}\right) \\ &= -5a^{3-2} \\ &= -5a\end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \frac{24x^3y^3}{-8y^2} &= \left(\frac{24}{-8}\right)(x^3)\left(\frac{y^3}{y^2}\right) \\
 &= -3x^3y^{3-2} \\
 &= -3x^3y
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \frac{15a^4b}{-5a^3b} &= \left(\frac{15}{-5}\right)\left(\frac{a^4}{a^3}\right)\left(\frac{b}{b}\right) \\
 &= -3a^{4-3}b^{1-1} \\
 &= -3a b^0 \\
 &= -3a \times 1 \quad (\because b^0 = 1) \\
 &= -3a
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad \frac{-24x^4d^3}{-2x^2d^5} &= \left(\frac{-24}{-2}\right)\left(\frac{x^4}{x^2}\right)\left(\frac{d^3}{d^5}\right) \\
 &= 12x^{4-2}d^{3-5} = 12x^2d^{-2} \\
 &= \frac{12x^2}{d^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad \frac{63a^4b^5c^6}{-9a^2b^4c^3} &= \left(\frac{63}{-9}\right)\left(\frac{a^4}{a^2}\right)\left(\frac{b^5}{b^4}\right)\left(\frac{c^6}{c^3}\right) \\
 &= -7a^{4-2} \cdot b^{5-4} \cdot c^{6-3} \\
 &= -7a^2bc^3
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad \frac{8x-10y+6c}{2} \\
 &= \frac{8x}{2} - \frac{10y}{2} + \frac{6c}{2} \\
 &= 4x-5y+3c
 \end{aligned}$$

$$\text{(vii)} \quad \frac{15a^3b^4-10a^4b^3-25a^3b^6}{-5a^3b^2}$$

$$\begin{aligned}
 &= \frac{15a^3b^4}{-5a^3b^2} - \frac{10a^4b^3}{-5a^3b^2} - \frac{25a^3b^6}{-5a^3b^2} \\
 &= -3b^{4-2} + 2a^{4-3}b^{3-2} + 5b^{6-2} \\
 &= -3b^2 + 2ab + 5b^4
 \end{aligned}$$

$$(viii) \frac{-14x^6y^3 - 21x^4y^5 + 7x^5y^4}{7x^2y^2}$$

$$\begin{aligned}
 &= \frac{-14x^6y^3}{7x^2y^2} - \frac{21x^4y^5}{7x^2y^2} + \frac{7x^5y^4}{7x^2y^2} \\
 &= -2x^{6-2}y^{3-2} - 3x^{4-2}y^{5-2} + x^{5-2}y^{4-2} \\
 &= -2x^4y - 3x^2y^3 + x^3y^2
 \end{aligned}$$

$$(ix) \quad a+4 \overline{) a^2+7a+12} (a+3$$

$$\begin{array}{r}
 a^2+4a \\
 \underline{- -} \\
 3a+12 \\
 3a+12 \\
 \underline{- -} \\
 \times
 \end{array}$$

$$\therefore \text{Answer} = a + 3$$

$$(x) \quad x-6 \overline{) x^2+3x-54} (x+9$$

$$\begin{array}{r}
 x^2-6x \\
 \underline{- +} \\
 9x-54 \\
 + 9x-54 \\
 \underline{- +} \\
 \times
 \end{array}$$

$$\therefore \text{Answer} = x + 9$$

$$\begin{array}{r}
 (xi) \quad 3x+4y \overline{)12x^2+7xy-12y^2} \quad (4x-3y \\
 \underline{12x^2+16xy} \\
 -9xy-12y^2 \\
 \underline{-9xy-12y^2} \\
 + + \\
 \hline \times
 \end{array}$$

$$\therefore \text{Answer} = 4x - 3y$$

(xii)

$$\begin{array}{r}
 x^2-2 \overline{)x^6-8} \quad (x^4+2x^2+4 \\
 \underline{x^6 } -2x^4 \\
 - + \\
 \hline 2x^4 - 8 \\
 2x^4 - 4x^2 \\
 - + \\
 \hline 4x^2 - 8 \\
 4x^2 - 8 \\
 - + \\
 \hline \times
 \end{array}$$

$$\therefore \text{Answer} = x^4+2x^2+4$$

$$\begin{array}{r}
 (xiii) \quad 2x^2-x-6 \overline{)6x^3-13x^2-13x+30} \quad (3x-5 \\
 \underline{6x^3-3x^2-18x} \\
 -10x^2+5x+30 \\
 \underline{-10x^2+5x+30} \\
 + - \\
 \hline \times
 \end{array}$$

$$\therefore \text{Answer} = 3x - 5$$

(xiv)

$$\begin{array}{r}
 \overline{2a+3b+5c) \ 4a^2+12ab+9b^2-25c^2} \quad (2a+3b-5c \\
 \underline{4a^2+6ab \qquad \qquad \qquad +10ca} \\
 - \quad - \qquad \qquad \qquad - \\
 \overline{6ab+9b^2-25c^2-10ca} \\
 \underline{6ab+9b^2 \qquad \qquad \qquad +15bc} \\
 - \quad - \qquad \qquad \qquad - \\
 \overline{-10ca-25c^2-15bc} \\
 \underline{-10ca-25c^2-15bc} \\
 + \quad + \quad + \\
 \overline{\qquad \qquad \qquad \times}
 \end{array}$$

$\therefore$ Answer = $2a+3b-5c$

(xv)

$$\begin{array}{r}
 \overline{-x^3+x+4) \ x^6-2x^4-8x^3+x^2+8x+16} \quad (-x^3+x+4 \\
 \underline{+x^6-x^4-4x^3} \\
 - \quad + \quad + \\
 \overline{-x^4-4x^3+x^2+8x+16} \\
 \underline{-x^4 \quad +x^2+4x} \\
 + \quad - \quad - \\
 \overline{-4x^3+4x+16} \\
 \underline{-4x^3+4x+16} \\
 + \quad - \quad - \\
 \overline{\qquad \qquad \qquad \times}
 \end{array}$$

$\therefore$ Answer = $-x^3 + x + 4$

Question 2.

Find the quotient and the remainder (if any) when :

(i) $a^3 - 5a^2 + 8a + 15$ is divided by $a + 1$.

(ii) $3x^4 + 6x^3 - 6x^2 + 2x - 7$ is divided by $x - 3$.

(iii) $6x^2 + x - 15$ is divided by $3x + 5$. In each case, verify your answer.

(iv) $6y^5 + 30y^4 + 18y^3 + 6y^2 + 15y + 3$ is divided by $2y^3 + 1$.

Solution:

$$(i) \ a + 1 \overline{) a^3 - 5a^2 + 8a + 15} \quad (a^2 - 6a + 14$$

$$\begin{array}{r} a^3 + a^2 \\ - \quad - \\ \hline -6a^2 + 8a + 15 \\ -6a^2 - 6a \\ + \quad + \\ \hline 14a + 15 \\ 14a + 14 \\ \hline 1 \end{array}$$

$\therefore$ Quotient $= a^2 - 6a + 14$ and remainder $= 1$

(ii)

$$x - 3 \overline{) 3x^4 + 6x^3 - 6x^2 + 2x - 7} \quad (3x^3 + 15x^2 + 39x + 119$$

$$\begin{array}{r} 3x^4 - 9x^3 \\ - \quad + \\ \hline 15x^3 - 6x^2 + 2x - 7 \\ 15x^3 - 45x^2 \\ - \quad + \\ \hline 39x^2 + 2x - 7 \\ 39x^2 - 117x \\ - \quad + \\ \hline 119x - 7 \\ 119x - 357 \\ - \quad + \\ \hline 350 \end{array}$$

$\therefore$ Quotient $= 3x^3 + 15x^2 + 39x + 119$ and remainder $= 350$

$$\begin{array}{r}
 \text{(iii) } 3x + 5 \overline{) 6x^2 + x - 15} \\
 \underline{6x^2 + 10x} \\
 -9x - 15 \\
 \underline{-9x - 15} \\
 + + \\
 \hline
 \times
 \end{array}$$

$\therefore$ Quotient = $2x - 3$ and remainder = 0

$$\begin{array}{r}
 \text{(iv) } 2y^3 + 1 \overline{) 6y^5 + 30y^4 + 18y^3 + 6y^2 + 15y + 3} \\
 \underline{6y^5 + 3y^2} \\
 30y^4 + 18y^3 + 3y^2 + 15y + 3 \\
 \underline{30y^4 + 15y} \\
 18y^3 + 3y^2 + 3 \\
 \underline{18y^3 + 9} \\
 3y^2 - 6
 \end{array}$$

$\therefore$ Quotient = $3y^2 + 15y + 9$ and remainder = $3y^2 - 6$

(i) Verification:

$$\begin{aligned}
 \text{Dividend} &= \text{Quotient} \times \text{Divisor} + \text{Remainder} \\
 &= (a^2 - 6a + 14) \times (a + 1) + 1 \\
 &= a^3 - 6a^2 + 14a + a^2 - 6a + 14 + 1 \\
 &= a^3 - 5a^2 + 8a + 15 \text{ which is given}
 \end{aligned}$$

(ii) Verification:

$$\begin{aligned}
 \text{Dividend} &= \text{Quotient} \times \text{Divisor} + \text{Remainder} \\
 &= (3x^3 + 15x^2 + 39x + 119) (x - 3) + 350 \\
 &= 3x^4 + 15x^3 + 39x^2 + 119x - 9x^3 - 45x^2 - 117x - 357 + 350 \\
 &= 3x^4 + 6x^3 - 6x^2 + 2x - 7 \text{ which is given}
 \end{aligned}$$

(iii) Verification:

$$\begin{aligned}
 \text{Dividend} &= \text{Quotient} \times \text{Divisor} + \text{Remainder} \\
 &= (2x - 3) (3x + 5) + 0 \\
 &= 6x^2 + 10x - 9x - 15 + 0 \\
 &= 6x^2 - x - 15 \text{ which is given}
 \end{aligned}$$

(iv) Verification:

$$\begin{aligned}
 \text{Dividend} &= \text{Quotient} \times \text{Divisor} + \text{Remainder} \\
 &= (3y^2 + 15y + 9) (2y^3 + 1) + 3y^2 - 6 \\
 &= 6y^5 + 30y^4 + 18y^3 + 3y^2 + 15y + 9 + 3y^2 - 6 \\
 &= 6y^5 + 30y^4 + 18y^3 + 6y^2 + 15y + 3 \text{ which is given}
 \end{aligned}$$

Question 3.

The area of a rectangle is $x^3 - 8x^2 + 7$ and one of its sides is $x - 1$. Find the length of the adjacent side.

Solution:

$$\text{Area} = x^3 - 8x^2 + 7$$

$$\text{One side} = x - 1$$

$$\therefore \text{Adjacent side} = (x^3 - 8x^2 + 7) \div (x - 1)$$

$$x - 1 \overline{) x^3 - 8x^2 + 7} \quad (x^2 - 7x - 7)$$

$$\begin{array}{r} x^3 - x^2 \\ - \quad + \\ \hline -7x^2 + 7 \\ -7x^2 + 7x \\ \hline + \quad - \\ \hline -7x + 7 \\ -7x + 7 \\ \hline + \quad - \\ \hline \quad \times \end{array}$$

$$\therefore \text{Other side} = x^2 - 7x - 7$$

Question 4.

The product of two numbers is $16x^4 - 1$. If one number is $2x - 1$, find the other.

Solution:

$$\text{Product of two numbers} = 16x^4 - 1$$

$$\text{One number} = 2x - 1$$

$$\text{Then second number} = \frac{16x^4 - 1}{2x - 1}$$

$$= 8x^3 + 4x^2 + 2x + 1$$

$$\begin{array}{r} 8x^3 + 4x^2 + 2x + 1 \\ 2x - 1 \overline{) 16x^4} \quad - 1(\\ \hline 16x^4 - 8x^3 \\ - \quad + \\ \hline 8x^3 \\ 8x^3 - 4x^2 \\ - \quad + \\ \hline 4x^2 \\ 4x^2 - 2x \\ - \quad + \\ \hline 2x - 1 \\ 2x - 1 \\ \hline - \quad + \\ \hline \quad \times \end{array}$$

Question 5.

Divide $x^6 - y^6$ by the product of $x^2 + xy + y^2$ and $x - y$.

Solution:

Product of $(x^2 + xy + y^2)$ and $(x - y)$

$$= (x - y)(x^2 + xy + y^2)$$

$$= x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$$

$$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$$

$$= x^3 - y^3$$

Now, $(x^6 - y^6) \div (x^3 - y^3)$

$$= x^3 + y^3$$

$$\begin{array}{r}
 x^3 - y^3 \overline{) x^6 - y^6} \\
 \underline{x^6 - x^3y^3} \\
 + y^6 \\
 \underline{x^3y^3 - y^6} \\
 y^6 \\
 \underline{- y^6} \\
 0
 \end{array}$$

Simplification**(Using removal of brackets)**

The signs for different types of brackets are :

1. ; Vinculum or bar brackets,
2. () ; Parenthesis or small brackets,
3. { } ; Curly brackets or middle brackets,
4. [] ; Square brackets or big brackets.

In a combined operation, the brackets must be removed in the same order as written above:

EXERCISE 11(E)

Simplify :

Question 1.

$$a^2 - 2a + \{5a^2 - (3a - 4a^2)\}$$

Solution:

$$= a^2 - 2a + \{5a^2 - 3a + 4a^2\}$$

$$= a^2 - 2a + \{9a - 3a\}$$

$$= a^2 - 2a + 9a^2 - 3a = 10a^2 - 5a$$

Question 2.

$$x - y - \{x - y - (x + y) - \overline{x - y}\}$$

Solution:

$$x - y - \{x - y - (x + y) - \overline{x - y}\}$$

$$= x - y - \{x - y - (x + y) - x + y\}$$

$$= x - y - \{x - y - x - y - x + y\}$$

$$= x - y - x + y + x + y + x - y = 2x$$

Question 3.

$$-3(1 - x^2) - 2\{x^2 - (3 - 2x^2)\}$$

Solution:

$$-3(1 - x^2) - 2\{x^2 - (3 - 2x^2)\}$$

$$= -3 + 3x^2 - 2\{x^2 - 3 + 2x^2\}$$

$$= -3 + 3x^2 - 2\{3x^2 - 3\}$$

$$= -3 + 3x^2 - 6x^2 + 6 = 3 - 3x^2$$

Question 4.

$$2\{m - 3(n + \overline{m - 2n})\}$$

Solution:

$$2\{m - 3(n + \overline{m - 2n})\}$$

$$= 2\{m - 3(n + m - 2n)\} = 2\{m - 3(m - n)\}$$

$$= 2\{m - 3m + 3n\} = 2\{3n - 2m\} = 6n - 4m$$

Question 5.

$$3x - [3x - \{3x - (3x - \overline{3x - y})\}]$$

Solution:

$$3x - [3x - \{3x - (3x - \overline{3x - y})\}]$$

$$= 3x - [3x - \{3x - (3x - 3x + y)\}]$$

$$= 3x - [3x - \{3x - y\}] = 3x - [3x - 3x + y]$$

$$= 3x - y$$

Question 6.

$$p^2x - 2\{px - 3x(x^2 - \overline{3a - x^2})\}$$

Solution:

$$\begin{aligned} p^2x - 2\{px - 3x(x^2 - \overline{3a - x^2})\} \\ &= p^2x - 2\{px - 3x(x^2 - 3a + x^2)\} \\ &= p^2x - 2\{px - 3x(2x^2 - 3a)\} \\ &= p^2x - 2\{px - 6x^3 + 9ax\} \\ &= p^2x - 2px + 12x^3 - 18ax \end{aligned}$$

Question 7.

$$2[6 + 4\{m - 6(7 - \overline{n + p}) + q\}]$$

Solution:

$$\begin{aligned} 2[6 + 4\{m - 6(7 - \overline{n + p}) + q\}] \\ &= 2[6 + 4\{m - 6(7 - n - p) + q\}] \\ &= 2[6 + 4\{m - 42 + 6n + 6p + q\}] \\ &= 2[6 + 4m - 168 + 24n + 24p + 4q] \\ &= 2[4m + 24n + 24p + 4q - 162] \\ &= 8m + 48n + 48p + 8q - 324 \end{aligned}$$

Question 8.

$$a - [a - \overline{b + a} - \{a - (a - \overline{b - a})\}]$$

Solution:

$$\begin{aligned} a - [a - \overline{b + a} - \{a - (a - \overline{b - a})\}] \\ &= a - [a - b - a - \{a - (a - b + a)\}] \\ &= a - [-b - \{a - a + b - a\}] \\ &= a - [-b - b + a] \\ &= a + b + b - a = 2b \end{aligned}$$

Question 9.

$$3x - [4x - \overline{3x-5y} - 3 \{2x - (3x - \overline{2x-3y})\}]$$

Solution:

$$\begin{aligned} & 3x - [4x - \overline{3x-5y} - 3 \{2x - (3x - \overline{2x-3y})\}] \\ &= 3x - [4x - 3x + 5y - 3 \{2x - (3x - 2x + 3y)\}] \\ &= 3x - [4x - 3x + 5y - 3 \{2x - (x + 3y)\}] \\ &= 3x - [4x - 3x + 5y - 3 \{2x - x - 3y\}] \\ &= 3x - [x + 5y - 6x + 3x + 9y] \\ &= 3x - [-2x + 14y] \\ &= 3x + 2x - 14y \\ &= 5x - 14y \end{aligned}$$

Question 10.

$$a^5 \div a^3 + 3a \times 2a$$

Solution:

$$a^5 \div a^3 + 3a \times 2a = a^{5-3} + 3a \times 2a = a^2 + 6a^2 = 7a^2$$

Question 11.

$$x^5 \div (x^2 \times y^2) \times y^3$$

Solution:

$$x^5 \div (x^2 \times y^2) \times y^3 = \frac{x^5}{x^2 y^2} \times y^3 = x^{5-2} \times y^{3-2} = x^3 y$$

Question 12.

$$(x^5 \div x^2) \times y^2 \times y^3$$

Solution:

$$(x^5 \div x^2) \times y^2 \times y^3 = x^{5-2} \times y^{2+3} = x^3 y^5$$

Question 13.

$$(y^3 - 5y^2) \div y \times (y - 1)$$

Solution:

$$\begin{aligned} (y^3 - 5y^2) \div y \times (y - 1) &= \frac{y^3 - 5y^2}{y} \times y - 1 \\ &= (y^2 - 5y) \times (y - 1) = y^2(y - 1) - 5y(y - 1) = y^3 - y^2 - 5y^2 + 5y = y^3 - 6y^2 + 5y \end{aligned}$$

Question 14.

$$3a \times [8b \div 4 - 6 \{a - (5a - \overline{3b - 2a})\}]$$

Solution:

$$3a \times [8b \div 4 - 6 \{a - (5a - \overline{3b - 2a})\}]$$

$$= 3a \times \left[\frac{8b}{4} - 6\{a - (5a - 3b + 2a)\} \right] = 3a \times [2b - 6 \{a - 7a + 3b\}] = 3a \times [2b - 6 \{-6a + 3b\}]$$

$$= 3a \times [2b + 36a - 18b] = 3a \times [36a - 16b] = 108a^2 - 48ab$$

Question 15.

$$7x + 4 \{x^2 \div (5x \div 10)\} - 3\{2 - x^3 \div (3x^2 \div x)\}$$

Solution:

$$7x + 4\{x^2 \div (5x \div 10)\} - 3\{2 - x^3 \div (3x^2 \div x)\}$$

$$= 7x + 4 \left\{ x^2 \div \left(\frac{5x}{10} \right) \right\} - 3 \left\{ 2 - x^3 \div \left(\frac{3x^2}{x} \right) \right\}$$

$$= 7x + 4 \left\{ x^2 \div \frac{x}{2} \right\} - 3 \{ 2 - x^3 \div 3x \} = 7x + 4 \left\{ x^2 \times \frac{2}{x} \right\} - 3 \left\{ 2 - \frac{x^3}{3x} \right\}$$

$$= 7x + 8x - 6 + x^2 = x^2 + 15x - 6$$