11. Trigonometric Equations

Exercise 11.1

1 A. Question

Find the general solutions of the following equations :

i. $\sin\, x = \frac{1}{2}$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

we have,

$$\sin x = \frac{1}{2}$$

We know that sin 30° = sin π/6 = 0.5

$\therefore \sin x = \sin\frac{\pi}{6}$

∵ it matches with the form sin x = sin y

Hence,

$\boldsymbol{x = n\pi + (-1)^n\frac{\pi}{3}}$ **,where n ϵ Z**

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∵ it matches with the form sin x = sin y

Hence,

$\boldsymbol{x = n\pi + (-1)^n \frac{\pi}{3}}$ **,where n ϵ Z**

1 B. Question

Find the general solutions of the following equations :

$$\cos x = -\frac{\sqrt{3}}{2}$$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$$\cos x = -\frac{\sqrt{3}}{2}$$

We know that, cos 150° $= \left(-\frac{\sqrt{3}}{2}\right) = \cos\frac{5\pi}{6}$

$\therefore \cos x = \cos\frac{5\pi}{6}$

If cos x = cos y then x = 2nπ ± y, where n ∈ Z.

For above equation y = 5π / 6

∴ **x = 2nπ ± 5π / 6 ,where n ϵ Z**

Thus, x gives the required general solution for the given trigonometric equation.

1 B. Question

Find the general solutions of the following equations :

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We know that, cos 150° $= \left(-\frac{\sqrt{3}}{2}\right) = \cos\frac{5\pi}{6}$

$\therefore \cos x = \cos \frac{5\pi}{6}$

If cos x = cos y then x = $2n\pi \pm y$, where n ∈ Z.

For above equation y = 5π / 6

∴ x = 2nπ ± 5π / 6 ,where n ϵ Z

Thus, x gives the required general solution for the given trigonometric equation.

1 C. Question

Find the general solutions of the following equations :

$\operatorname{cosec} x = -\sqrt{2}$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = $n\pi + (-1)^n y$, where n ∈ Z.

• cos x = cos y, implies x = $2n\pi \pm y$, where n ∈ Z.

• tan x = tan y, implies x = $n\pi + y$, where n ∈ Z.

Given,

$\operatorname{cosec} x = -\sqrt{2}$

We know that sin x, and cosec x have negative values in the 3rd and 4th quadrant.

While giving a solution, we always try to take the least value of y

The fourth quadrant will give the least magnitude of y as we are taking an angle in a clockwise sense (i.e., negative angle)

-√2 = -cosec (π/4) = cosec (-π/4) { ∵ sin -θ = -sin θ }

$\therefore \operatorname{cosec} x = \operatorname{cosec}\left(-\frac{\pi}{4}\right)$

$\Rightarrow \sin x = \sin\left(\frac{-\pi}{4}\right)$

If sin x = sin y ,then x = $n\pi + (-1)^n y$, where n ∈ Z.

For above equation y = $-\frac{\pi}{4}$

∴ x = nπ + $(-1)^n\left(-\frac{\pi}{4}\right)$,where n ϵ Z

Or x = nπ + $(-1)^{n+1}\left(\frac{\pi}{4}\right)$,where n ϵ Z

Thus, x gives the required general solution for given trigonometric equation.

1 C. Question

Find the general solutions of the following equations :

$\operatorname{cosec} x = -\sqrt{2}$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

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• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\text{cosec}x = -\sqrt{2}$

We know that sin x, and cosec x have negative values in the 3rd and 4th quadrant.

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The fourth quadrant will give the least magnitude of y as we are taking an angle in a clockwise sense (i.e., negative angle)

-√2 = -cosec (π/4) = cosec (-π/4) { ∵ sin -θ = -sin θ }

$\therefore cosec\ x = cosec\ \left(-\frac{\pi}{4}\right)$

$\Rightarrow \sin x = \sin\left(\frac{-\pi}{4}\right)$

If sin x = sin y ,then x = nπ + $(-1)^n$y , where n ∈ Z.

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∴ x = nπ + $(-1)^n\left(-\frac{\pi}{4}\right)$,where n ϵ Z

Or x = nπ + $(-1)^{n+1}\left(\frac{\pi}{4}\right)$,where n ϵ Z

Thus, x gives the required general solution for given trigonometric equation.

1 D. Question

Find the general solutions of the following equations :

$\sec\ x = \sqrt{2}$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$sec\ x = \sqrt{2}$

We know that sec x and cos x have positive values in the 1st and 4th quadrant.

While giving a solution, we always try to take the least value of y

both quadrants will give the least magnitude of y.

We can choose any one, in this solution we are assuming a positive value.

$sec\ x = \sec\frac{\pi}{4}$

$\Rightarrow \cos x = \cos\frac{\pi}{4}$

If cos x = cos y then x = 2nπ ± y, where n ∈ Z.

For above equation y = π / 4

∴ **x = 2nπ ± $\frac{\pi}{4}$,where n ϵ Z**

Thus, x gives the required general solution for the given trigonometric equation.

1 D. Question

Find the general solutions of the following equations :

$\sec x = \sqrt{2}$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

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Given,

$sec\, x = \sqrt{2}$

We know that sec x and cos x have positive values in the 1st and 4th quadrant.

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$sec\, x = \sec \frac{\pi}{4}$

⇒ $\cos x = \cos\frac{\pi}{4}$

If cos x = cos y then x = 2nπ ± y, where n ∈ Z.

For above equation y = π / 4

∴ **x = 2nπ ± $\frac{\pi}{4}$,where n ϵ Z**

Thus, x gives the required general solution for the given trigonometric equation.

1 E. Question

Find the general solutions of the following equations :

$\tan x = -\frac{1}{\sqrt{3}}$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$$tan\, x = -\frac{1}{\sqrt{3}}$$

We know that tan x and cot x have negative values in the 2nd and 4th quadrant.

While giving solution, we always try to take the least value of y.

The fourth quadrant will give the least magnitude of y as we are taking an angle in a clockwise sense (i.e. negative angle)

$$tan\, x = \tan\left(-\frac{\pi}{6}\right)$$

If tan x = tan y then x = nπ + y, where n ∈ Z.

For above equation y = $-\frac{\pi}{6}$

∴ x = nπ + $\left(-\frac{\pi}{6}\right)$,w here n ϵ Z

Or x = nπ - $\frac{\pi}{6}$,wh ere n ϵ Z

Thus, x gives the required general solution for the given trigonometric equation.

1 E. Question

Find the general solutions of the following equations :

$$\tan\ x = -\frac{1}{\sqrt{3}}$$

Answer

Ideas required to solve the problem:

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• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$$tan\, x = -\frac{1}{\sqrt{3}}$$

We know that tan x and cot x have negative values in the 2nd and 4th quadrant.

While giving solution, we always try to take the least value of y.

The fourth quadrant will give the least magnitude of y as we are taking an angle in a clockwise sense (i.e. negative angle)

$$tan\, x = \tan\left(-\frac{\pi}{6}\right)$$

If tan x = tan y then x = nπ + y, where n ∈ Z.

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Or x = nπ - $\frac{\pi}{6}$,wh ere n ϵ Z

Thus, x gives the required general solution for the given trigonometric equation.

1 F. Question

Find the general solutions of the following equations :

$\sqrt{3}\ \sec\ x = 2$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\sqrt{3}\, sec\, x = 2$

$\Rightarrow sec\, x = \frac{2}{\sqrt{3}}$

We know that sec x and cos x have positive values in the 1st and 4th quadrant.

While giving solution, we always try to take the least value of y

both quadrants will give the least magnitude of y.

We can choose any one, in this solution we are assuming a positive value.

$sec\, x = \sec \frac{\pi}{6}$

$\Rightarrow \cos x = \cos \frac{\pi}{6}$

If cos x = cos y then x = 2nπ ± y, where n ∈ Z.

For above equation y = π / 6

∴ **x = 2nπ ± $\frac{\pi}{6}$,where n ϵ Z**

Thus, x gives the required general solution for the given trigonometric equation.

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• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\sqrt{3}\, sec\, x = 2$

$\Rightarrow sec\, x = \frac{2}{\sqrt{3}}$

We know that sec x and cos x have positive values in the 1st and 4th quadrant.

While giving solution, we always try to take the least value of y

both quadrants will give the least magnitude of y.

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For above equation y = π / 6

∴ **x = 2nπ ± $\frac{\pi}{6}$,where n ϵ Z**

Thus, x gives the required general solution for the given trigonometric equation.

2 A. Question

Find the general solutions of the following equations :

$$\sin 2x = \frac{\sqrt{3}}{2}$$

Answer

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• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$$sin\, 2x = \frac{\sqrt{3}}{2}$$

We know that sin x, and cos x have positive values in the 1st and 2nd quadrant.

While giving solution, we always try to take the least value of y

The first quadrant will give the least magnitude of y.

$\therefore sin\, 2x = \sin\frac{\pi}{3}$

If sin x = sin y then x = nπ + (– 1)^n y, where n ∈ Z

Clearly on comparing we have y = π/3

$$\therefore\ 2x = n\pi + (-1)^n\frac{\pi}{3}$$

$\Rightarrow \boldsymbol{x = \frac{n\pi}{2} + (-1)^n\frac{\pi}{6}}$ **,where n ϵ Z ….ans**

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Given,

$$sin\ 2x = \frac{\sqrt{3}}{2}$$

We know that sin x, and cos x have positive values in the 1st and 2nd quadrant.

While giving solution, we always try to take the least value of y

The first quadrant will give the least magnitude of y.

$\therefore sin\ 2x = \sin\frac{\pi}{3}$

If sin x = sin y then x = nπ + (– 1)^n y, where n ∈ Z

Clearly on comparing we have y = π/3

$$\therefore\ 2x\ =\ n\pi\ +\ (-\ 1)^n\frac{\pi}{3}$$

$\Rightarrow \boldsymbol{x\ = \frac{n\pi}{2}\ +\ (-\ 1)^n\frac{\pi}{6}}$ **,where n ϵ Z ….ans**

2 B. Question

Find the general solutions of the following equations :

$$\cos 3x = \frac{1}{2}$$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

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• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$$cos\ 3x = \frac{1}{2}$$

We know that cos x and sec x have positive values in the 1st and 4th quadrant.

While giving solution, we always try to take the least value of y

both quadrant will give the least magnitude of y. We prefer the first quadrant.

$\therefore cos\ 3x = \cos\ \frac{\pi}{3}$

If cos x = cos y then x = 2nπ ± y, where n ∈ Z

Clearly on comparing we have y = π/3

$\therefore\ 3x = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow x = \frac{2n\pi}{3} \pm \frac{\pi}{9}$,where n ϵ Z ....ans

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• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$$\cos 3x = \frac{1}{2}$$

We know that cos x and sec x have positive values in the 1st and 4th quadrant.

While giving solution, we always try to take the least value of y

both quadrant will give the least magnitude of y. We prefer the first quadrant.

$\therefore \cos 3x = \cos \frac{\pi}{3}$

If cos x = cos y then x = 2nπ ± y, where n ∈ Z

Clearly on comparing we have y = π/3

$\therefore\ 3x = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow x = \frac{2n\pi}{3} \pm \frac{\pi}{9}$,where n ϵ Z ....ans

2 C. Question

Find the general solutions of the following equations :

sin 9x = sin x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$sin\, 9x = \sin x$

$\Rightarrow \sin 9x - \sin x = 0$

Using transformation formula: $sin\ A - \ sin\ B\ = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

$\therefore 2\cos\frac{9x+x}{2}\sin\frac{9x-x}{2} = 0$

$\Rightarrow \cos 5x \sin 4x = 0$

∴ cos 5x = 0 or sin 4x = 0

If either of the equation is satisfied, the result will be 0

So we will find the solution individually and then finally combined the solution.

∴ cos 5x = 0

⇒ cos 5x = cos π/2

$\therefore 5x = (2n+1)\frac{\pi}{2}$

$\boldsymbol{x = (2n+1)\frac{\pi}{10}}$,where n ϵ Z ....eqn 1

Also,

sin 4x = sin 0

$\therefore 4x = n\pi$

Or $\boldsymbol{x = \frac{n\pi}{4}}$,where n ϵ Z ....eqn 2

From equation 1 and eqn 2,

$\boldsymbol{x = (2n+1)\frac{\pi}{10}}$ or $\boldsymbol{x = \frac{n\pi}{4}}$,where n ϵ Z ...ans

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Find the general solutions of the following equations :

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⇒ cos 5x = cos π/2

$\therefore 5x = (2n+1)\frac{\pi}{2}$

$\boldsymbol{x = (2n+1)\frac{\pi}{10}}$,where n ϵ Z ....eqn 1

Also,

sin 4x = sin 0

$\therefore 4x = n\pi$

Or $\boldsymbol{x = \frac{n\pi}{4}}$,where n ϵ Z ....eqn 2

From equation 1 and eqn 2,

$x = (2n+1)\frac{\pi}{10}$ or $x = \frac{n\pi}{4}$,where n ϵ Z ...ans

2 D. Question

Find the general solutions of the following equations :

sin 2x = cos 3x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$sin\,2x = \cos 3x$

⇒ $\cos(\frac{\pi}{2} - 2x) = \cos 3x$ {∵ sin θ = cos (π/2 – θ) }

If cos x = cos y then x = 2nπ ± y, where n ∈ Z

Clearly on comparing we have y = 3x

$\therefore \frac{\pi}{2} - 2x = 2n\pi \ \pm 3x$

, $\frac{\pi}{2} - 2x = 2n\pi + 3x$ or $\frac{\pi}{2} - 2x = 2n\pi - 3x$

$\therefore 5x = \frac{\pi}{2} - 2n\pi = \frac{\pi}{2}\ (1-4n)$ or $x = 2n\pi - \frac{\pi}{2}\ = \frac{\pi}{2}(4n-1)$

$x = \frac{\pi}{10}\ (1-4n)\ , where\ n\ \epsilon\ Z$

Hence,

$x = \frac{\pi}{10}\ (1-4n)\ or\ \frac{\pi}{2}(4n-1)\ , where\ n\ \epsilon\ Z$ ...ans

2 D. Question

Find the general solutions of the following equations :

sin 2x = cos 3x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$sin\,2x = \cos 3x$

⇒ $\cos(\frac{\pi}{2} - 2x) = \cos 3x$ {∵ sin θ = cos (π/2 – θ) }

If cos x = cos y then x = 2nπ ± y, where n ∈ Z

Clearly on comparing we have y = 3x

$\therefore \frac{\pi}{2} - 2x = 2n\pi\ \pm 3x$

, $\frac{\pi}{2} - 2x = 2n\pi + 3x$ or $\frac{\pi}{2} - 2x = 2n\pi - 3x$

$\therefore 5x = \frac{\pi}{2} - 2n\pi = \frac{\pi}{2}\ (1 - 4n)$ or $x = 2n\pi - \frac{\pi}{2}\ = \frac{\pi}{2}(4n - 1)$

$x = \frac{\pi}{10}\ (1 - 4n)\ , where\ n\ \epsilon\ Z$

Hence,

$\boldsymbol{x = \frac{\pi}{10}\ (1 - 4n)\ or\ \frac{\pi}{2}(4n - 1)\ , where\ n\ \epsilon\ Z}$ **...ans**

2 E. Question

Find the general solutions of the following equations :

tan x + cot 2x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\tan x + \cot 2x = 0$

⇒ $\tan x = \ -\cot 2x$

We know that: cot θ = tan (π/2 – θ)

$\therefore \tan x = \ -\tan\ (\frac{\pi}{2} - 2x)$

⇒ $\tan x = \tan(2x - \frac{\pi}{2})$ { ∵ - tan θ = tan -θ }

If tan x = tan y, then x is given by x = nπ + y, where n ∈ Z.

From above expression, on comparison with standard equation we have

y = $(2x - \frac{\pi}{2})$

$\therefore x = n\pi + 2x - \frac{\pi}{2}$

$\Rightarrow x = \frac{\pi}{2} - n\pi = \frac{\pi}{2}(1 - 2n)$,where n ϵ Z **...ans**

2 E. Question

Find the general solutions of the following equations :

tan x + cot 2x = 0

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Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

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• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\tan x + \cot 2x = 0$

$\Rightarrow \tan x = -\cot 2x$

We know that: cot θ = tan (π/2 – θ)

$\therefore \tan x = -\tan\left(\frac{\pi}{2} - 2x\right)$

$\Rightarrow \tan x = \tan(2x - \frac{\pi}{2})$ { ∵ - tan θ = tan -θ }

If tan x = tan y, then x is given by x = nπ + y, where n ∈ Z.

From above expression, on comparison with standard equation we have

y = $(2x - \frac{\pi}{2})$

$\therefore x = n\pi + 2x - \frac{\pi}{2}$

$\Rightarrow x = \frac{\pi}{2} - n\pi = \frac{\pi}{2}(1 - 2n)$,where n ϵ Z **...ans**

2 F. Question

Find the general solutions of the following equations :

tan 3x = cot x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\tan 3x = \cot x$

We know that: cot θ = tan (π/2 – θ)

$\therefore \tan 3x = \tan\left(\frac{\pi}{2} - x\right)$

If tan x = tan y, then x is given by x = nπ + y, where n ∈ Z.

From above expression, on comparison with standard equation we have

y = $\left(\frac{\pi}{2} - x\right)$

$\therefore 3x = n\pi + \frac{\pi}{2} - x$

$\Rightarrow 4x = n\pi + \frac{\pi}{2}$

$\therefore x = \frac{n\pi}{4} + \frac{\pi}{8}$ **,where n ϵ Z .....ans**

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Answer

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The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\tan 3x = \cot x$

We know that: cot θ = tan (π/2 – θ)

$\therefore \tan 3x = \tan\left(\frac{\pi}{2} - x\right)$

If tan x = tan y, then x is given by x = nπ + y, where n ∈ Z.

From above expression, on comparison with standard equation we have

y = $\left(\frac{\pi}{2} - x\right)$

$\therefore 3x = n\pi + \frac{\pi}{2} - x$

$\Rightarrow 4x = n\pi + \frac{\pi}{2}$

$\therefore x = \frac{n\pi}{4} + \frac{\pi}{8}$ **,where n ϵ Z .....ans**

2 G. Question

Find the general solutions of the following equations :

tan 2x tan x = 1

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\tan 2x \tan x = 1$

$\Rightarrow \tan 2x = \frac{1}{\tan x}$

$\Rightarrow \tan 2x = \cot x$

We know that: cot θ = tan (π/2 – θ)

$\therefore \tan 2x = \tan(\frac{\pi}{2} - x)$

If tan x = tan y, then x is given by x = nπ + y, where n ∈ Z.

From above expression, on comparison with standard equation we have

y = $(\frac{\pi}{2} - x)$

$\therefore 2x = n\pi + \frac{\pi}{2} - x$

$\Rightarrow 3x = \ n\pi + \frac{\pi}{2}$

$\Rightarrow \boldsymbol{x = \frac{n\pi}{3} + \frac{\pi}{6}}$ **,where n ϵ Z ….ans**

2 G. Question

Find the general solutions of the following equations :

tan 2x tan x = 1

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\tan 2x \tan x = 1$

$\Rightarrow \tan 2x = \frac{1}{\tan x}$

$\Rightarrow \tan 2x = \cot x$

We know that: cot θ = tan (π/2 – θ)

$\therefore \tan 2x = \tan(\frac{\pi}{2} - x)$

If tan x = tan y, then x is given by x = nπ + y, where n ∈ Z.

From above expression, on comparison with standard equation we have

y = $(\frac{\pi}{2} - x)$

$\therefore 2x = n\pi + \frac{\pi}{2} - x$

$\Rightarrow 3x = \ n\pi + \frac{\pi}{2}$

$\Rightarrow \boldsymbol{x = \frac{n\pi}{3} + \frac{\pi}{6}}$ **,where n ϵ Z ….ans**

2 H. Question

Find the general solutions of the following equations :

tan mx + cot nx = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$tan\ mx + cot\ nx = 0$

$\Rightarrow \tan mx = \ -\cot nx$

We know that: cot θ = tan (π/2 – θ)

$\therefore \tan mx = \ -\tan\ (\frac{\pi}{2} - nx)$

$\Rightarrow \tan mx = \tan(nx - \frac{\pi}{2})$ { ∵ - tan θ = tan -θ }

If tan x = tan y, then x is given by x = kπ + y, where k ∈ Z.

From above expression, on comparison with standard equation we have

y = $(nx - \frac{\pi}{2})$

$\therefore mx = k\pi + nx - \frac{\pi}{2}$

$\Rightarrow (m - n)x = k\pi - \frac{\pi}{2}$

$\Rightarrow \ x = \frac{\pi}{2}\left(\frac{2k-1}{m-n}\right)$,where k ϵ Z **…ans**

2 H. Question

Find the general solutions of the following equations :

tan mx + cot nx = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$tan\ mx + cot\ nx = 0$

$\Rightarrow \tan mx = -\cot nx$

We know that: cot θ = tan (π/2 – θ)

$\therefore \tan mx = -\tan\left(\frac{\pi}{2} - nx\right)$

$\Rightarrow \tan mx = \tan\left(nx - \frac{\pi}{2}\right)$ { ∵ - tan θ = tan -θ }

If tan x = tan y, then x is given by x = kπ + y, where k ∈ Z.

From above expression, on comparison with standard equation we have

y = $\left(nx - \frac{\pi}{2}\right)$

$\therefore mx = k\pi + nx - \frac{\pi}{2}$

$\Rightarrow (m-n)x = k\pi - \frac{\pi}{2}$

$\Rightarrow\ x = \frac{\pi}{2}\left(\frac{2k-1}{m-n}\right)$,where k ϵ Z **...ans**

2 I. Question

Find the general solutions of the following equations :

tan px = cot qx

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\tan px = \cot qx$

We know that: cot θ = tan (π/2 – θ)

$\therefore \tan px = \tan\left(\frac{\pi}{2} - qx\right)$

If tan x = tan y, then x is given by x = nπ + y, where n ∈ Z.

From above expression, on comparison with standard equation we have

y = $\left(\frac{\pi}{2} - qx\right)$

$\therefore px = n\pi + \frac{\pi}{2} - qx$

$\Rightarrow (p+q)x = n\pi + \frac{\pi}{2}$

$\therefore x = \frac{n\pi}{(p+q)} + \frac{\pi}{2(p+q)}$ **,where n ϵ Z**

2 I. Question

Find the general solutions of the following equations :

tan px = cot qx

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\tan px = \cot qx$

We know that: cot θ = tan (π/2 – θ)

$\therefore \tan px = \tan\left(\frac{\pi}{2} - qx\right)$

If tan x = tan y, then x is given by x = nπ + y, where n ∈ Z.

From above expression, on comparison with standard equation we have

y = $\left(\frac{\pi}{2} - qx\right)$

$\therefore px = n\pi + \frac{\pi}{2} - qx$

$\Rightarrow (p+q)x = n\pi + \frac{\pi}{2}$

$\therefore x = \frac{n\pi}{(p+q)} + \frac{\pi}{2(p+q)}$ **,where n ϵ Z**

2 J. Question

Find the general solutions of the following equations :

sin 2x + cos x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\sin 2x + \cos x = 0$

We know that: sin θ = cos (π/2 – θ)

$\therefore \cos x = -\sin 2x$

$\Rightarrow \cos x = -\cos\left(\frac{\pi}{2} - 2x\right)$

We know that: -cos θ = cos (π – θ)

$\therefore \cos x = \cos\left(\pi - \left(\frac{\pi}{2} - 2x\right)\right)$

$\Rightarrow \cos x = \cos\left(\frac{\pi}{2} + 2x\right)$

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

From above expression and on comparison with standard equation we have:

y = $(\frac{\pi}{2} + 2x)$

$\therefore x = 2n\pi \pm (\frac{\pi}{2} + 2x)$

Hence,

$x = 2n\pi + \frac{\pi}{2} + 2x$ or $x = 2n\pi - \frac{\pi}{2} - 2x$

$\therefore x = -\frac{\pi}{2} - 2n\pi$ or $3x = 2n\pi - \frac{\pi}{2}$

$\Rightarrow x = -\frac{\pi}{2}\ (1 + 4n)$ or $x = \frac{\pi}{6}(4n - 1)$

$\therefore x = -\frac{\pi}{2}\ (4n + 1)$ *or* $\frac{\pi}{6}\ (4n - 1)$,where n ϵ Z

2 J. Question

Find the general solutions of the following equations :

sin 2x + cos x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\sin 2x + \cos x = 0$

We know that: sin θ = cos (π/2 – θ)

$\therefore \cos x = -\sin 2x$

$\Rightarrow \cos x = -\cos(\frac{\pi}{2} - 2x)$

We know that: -cos θ = cos (π – θ)

$\therefore \cos x = \cos(\pi - (\frac{\pi}{2} - 2x))$

$\Rightarrow \cos x = \cos\ (\frac{\pi}{2} + 2x)$

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

From above expression and on comparison with standard equation we have:

y = $(\frac{\pi}{2} + 2x)$

$\therefore x = 2n\pi \pm (\frac{\pi}{2} + 2x)$

Hence,

$x = 2n\pi + \frac{\pi}{2} + 2x$ or $x = 2n\pi - \frac{\pi}{2} - 2x$

$\therefore x = -\frac{\pi}{2} - 2n\pi$ or $3x = 2n\pi - \frac{\pi}{2}$

$\Rightarrow x = -\frac{\pi}{2}\ (1 + 4n)$ or $x = \frac{\pi}{6}(4n - 1)$

$\therefore x = -\frac{\pi}{2}(4n+1)\ or\ \frac{\pi}{6}(4n-1)$ **,where n ϵ Z**

2 K. Question

Find the general solutions of the following equations :

sin x = tan x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\sin x = \tan x$

$\Rightarrow \sin x = \frac{\sin x}{\cos x}$

$\Rightarrow \sin x \cos x = \sin x$

$\Rightarrow \sin x(\cos x - 1) = 0$

either,

sin x = 0 or cos x = 1

⇒ sin x = sin 0 or cos x = cos 0

We know that,

If sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z

∵ sin x = sin 0

∴ y = 0

And hence,

x = nπ where n ϵ Z

Also,

If cos x = cos y, implies x = 2mπ ±y, where m∈ Z

∵ cos x = cos 0

∴ y = 0

Hence, x is given by

x = 2mπ where m ϵ Z

∴ x = nπ or 2mπ ,where m,n ϵ Z …ans

2 K. Question

Find the general solutions of the following equations :

sin x = tan x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\sin x = \tan x$

⇒ $\sin x = \frac{\sin x}{\cos x}$

⇒ $\sin x \cos x = \sin x$

⇒ $\sin x\,(\cos x - 1) = 0$

either,

sin x = 0 or cos x = 1

⇒ sin x = sin 0 or cos x = cos 0

We know that,

If sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z

∵ sin x = sin 0

∴ y = 0

And hence,

x = nπ where n ϵ Z

Also,

If cos x = cos y, implies x = 2mπ ±y, where m∈ Z

∵ cos x = cos 0

∴ y = 0

Hence, x is given by

x = 2mπ where m ϵ Z

∴ x = nπ or 2mπ ,where m,n ϵ Z …ans

2 L. Question

Find the general solutions of the following equations :

sin 3x + cos 2x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\sin 3x + \cos 2x = 0$

We know that: sin θ = cos (π/2 – θ)

$\therefore \cos 2x = -\sin 3x$

⇒ $\cos 2x = -\cos(\frac{\pi}{2} - 3x)$

We know that: -cos θ = cos (π – θ)

$\therefore \cos 2x = \cos(\pi - (\frac{\pi}{2} - 3x))$

⇒ $\cos 2x = \cos(\frac{\pi}{2} + 3x)$

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

From above expression and on comparison with standard equation we have:

y = $(\frac{\pi}{2} + 3x)$

$\therefore 2x = 2n\pi \pm (\frac{\pi}{2} + 3x)$

Hence,

$2x = 2n\pi + \frac{\pi}{2} + 3x$ or $2x = 2n\pi - \frac{\pi}{2} - 3x$

$\therefore x = -\frac{\pi}{2} - 2n\pi$ or $5x = 2n\pi - \frac{\pi}{2}$

⇒ $x = -\frac{\pi}{2}(1 + 4n)$ or $x = \frac{\pi}{10}(4n - 1)$

$\therefore x = -\frac{\pi}{2}(4n + 1)$ *or* $\frac{\pi}{10}(4n - 1)$,where n ϵ Z

2 L. Question

Find the general solutions of the following equations :

sin 3x + cos 2x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\sin 3x + \cos 2x = 0$

We know that: sin θ = cos (π/2 – θ)

$\therefore \cos 2x = -\sin 3x$

⇒ $\cos 2x = -\cos(\frac{\pi}{2} - 3x)$

We know that: -cos θ = cos (π – θ)

$\therefore \cos 2x = \cos(\pi - (\frac{\pi}{2} - 3x))$

⇒ $\cos 2x = \cos(\frac{\pi}{2} + 3x)$

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

From above expression and on comparison with standard equation we have:

y = $(\frac{\pi}{2} + 3x)$

$\therefore 2x = 2n\pi \pm (\frac{\pi}{2} + 3x)$

Hence,

$2x = 2n\pi + \frac{\pi}{2} + 3x$ or $2x = 2n\pi - \frac{\pi}{2} - 3x$

$\therefore x = -\frac{\pi}{2} - 2n\pi$ or $5x = 2n\pi - \frac{\pi}{2}$

$\Rightarrow x = -\frac{\pi}{2}\ (1 + 4n)$ or $x = \frac{\pi}{10}(4n - 1)$

$\therefore x = -\frac{\pi}{2}\ (4n + 1)\ or\ \frac{\pi}{10}\ (4n - 1)$,where n ϵ Z

3 A. Question

Solve the following equations :

$$\sin^2 x - \cos x = \frac{1}{4}$$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

$$sin^2\ x - cos\ x = \frac{1}{4}$$

As the equation is of 2nd degree, so we need to solve a quadratic equation.

First we will substitute trigonometric ratio with some variable k and we will solve for k

As, $\sin^2 x = 1 - \cos^2 x$

∴ we have,

$$1 - cos^2\ x - cos\ x = \frac{1}{4}$$

$\Rightarrow 4 - 4\cos^2 x - 4\cos x = 1$

$\Rightarrow 4\cos^2 x + 4\cos x - 3 = 0$

Let, cos x = k

$\therefore 4k^2 + 4k - 3 = 0$

$\Rightarrow 4k^2 - 2k + 6k - 3$

$\Rightarrow 2k(2k - 1) + 3(2k - 1) = 0$

$\Rightarrow (2k - 1)(2k + 3) = 0$

$\therefore k = 1/2$ or $k = -3/2$

⇒ cos x = � or cos x = -3/2

As cos x lies between -1 and 1

∴ cos x can’t be -3/2

So we ignore that value.

∴ cos x = �

⇒ cos x = cos 60° = cos π/3

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

On comparing our equation with standard form, we have

y = π/3

∴ $\boldsymbol{x = 2n\pi \pm \frac{\pi}{3}}$ **where n ϵ Z ..ans**

3 A. Question

Solve the following equations :

$$\sin^2 x - \cos x = \frac{1}{4}$$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

$$sin^2\,x - cos\,x = \frac{1}{4}$$

As the equation is of 2nd degree, so we need to solve a quadratic equation.

First we will substitute trigonometric ratio with some variable k and we will solve for k

As, $\sin^2 x = 1 - \cos^2 x$

∴ we have,

$$1 - cos^2\,x - cos\,x = \frac{1}{4}$$

⇒ $4 - 4\cos^2 x - 4\cos x = 1$

⇒ $4\cos^2 x + 4\cos x - 3 = 0$

Let, cos x = k

∴ $4k^2 + 4k - 3 = 0$

⇒ $4k^2 - 2k + 6k - 3$

⇒ 2k(2k – 1) +3(2k – 1) = 0

⇒ (2k – 1)(2k + 3) = 0

∴ k =1/2 or k = -3/2

⇒ cos x = � or cos x = -3/2

As cos x lies between -1 and 1

∴ cos x can't be -3/2

So we ignore that value.

∴ cos x = �

⇒ cos x = cos 60° = cos π/3

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

On comparing our equation with standard form, we have

y = π/3

∴ $\boldsymbol{x = 2n\pi \pm \frac{\pi}{3}}$ **where n ϵ Z ..ans**

3 B. Question

Solve the following equations :

2 $\cos^2$ x – 5 cos x + 2 =0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(– 1)^n$ y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

2 $\cos^2$ x – 5 cos x + 2 =0

As the equation is of 2^{nd} degree, so we need to solve a quadratic equation.

First we will substitute trigonometric ratio with some variable k and we will solve for k

Let, cos x = k

∴ $2k^2$ – 5k + 2 = 0

⇒ $2k^2$ – 4k – k +2 = 0

⇒ 2k(k – 2) -1(k -2) = 0

⇒ (k – 2)(2k - 1) = 0

∴ k = 2 or k = �

⇒ cos x = 2 {which is not possible} or cos x = � (acceptable)

∴ cos x = �

⇒ cos x = cos 60° = cos π/3

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

On comparing our equation with standard form, we have

y = π/3

∴ $\boldsymbol{x = 2n\pi \pm \frac{\pi}{3}}$ **where n ϵ Z ..ans**

3 B. Question

Solve the following equations :

$2\cos^2 x - 5\cos x + 2 = 0$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies $x = n\pi + (-1)^n y$, where $n \in Z$.

• cos x = cos y, implies $x = 2n\pi \pm y$, where $n \in Z$.

• tan x = tan y, implies $x = n\pi + y$, where $n \in Z$.

given,

$2\cos^2 x - 5\cos x + 2 = 0$

As the equation is of 2nd degree, so we need to solve a quadratic equation.

First we will substitute trigonometric ratio with some variable k and we will solve for k

Let, cos x = k

$\therefore 2k^2 - 5k + 2 = 0$

$\Rightarrow 2k^2 - 4k - k + 2 = 0$

$\Rightarrow 2k(k-2) - 1(k-2) = 0$

$\Rightarrow (k-2)(2k-1) = 0$

∴ k = 2 or k = �

⇒ cos x = 2 {which is not possible} or cos x = � (acceptable)

∴ cos x = �

$\Rightarrow \cos x = \cos 60° = \cos \pi/3$

If cos x = cos y, implies $x = 2n\pi \pm y$, where $n \in Z$.

On comparing our equation with standard form, we have

$y = \pi/3$

$\therefore \boldsymbol{x = 2n\pi \pm \frac{\pi}{3}}$ **where n ϵ Z ..ans**

3 C. Question

Solve the following equations :

$2\sin^2 x + \sqrt{3}\cos x + 1 = 0$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies $x = n\pi + (-1)^n y$, where $n \in Z$.

• cos x = cos y, implies $x = 2n\pi \pm y$, where $n \in Z$.

• tan x = tan y, implies $x = n\pi + y$, where $n \in Z$.

given,

$2\sin^2 x + \sqrt{3}\cos x + 1 = 0$

As the equation is of 2^{nd} degree, so we need to solve a quadratic equation.

First we will substitute trigonometric ratio with some variable k and we will solve for k

As, $\sin^2 x = 1 - \cos^2 x$

∴ we have,

$2(1 - cos^2 x) + \sqrt{3}\cos x + 1 = 0$

⇒ $2 - 2\cos^2 x + \sqrt{3}\cos x + 1 = 0$

⇒ $2\cos^2 x - \sqrt{3}\cos x - 3 = 0$

Let, cos x = k

∴ $2k^2 - \sqrt{3}k - 3 = 0$

⇒ $2k^2 - 2\sqrt{3}k + \sqrt{3}k - 3 = 0$

⇒ $2k(k - \sqrt{3}) + \sqrt{3}(k - \sqrt{3}) = 0$

⇒ $(2k + \sqrt{3})(k - \sqrt{3}) = 0$

∴ $k = \sqrt{3}$ or $k = -\sqrt{3}/2$

⇒ $\cos x = \sqrt{3}$ or $\cos x = -\sqrt{3}/2$

As cos x lies between -1 and 1

∴ cos x can't be √3

So we ignore that value.

∴ $\cos x = -\sqrt{3}/2$

⇒ $\cos x = \cos 150° = \cos 5\pi/6$

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

On comparing our equation with standard form, we have

$y = 5\pi/6$

∴ $\boldsymbol{x = 2n\pi \pm \frac{5\pi}{6}}$ **where n ϵ Z ..ans**

3 C. Question

Solve the following equations :

$$2\sin^2 x + \sqrt{3}\cos x + 1 = 0$$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

$2\sin^2 x + \sqrt{3}\cos x + 1 = 0$

As the equation is of 2nd degree, so we need to solve a quadratic equation.

First we will substitute trigonometric ratio with some variable k and we will solve for k

As, $\sin^2 x = 1 - \cos^2 x$

∴ we have,

$2(1 - cos^2 x) + \sqrt{3}\cos x + 1 = 0$

⇒ $2 - 2\cos^2 x + \sqrt{3}\cos x + 1 = 0$

⇒ $2\cos^2 x - \sqrt{3}\cos x - 3 = 0$

Let, cos x = k

∴ $2k^2 - \sqrt{3}\,k - 3 = 0$

⇒ $2k^2 - 2\sqrt{3}\,k + \sqrt{3}\,k - 3 = 0$

⇒ $2k(k - \sqrt{3}) + \sqrt{3}(k - \sqrt{3}) = 0$

⇒ $(2k + \sqrt{3})(k - \sqrt{3}) = 0$

∴ $k = \sqrt{3}$ or $k = -\sqrt{3}/2$

⇒ $\cos x = \sqrt{3}$ or $\cos x = -\sqrt{3}/2$

As cos x lies between -1 and 1

∴ cos x can't be √3

So we ignore that value.

∴ $\cos x = -\sqrt{3}/2$

⇒ $\cos x = \cos 150° = \cos 5\pi/6$

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

On comparing our equation with standard form, we have

$y = 5\pi/6$

∴ $\boldsymbol{x = 2n\pi \pm \frac{5\pi}{6}}$ **where n ϵ Z ..ans**

3 D. Question

Solve the following equations :

$4\sin^2 x - 8\cos x + 1 = 0$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

- sin x = sin y, implies $x = n\pi + (-1)^n y$, where n ∈ Z.
- cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.
- tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

$4\sin^2 x - 8\cos x + 1 = 0$

As the equation is of 2nd degree, so we need to solve a quadratic equation.

First we will substitute trigonometric ratio with some variable k and we will solve for k

As, $\sin^2 x = 1 - \cos^2 x$

∴ we have,

$4(1 - cos^2 x) - 8\cos x + 1 = 0$

⇒ $4 - 4\cos^2 x - 8\cos x + 1 = 0$

⇒ $4\cos^2 x + 8\cos x - 5 = 0$

Let, cos x = k

∴ $4k^2 + 8k - 5 = 0$

⇒ $4k^2 - 2k + 10k - 5 = 0$

⇒ 2k(2k – 1) +5(2k – 1) = 0

⇒ (2k + 5)(2k - 1) = 0

∴ k = -5/2 = -2.5 or k = 1/2

⇒ cos x = -2.5 or cos x = 1/2

As cos x lies between -1 and 1

∴ cos x can't be -2.5

So we ignore that value.

∴ cos x = 1/2

⇒ cos x = cos 60° = cos π/3

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

On comparing our equation with standard form, we have

y = π/3

∴ $\boldsymbol{x = 2n\pi \pm \frac{\pi}{3}}$ **where n ϵ Z ..ans**

3 D. Question

Solve the following equations :

$4\sin^2 x - 8\cos x + 1 = 0$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

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⇒ $4k^2 - 2k + 10k - 5 = 0$

⇒ 2k(2k – 1) +5(2k – 1) = 0

⇒ (2k + 5)(2k - 1) = 0

∴ k = -5/2 = -2.5 or k = 1/2

⇒ cos x = -2.5 or cos x = 1/2

As cos x lies between -1 and 1

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So we ignore that value.

∴ cos x = 1/2

⇒ cos x = cos 60° = cos π/3

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

On comparing our equation with standard form, we have

y = π/3

∴ $\boldsymbol{x = 2n\pi \pm \frac{\pi}{3}}$ **where n ϵ Z ..ans**

3 E. Question

Solve the following equations :

$$\tan^2 x + \left(1 - \sqrt{3}\right)\tan x - \sqrt{3} = 0$$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$tan^2 x + \left(1 - \sqrt{3}\right) tan\, x - \sqrt{3} = 0$

⇒ $\tan^2 x + \tan x - \sqrt{3}\tan x - \sqrt{3} = 0$

⇒ $\tan x (\tan x + 1) - \sqrt{3}(\tan x + 1) = 0$

⇒ $(\tan x + 1)(\tan x - \sqrt{3}) = 0$

∴ tan x = -1 or tan x = √3

As, tan x ϵ (-∞ , ∞) so both values are valid and acceptable.

⇒ tan x = tan (-π/4) or tan x = tan (π/3)

If tan x = tan y, implies x = nπ + y, where n ∈ Z.

Clearly by comparing standard form with obtained equation we have

y = -π/4 or y = π/3

$\therefore x = m\pi - \frac{\pi}{4}$ or $x = n\pi + \frac{\pi}{3}$

Hence,

$\boldsymbol{x = m\pi - \frac{\pi}{4}\ or\ n\pi + \frac{\pi}{3}}$ **,where m,n ϵ Z**

3 E. Question

Solve the following equations :

$$\tan^2 x + \left(1 - \sqrt{3}\right)\tan x - \sqrt{3} = 0$$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)ⁿ y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$$tan^2\, x + \left(1 - \sqrt{3}\right) tan\, x - \sqrt{3} = 0$$

⇒ $\tan^2 x\ + \tan x\ - \sqrt{3}\tan x - \sqrt{3} = 0$

⇒ $\tan x\ (\tan x + 1) - \sqrt{3}(\tan x + 1) = 0$

⇒ $(\tan x + 1)(\tan x - \sqrt{3}) = 0$

∴ tan x = -1 or tan x = √3

As, tan x ϵ (-∞ , ∞) so both values are valid and acceptable.

⇒ tan x = tan (-π/4) or tan x = tan (π/3)

If tan x = tan y, implies x = nπ + y, where n ∈ Z.

Clearly by comparing standard form with obtained equation we have

y = -π/4 or y = π/3

$\therefore x = m\pi - \frac{\pi}{4}$ or $x = n\pi + \frac{\pi}{3}$

Hence,

$\boldsymbol{x = m\pi - \frac{\pi}{4}\ or\ n\pi + \frac{\pi}{3}}$ **,where m,n ϵ Z**

3 F. Question

Solve the following equations :

$$3\cos^2 x - 2\sqrt{3}\sin x\cos x - 3\sin^2 x = 0$$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

$3\,cos^2 x - 2\sqrt{3}\,sin\,x\,cos\,x - 3\,sin^2 x = 0$

⇒ $3\,cos^2 x - 3\sqrt{3}\,sin\,x\,cos\,x + \sqrt{3}\sin x\cos x - 3\,sin^2 x = 0$

⇒ $3\cos x\,(\cos x - \sqrt{3}\sin x) + \sqrt{3}\sin x\,(\cos x - \sqrt{3}\,sinx) = 0$

⇒ $\sqrt{3}\,(\cos x - \sqrt{3}\sin x)(\sqrt{3}\cos x + \sin x) = 0$

∴ either, $\cos x\ - \sqrt{3}\sin x = 0$ or $\sin x + \sqrt{3}\cos x = 0$

⇒ $\cos x = \sqrt{3}\sin x$ or $\sin x = -\sqrt{3}\cos x$

⇒ $\tan x = \frac{1}{\sqrt{3}}$ or $\tan x = -\sqrt{3}$

⇒ $\tan x = \tan\frac{\pi}{6}$ or $\tan x = \tan(-\frac{\pi}{3})$

If tan x = tan y, implies x = nπ + y, where n ∈ Z.

Clearly by comparing standard form with obtained equation we have:

y = π/6 or y = -π/3

∴ $x = m\pi + \frac{\pi}{6}$ or $x = n\pi - \frac{\pi}{3}$

Hence,

$\boldsymbol{x = m\pi + \frac{\pi}{6}\ or\ n\pi - \frac{\pi}{3}}$ **,where m,n ϵ Z**

3 F. Question

Solve the following equations :

$$3\cos^2 x - 2\sqrt{3}\sin x\cos x - 3\sin^2 x = 0$$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

$3\,cos^2 x - 2\sqrt{3}\,sin\,x\,cos\,x - 3\,sin^2 x = 0$

⇒ $3\,cos^2 x - 3\sqrt{3}\,sin\,x\,cos\,x + \sqrt{3}\sin x\cos x - 3\,sin^2 x = 0$

⇒ $3\cos x\,(\cos x - \sqrt{3}\sin x) + \sqrt{3}\sin x\,(\cos x - \sqrt{3}\,sinx) = 0$

⇒ $\sqrt{3}\,(\cos x - \sqrt{3}\sin x)(\sqrt{3}\cos x + \sin x) = 0$

∴ either, $\cos x - \sqrt{3}\sin x = 0$ or $\sin x + \sqrt{3}\cos x = 0$

⇒ $\cos x = \sqrt{3}\sin x$ or $\sin x = -\sqrt{3}\cos x$

⇒ $\tan x = \frac{1}{\sqrt{3}}$ or $\tan x = -\sqrt{3}$

⇒ $\tan x = \tan\frac{\pi}{6}$ or $\tan x = \tan(-\frac{\pi}{3})$

If tan x = tan y, implies x = nπ + y, where n ∈ Z.

Clearly by comparing standard form with obtained equation we have:

y = π/6 or y = -π/3

∴ $x = m\pi + \frac{\pi}{6}$ or $x = n\pi - \frac{\pi}{3}$

Hence,

$\boldsymbol{x = m\pi + \frac{\pi}{6} \; or \; n\pi - \frac{\pi}{3}}$ **,where m,n ϵ Z**

3 G. Question

Solve the following equations :

cos 4x = cos 2x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\cos 4x = \cos 2x$

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

From above expression and on comparison with standard equation we have:

y = 2x

∴ $4x = 2n\pi \pm 2x$

Hence,

$4x = 2n\pi + 2x$ or $4x = 2m\pi - 2x$

∴ $2x = 2n\pi$ or $6x = 2m\pi$

⇒ x = nπ or $x = \frac{2m\pi}{6} = \frac{m\pi}{3}$

∴ $\boldsymbol{x = n\pi \; or \; \frac{m\pi}{3}}$ **where m, n ϵ Z ..ans**

3 G. Question

Solve the following equations :

cos 4x = cos 2x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$\cos 4x = \cos 2x$

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

From above expression and on comparison with standard equation we have:

y = 2x

∴ $4x = 2n\pi \pm 2x$

Hence,

$4x = 2n\pi + 2x$ or $4x = 2m\pi - 2x$

∴ $2x = 2n\pi$ or $6x = 2m\pi$

⇒ x = nπ or $x = \frac{2m\pi}{6} = \frac{m\pi}{3}$

∴ $x = n\pi$ or $\frac{m\pi}{3}$ where m, n ϵ Z ..ans

4 A. Question

Solve the following equations :

cos x + cos 2x + cos 3x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

cos x + cos 2x + cos 3x = 0

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

As, cos x + cos 2x + cos 3x = 0

∴ we will use cos x and cos 2x for transformation as after transformation it will give cos 2x term which can be taken common.

∴ cos x + cos 2x + cos 3x = 0

⇒ cos 2x + (cos x + cos 3x) = 0

{$\because$ cos A + cos B = $2\cos\left(\frac{A+B}{2}\right)\cos(\frac{A-B}{2})$ }

⇒ cos 2x + 2 $\cos(\frac{3x+x}{2})\ \cos\frac{3x-x}{2} = 0$

⇒ cos 2x + 2cos 2x cos x = 0

⇒ cos 2x (1 + 2 cos x) = 0

∴ cos 2x = 0 or 1 + 2cos x = 0

⇒ cos 2x = cos π/2 or cos x = -1/2

⇒ cos 2x = cos π/2 or cos x = cos (π - π/3) = cos (2π /3)

If cos x = cos y implies x = 2nπ ± y, where n ∈ Z.

From above expression and on comparison with standard equation we have:

y = π/2 or y = 2π/3

∴ 2x = 2nπ ± π/2 or x = 2mπ ± 2π/3

∴ $\boldsymbol{x = n\pi \pm \frac{\pi}{4}\ or\ x =\ 2m\pi\ \pm\ \frac{2\pi}{3}}$ **where m, n ϵ Z**

4 A. Question

Solve the following equations :

cos x + cos 2x + cos 3x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

cos x + cos 2x + cos 3x = 0

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

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As, cos x + cos 2x + cos 3x = 0

∴ we will use cos x and cos 2x for transformation as after transformation it will give cos 2x term which can be taken common.

∴ cos x + cos 2x + cos 3x = 0

⇒ cos 2x + (cos x + cos 3x) = 0

{$\because$ cos A + cos B = $2\cos\left(\frac{A+B}{2}\right)\cos(\frac{A-B}{2})$ }

⇒ cos 2x + 2 $\cos(\frac{3x+x}{2})\ \cos\frac{3x-x}{2} = 0$

⇒ cos 2x + 2cos 2x cos x = 0

⇒ cos 2x (1 + 2 cos x) = 0

∴ cos 2x = 0 or 1 + 2cos x = 0

⇒ cos 2x = cos π/2 or cos x = -1/2

⇒ cos 2x = cos π/2 or cos x = cos (π - π/3) = cos (2π /3)

If cos x = cos y implies x = 2nπ ± y, where n ∈ Z.

From above expression and on comparison with standard equation we have:

y = π/2 or y = 2π/3

∴ 2x = 2nπ ± π/2 or x = 2mπ ± 2π/3

∴ $x = n\pi \pm \frac{\pi}{4}$ $or\ x = \ 2m\pi\ \pm\ \frac{2\pi}{3}$ **where m, n ϵ Z**

4 B. Question

Solve the following equations :

cos x + cos 3x – cos 2x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(–1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

cos x - cos 2x + cos 3x = 0

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

As, cos x - cos 2x + cos 3x = 0

∴ we will use cos x and cos 2x for transformation as after transformation it will give cos 2x term which can be taken common.

∴ cos x - cos 2x + cos 3x = 0

⇒ -cos 2x + (cos x + cos 3x) = 0

{∵ cos A + cos B = $2\cos\left(\frac{A+B}{2}\right)\cos(\frac{A-B}{2})$ }

⇒ -cos 2x + 2 $\cos(\frac{3x+x}{2})\ \cos\frac{3x-x}{2} = 0$

⇒ -cos 2x + 2cos 2x cos x = 0

⇒ cos 2x (-1 + 2 cos x) = 0

∴ cos 2x = 0 or 1 + 2cos x = 0

⇒ cos 2x = cos π/2 or cos x = 1/2

⇒ cos 2x = cos π/2 or cos x = cos (π/3) = cos (π /3)

If cos x = cos y implies x = 2nπ ± y, where n ∈ Z.

From above expression and on comparison with standard equation we have:

y = π/2 or y = π/3

∴ 2x = 2nπ ± π/2 or x = 2mπ ± π/3

∴ $x = n\pi \pm \frac{\pi}{4}$ $or\ x = \ 2m\pi\ \pm \frac{\pi}{3}$ where m, n ϵ Z

4 B. Question

Solve the following equations :

cos x + cos 3x – cos 2x = 0

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• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

cos x - cos 2x + cos 3x = 0

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

As, cos x - cos 2x + cos 3x = 0

∴ we will use cos x and cos 2x for transformation as after transformation it will give cos 2x term which can be taken common.

∴ cos x - cos 2x + cos 3x = 0

⇒ -cos 2x + (cos x + cos 3x) = 0

{∵ cos A + cos B = 2$\cos\left(\frac{A+B}{2}\right)\cos(\frac{A-B}{2})$ }

⇒ -cos 2x + 2 cos$(\frac{3x+x}{2})$ $\cos\frac{3x-x}{2} = 0$

⇒ -cos 2x + 2cos 2x cos x = 0

⇒ cos 2x (-1 + 2 cos x) = 0

∴ cos 2x = 0 or 1 + 2cos x = 0

⇒ cos 2x = cos π/2 or cos x = 1/2

⇒ cos 2x = cos π/2 or cos x = cos (π/3) = cos (π /3)

If cos x = cos y implies x = 2nπ ± y, where n ∈ Z.

From above expression and on comparison with standard equation we have:

y = π/2 or y = π/3

∴ 2x = 2nπ ± π/2 or x = 2mπ ± π/3

∴ $x = n\pi \pm \frac{\pi}{4}$ $or\ x = \ 2m\pi\ \pm \frac{\pi}{3}$ where m, n ϵ Z

4 C. Question

Solve the following equations :

sin x + sin 5x = sin 3x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

sin x + sin 5x = sin 3x

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

As, sin x + sin 5x = sin 3x

∴ sin x + sin 5x – sin 3x = 0

∴ we will use sin x and sin 5x for transformation as after transformation it will give sin 3x term which can be taken common.

{∵ sin A + sin B = $2\sin\left(\frac{A+B}{2}\right)\cos(\frac{A-B}{2})$ }

⇒ -sin 3x + 2 sin$(\frac{5x+x}{2})$ cos$\frac{5x-x}{2} = 0$

⇒ 2sin 3x cos 2x – sin 3x = 0

⇒ sin 3x (2cos 2x – 1) = 0

∴ either, sin 3x = 0 or 2cos 2x – 1 = 0

⇒ sin 3x = sin 0 or cos 2x = � = cos π/3

If sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

Comparing obtained equation with standard equation, we have:

3x = nπ or 2x = 2mπ ± π/3

∴ $\boldsymbol{x = \frac{n\pi}{3}\ or\ x = m\pi \pm \frac{\pi}{6}}$ **where m,n ϵ Z ..ans**

4 C. Question

Solve the following equations :

sin x + sin 5x = sin 3x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

sin x + sin 5x = sin 3x

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

As, sin x + sin 5x = sin 3x

∴ sin x + sin 5x – sin 3x = 0

∴ we will use sin x and sin 5x for transformation as after transformation it will give sin 3x term which can be taken common.

{∵ sin A + sin B = $2\sin\left(\frac{A+B}{2}\right)\cos(\frac{A-B}{2})$ }

⇒ -sin 3x + 2 sin$(\frac{5x+x}{2})$ $\cos\frac{5x-x}{2} = 0$

⇒ 2sin 3x cos 2x – sin 3x = 0

⇒ sin 3x (2cos 2x – 1) = 0

∴ either, sin 3x = 0 or 2cos 2x – 1 = 0

⇒ sin 3x = sin 0 or cos 2x = � = cos π/3

If sin x = sin y, implies x = nπ + $(– 1)^n$y, where n ∈ Z.

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

Comparing obtained equation with standard equation, we have:

3x = nπ or 2x = 2mπ ± π/3

∴ $\boldsymbol{x = \frac{n\pi}{3}\ or\ x = m\pi\ \pm\ \frac{\pi}{6}}$ **where m,n ϵ Z ..ans**

4 D. Question

Solve the following equations :

cos x cos 2x cos 3x = �

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(– 1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

cos x cos 2x cos 3x = �

⇒ 4cos x cos 2x cos 3x – 1 = 0

{∵ 2 cos A cos B = cos (A + B) + cos (A – B)}

∴ 2(2cos x cos 3x)cos 2x – 1 = 0

⇒ 2(cos 4x + cos 2x)cos2x – 1 = 0

⇒ $2(2\cos^2 2x - 1 + \cos 2x)\cos 2x - 1 = 0$ {using $\cos 2\theta = 2\cos^2\theta - 1$ }

⇒ $4\cos^3 2x - 2\cos 2x + 2\cos^2 2x - 1 = 0$

⇒ $2\cos^2 2x (2\cos 2x + 1) -1(2\cos 2x + 1) = 0$

⇒ $(2\cos^2 2x - 1)(2 \cos 2x + 1) = 0$

∴ either, 2cos 2x + 1 = 0 or $(2\cos^2 2x - 1) = 0$

⇒ cos 2x = -1/2 or cos 4x = 0 {using $\cos 2\theta = 2\cos^2\theta - 1$}

⇒ cos 2x = cos (π - π/3) = cos 2π /3 or cos 4x = cos π/2

If cos x = cos y implies x = 2nπ ± y, where n ∈ Z.

In case of cos x = 0 we can give solution directly as cos x = 0 is true for x = odd multiple of π/2

Comparing obtained equation with standard equation, we have:

y = 2π / 3 or y = π/2

∴ 2x = 2mπ ± 2π/3 or 4x = (2n+1)π/2

∴ $x = m\pi \pm \frac{\pi}{3}$ or $x = (2n+1)\frac{\pi}{8}$ where m,n ϵ Z ….ans

4 D. Question

Solve the following equations :

cos x cos 2x cos 3x = �

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies $x = n\pi + (-1)^n y$, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

cos x cos 2x cos 3x = �

⇒ 4cos x cos 2x cos 3x – 1 = 0

{∵ 2 cos A cos B = cos (A + B) + cos (A – B)}

∴ 2(2cos x cos 3x)cos 2x – 1 = 0

⇒ 2(cos 4x + cos 2x)cos2x – 1 = 0

⇒ $2(2\cos^2 2x - 1 + \cos 2x)\cos 2x - 1 = 0$ {using $\cos 2\theta = 2\cos^2\theta - 1$ }

⇒ $4\cos^3 2x - 2\cos 2x + 2\cos^2 2x - 1 = 0$

⇒ $2\cos^2 2x (2\cos 2x + 1) -1(2\cos 2x + 1) = 0$

⇒ $(2\cos^2 2x - 1)(2 \cos 2x + 1) = 0$

∴ either, 2cos 2x + 1 = 0 or $(2\cos^2 2x - 1) = 0$

⇒ cos 2x = -1/2 or cos 4x = 0 {using $\cos 2\theta = 2\cos^2\theta - 1$}

⇒ cos 2x = cos (π - π/3) = cos 2π /3 or cos 4x = cos π/2

If cos x = cos y implies x = 2nπ ± y, where n ∈ Z.

In case of cos x = 0 we can give solution directly as cos x = 0 is true for x = odd multiple of π/2

Comparing obtained equation with standard equation, we have:

y = 2π / 3 or y = π/2

∴ 2x = 2mπ ± 2π/3 or 4x = (2n+1)π/2

∴ $\boldsymbol{x = m\pi \pm \frac{\pi}{3} \; or \; x = (2n+1)\frac{\pi}{8}}$ where m,n ϵ Z ….ans

4 E. Question

Solve the following equations :

cos x + sin x = cos 2x + sin 2x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

cos x + sin x = cos 2x + sin 2x

cos x – cos 2x = sin 2x – sin x

{∵ sin A - sin B = $2\cos\left(\frac{A+B}{2}\right)\sin(\frac{A-B}{2})$ & $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin(\frac{A-B}{2})$ }

$\therefore -2\sin\left(\frac{x+2x}{2}\right)\sin\left(\frac{x-2x}{2}\right) = 2\cos(\frac{2x+x}{2})\sin(\frac{2x-x}{2})$

$\Rightarrow 2\sin\frac{3x}{2}\sin\frac{x}{2} = 2\cos\frac{3x}{2}\sin\frac{x}{2}$

$\therefore \sin\frac{x}{2}\,(\sin\frac{3x}{2} - \cos\frac{3x}{2}) = 0$

Hence,

Either, $\sin\frac{x}{2} = 0 \; or \; \sin\frac{3x}{2} = \cos\frac{3x}{2}$

$\Rightarrow \sin\frac{x}{2} = \sin m\pi \; or \; \tan\frac{3x}{2} = 1 = \tan\frac{\pi}{4}$

If tan x = tan y, implies x = nπ + y, where n ∈ Z.

$\therefore \frac{x}{2} = m\pi \; or \; \frac{3x}{2} = n\pi + \frac{\pi}{4}$

$\Rightarrow \boldsymbol{x = 2m\pi \; or \; x} = \frac{2n\pi}{3} + \frac{\pi}{6}$ where m,n ϵ Z ….**ans**

4 E. Question

Solve the following equations :

cos x + sin x = cos 2x + sin 2x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

cos x + sin x = cos 2x + sin 2x

cos x – cos 2x = sin 2x – sin x

{∵ sin A - sin B = $2\cos\left(\frac{A+B}{2}\right)\sin(\frac{A-B}{2})$ & $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin(\frac{A-B}{2})$ }

$\therefore -2\sin\left(\frac{x+2x}{2}\right)\sin\left(\frac{x-2x}{2}\right) = 2\cos(\frac{2x+x}{2})\sin(\frac{2x-x}{2})$

$\Rightarrow 2\sin\frac{3x}{2}\sin\frac{x}{2} = 2\cos\frac{3x}{2}\sin\frac{x}{2}$

$\therefore \sin\frac{x}{2}\ (\sin\frac{3x}{2} - \cos\frac{3x}{2}) = 0$

Hence,

Either, $\sin\frac{x}{2} = 0\ or\ \sin\frac{3x}{2} = \cos\frac{3x}{2}$

$\Rightarrow \sin\frac{x}{2} = \sin m\pi\ or\ \tan\frac{3x}{2} = 1 = \tan\frac{\pi}{4}$

If tan x = tan y, implies x = nπ + y, where n ∈ Z.

$\therefore \frac{x}{2} = m\pi\ or\ \frac{3x}{2} = n\pi + \frac{\pi}{4}$

$\Rightarrow \boldsymbol{x = 2m\pi\ or\ x = \frac{2n\pi}{3} + \frac{\pi}{6}}$ where m,n ϵ Z ....**ans**

4 F. Question

Solve the following equations :

sin x + sin 2x + sin 3x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

sin x + sin 2x + sin 3x = 0

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

As, sin x + sin 2x + sin 3x = 0

∴ we will use sin x and sin 3x for transformation as after transformation it will give sin 2x term which can be taken common.

{∵ sin A + sin B = $2\sin\left(\frac{A+B}{2}\right)\cos(\frac{A-B}{2})$ }

⇒ sin 2x + 2 sin($\frac{3x+x}{2}$) $\cos\frac{3x-x}{2} = 0$

⇒ 2sin 2x cos x + sin 2x = 0

⇒ sin 2x (2cos x + 1) = 0

∴ either, sin 2x = 0 or 2cos x + 1 = 0

⇒ sin 2x = sin 0 or cos x = - � = cos (π-π/3) = cos 2π/3

If sin x = sin y, implies x = nπ + $(– 1)^n$y, where n ∈ Z.

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

Comparing obtained equation with standard equation, we have:

2x = nπ or x = 2mπ ± 2π/3

∴ $x = \frac{n\pi}{2}$ $or\ x = 2m\pi\ \pm\ \frac{2\pi}{3}$ **where m,n ϵ Z ..ans**

4 F. Question

Solve the following equations :

sin x + sin 2x + sin 3x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(– 1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

sin x + sin 2x + sin 3x = 0

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

As, sin x + sin 2x + sin 3x = 0

∴ we will use sin x and sin 3x for transformation as after transformation it will give sin 2x term which can be taken common.

{∵ sin A + sin B = $2\sin\left(\frac{A+B}{2}\right)\cos(\frac{A-B}{2})$ }

⇒ sin 2x + 2 sin($\frac{3x+x}{2}$) $\cos\frac{3x-x}{2} = 0$

⇒ 2sin 2x cos x + sin 2x = 0

⇒ sin 2x (2cos x + 1) = 0

∴ either, sin 2x = 0 or 2cos x + 1 = 0

⇒ sin 2x = sin 0 or cos x = - � = cos (π-π/3) = cos 2π/3

If sin x = sin y, implies x = nπ + $(– 1)^n$y, where n ∈ Z.

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

Comparing obtained equation with standard equation, we have:

2x = nπ or x = 2mπ ± 2π/3

$\therefore \boldsymbol{x = \frac{n\pi}{2}\ or\ x = 2m\pi \pm \frac{2\pi}{3}}$ **where m,n ϵ Z ..ans**

4 G. Question

Solve the following equations :

sin x + sin 2x + sin 3x + sin 4x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

sin x + sin 2x + sin 3x + sin 4x = 0

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

As, sin x + sin 2x + sin 3x + sin 4x= 0

∴ we will use sin x and sin 3x together in 1 group for transformation and sin 4x and sin 2x common in other group as after transformation both will give cos x term which can be taken common.

{∵ sin A + sin B = $2\sin\left(\frac{A+B}{2}\right)\cos(\frac{A-B}{2})$ }

(sin x + sin 3x)+(sin 2x + sin 4x) = 0

⇒ 2 sin$(\frac{4x+2x}{2})$ cos $\frac{4x-2x}{2}$ + 2 sin$(\frac{3x+x}{2})$ cos$\frac{3x-x}{2} = 0$

⇒ 2sin 2x cos x + 2sin 3x cos x= 0

⇒ 2cos x (sin 2x + sin 3x) = 0

Again using transformation formula, we have:

⇒ 2 cos x 2$\sin\frac{3x+2x}{2}\cos\frac{3x-2x}{2} = 0$

⇒ $4\cos x\sin\frac{5x}{2}\cos\frac{x}{2}\ = 0$

∴ either, cos x = 0 or $\sin\frac{5x}{2} = 0$ or $\cos\frac{x}{2} = 0$

In case of cos x = 0 we can give solution directly as cos x = 0 is true for x = odd multiple of π/2

In case of sin x = 0 we can give solution directly as sin x = 0 is true for x = integral multiple of π

$\therefore x = (2n+1)\frac{\pi}{2}\ or\ \frac{5x}{2} = k\pi\ or\ \frac{x}{2} = (2p+1)\frac{\pi}{2}$

⇒ $\boldsymbol{x = (2n+1)\frac{\pi}{2}\ or\ x = \frac{2k\pi}{5}\ or\ x = (2p+1)\pi}$ **where n,p,m ϵ Z**

4 G. Question

Solve the following equations :

sin x + sin 2x + sin 3x + sin 4x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

sin x + sin 2x + sin 3x + sin 4x = 0

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

As, sin x + sin 2x + sin 3x + sin 4x= 0

∴ we will use sin x and sin 3x together in 1 group for transformation and sin 4x and sin 2x common in other group as after transformation both will give cos x term which can be taken common.

{∵ sin A + sin B = $2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ }

(sin x + sin 3x)+(sin 2x + sin 4x) = 0

⇒ 2 sin$\left(\frac{4x+2x}{2}\right)$ $\cos\frac{4x-2x}{2}$ + 2 sin$\left(\frac{3x+x}{2}\right)$ $\cos\frac{3x-x}{2} = 0$

⇒ 2sin 2x cos x + 2sin 3x cos x= 0

⇒ 2cos x (sin 2x + sin 3x) = 0

Again using transformation formula, we have:

⇒ 2 cos x $2\sin\frac{3x+2x}{2}\cos\frac{3x-2x}{2} = 0$

⇒ $4\cos x\sin\frac{5x}{2}\cos\frac{x}{2} = 0$

∴ either, cos x = 0 or $\sin\frac{5x}{2} = 0$ or $\cos\frac{x}{2} = 0$

In case of cos x = 0 we can give solution directly as cos x = 0 is true for x = odd multiple of π/2

In case of sin x = 0 we can give solution directly as sin x = 0 is true for x = integral multiple of π

$\therefore x = (2n+1)\frac{\pi}{2}\ or\ \frac{5x}{2} = k\pi\ or\ \frac{x}{2} = (2p+1)\frac{\pi}{2}$

⇒ $\boldsymbol{x = (2n+1)\frac{\pi}{2}\ or\ x = \frac{2k\pi}{5}\ or\ x = (2p+1)\pi}$ **where n,p,m ϵ Z**

4 H. Question

Solve the following equations :

sin 3x – sin x = 4 $\cos^2$ x – 2

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

sin 3x – sin x = 4 $\cos^2$ x – 2

⇒ sin 3x – sin x = 2(2 $\cos^2$ x – 1)

⇒ sin 3x – sin x = 2 cos 2x {∵ cos 2θ = $2\cos^2$ θ – 1}

{∵ sin A - sin B = $2\cos\left(\frac{A+B}{2}\right)\sin(\frac{A-B}{2})$ }

⇒ $2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right) = 2\cos 2x$

⇒ $2\cos 2x \sin x - 2\cos 2x = 0$

⇒ $2\cos 2x\,(\sin x - 1) = 0$

∴ either, cos 2x = 0 or sin x = 1 = sin π/2

In case of cos x = 0 we can give solution directly as cos x = 0 is true for x = odd multiple of π/2

If sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

∴ $2x = (2n+1)\frac{\pi}{2}$ $or\ x = m\pi + (-1)^m\frac{\pi}{2}$

⇒ $\boldsymbol{x = (2n+1)\frac{\pi}{4}\ or\ x = m\pi + (-1)^m\frac{\pi}{2}}$ **where m, n ϵ Z**

4 H. Question

Solve the following equations :

sin 3x – sin x = 4 $\cos^2$ x – 2

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

sin 3x – sin x = 4 $\cos^2$ x – 2

⇒ sin 3x – sin x = 2(2 $\cos^2$ x – 1)

⇒ sin 3x – sin x = 2 cos 2x {∵ cos 2θ = $2\cos^2$ θ – 1}

{∵ sin A - sin B = $2\cos\left(\frac{A+B}{2}\right)\sin(\frac{A-B}{2})$ }

⇒ $2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right) = 2\cos 2x$

⇒ $2\cos 2x \sin x - 2\cos 2x = 0$

$\Rightarrow 2\cos 2x\,(\sin x - 1) = 0$

∴ either, cos 2x = 0 or sin x = 1 = sin π/2

In case of cos x = 0 we can give solution directly as cos x = 0 is true for x = odd multiple of π/2

If sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

$\therefore 2x = (2n+1)\frac{\pi}{2}\ or\ x = m\pi + (-1)^m\frac{\pi}{2}$

$\Rightarrow \boldsymbol{x = (2n+1)\frac{\pi}{4}\ or\ x = m\pi + (-1)^m\frac{\pi}{2}}$ **where m, n ϵ Z**

4 I. Question

Solve the following equations :

sin 2x – sin 4x + sin 6x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

sin 2x - sin 4x + sin 6x = 0

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

we have, sin 2x - sin 4x + sin 6x = 0

∴ we will use sin 6x and sin 2x for transformation as after transformation it will give sin 4x term which can be taken common.

{∵ sin A + sin B = $2\sin\left(\frac{A+B}{2}\right)\cos(\frac{A-B}{2})$ }

⇒ -sin 4x + 2 sin$(\frac{2x+6x}{2})$ $\cos\frac{6x-2x}{2} = 0$

⇒ 2sin 4x cos 2x – sin 4x = 0

⇒ sin 4x (2cos 2x – 1) = 0

∴ either, sin 4x = 0 or 2cos 2x – 1 = 0

⇒ sin 4x = sin 0 or cos 2x = � = cos π/3

If sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

Comparing obtained equation with standard equation, we have:

4x = nπ or 2x = 2mπ ± π/3

$\therefore \boldsymbol{x = \frac{n\pi}{4}\ or\ x = m\pi \pm \frac{\pi}{6}}$ **where m,n ϵ Z ..ans**

4 I. Question

Solve the following equations :

sin 2x – sin 4x + sin 6x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

sin 2x - sin 4x + sin 6x = 0

To solve the equation we need to change its form so that we can equate the t-ratios individually.

For this we will be applying transformation formulae. While applying the

Transformation formula we need to select the terms wisely which we want

to transform.

we have, sin 2x - sin 4x + sin 6x = 0

∴ we will use sin 6x and sin 2x for transformation as after transformation it will give sin 4x term which can be taken common.

{∵ sin A + sin B = $2\sin\left(\frac{A+B}{2}\right)\cos(\frac{A-B}{2})$ }

⇒ -sin 4x + 2 sin$(\frac{2x+6x}{2})$ $\cos\frac{6x-2x}{2} = 0$

⇒ 2sin 4x cos 2x – sin 4x = 0

⇒ sin 4x (2cos 2x – 1) = 0

∴ either, sin 4x = 0 or 2cos 2x – 1 = 0

⇒ sin 4x = sin 0 or cos 2x = � = cos π/3

If sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

Comparing obtained equation with standard equation, we have:

4x = nπ or 2x = 2mπ ± π/3

∴ $\boldsymbol{x = \frac{n\pi}{4}\ or\ x = m\pi\ \pm\ \frac{\pi}{6}}$ **where m,n ϵ Z ..ans**

5 A. Question

Solve the following equations :

tan x + tan 2x + tan 3x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

tan x + tan 2x + tan 3x = 0

In order to solve the equation we need to reduce the equation into factor form so that we can equate the ratios with 0 and can solve the equation easily

As if we expand tan 3x = tan (x + 2x) we will get tan x + tan 2x common.

∴ tan x + tan 2x + tan 3x = 0

⇒ tan x + tan 2x + tan (x + 2x) = 0

As, tan (A + B) = $\frac{\tan A + \tan B}{1 - \tan A \tan B}$

∴ tan x + tan 2x + $\frac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = 0$

⇒ $(\tan x + \tan 2x)(1 + \frac{1}{1 - \tan x \tan 2x}) = 0$

⇒ $(\tan x + \tan 2x)(\frac{2 - \tan x \tan 2x}{1 - \tan x \tan 2x}) = 0$

∴ tan x + tan 2x = 0 or 2 – tan x tan 2x = 0

Using, tan 2x = $\frac{2 \tan x}{1 - \tan^2 x}$ we have,

⇒ tan x = tan (-2x) or 2 - $\frac{2 \tan^2 x}{1 - \tan^2 x} = 0$

⇒ tan x = tan(-2x) or 2 – 4$\tan^2$ x = 0 ⇒ tan x = 1/ √2

Let 1/√2 = tan α and if tan x = tan y, implies x = nπ + y, where n ∈ Z

∴ x = nπ + (-2x) or tan x = tan α ⇒ x = mπ + α

⇒ 3x = nπ or x = mπ + α

∴ $\boldsymbol{x = \frac{n\pi}{3} \text{ or } x = m\pi + \alpha \text{ where } \tan \alpha = \frac{1}{\sqrt{2}} \text{ and } m, n \in Z}$

5 A. Question

Solve the following equations :

tan x + tan 2x + tan 3x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

tan x + tan 2x + tan 3x = 0

In order to solve the equation we need to reduce the equation into factor form so that we can equate the ratios with 0 and can solve the equation easily

As if we expand tan 3x = tan (x + 2x) we will get tan x + tan 2x common.

∴ tan x + tan 2x + tan 3x = 0

⇒ tan x + tan 2x + tan (x + 2x) = 0

As, tan (A + B) = $\frac{\tan A+\tan B}{1-\tan A\tan B}$

∴ tan x + tan 2x + $\frac{\tan x+\tan 2x}{1-\tan x\tan 2x} = 0$

⇒ $(\tan x + \tan 2x)(1 + \frac{1}{1-\tan x\tan 2x}) = 0$

⇒ $(\tan x + \tan 2x)(\frac{2-\tan x\tan 2x}{1-\tan x tan\, 2x}) = 0$

∴ tan x + tan 2x = 0 or 2 – tan x tan 2x = 0

Using, tan 2x = $\frac{2\tan x}{1-\tan^2 x}$ we have,

⇒ tan x = tan (-2x) or 2 -$\frac{2\tan^2 x}{1-\tan^2 x} = 0$

⇒ tan x = tan(-2x) or 2 – 4$\tan^2$ x = 0 ⇒ tan x = 1/ √2

Let 1/√2 = tan α and if tan x = tan y, implies x = nπ + y, where n ∈ Z

∴ x = nπ + (-2x) or tan x = tan α ⇒ x = mπ + α

⇒ 3x = nπ or x = mπ + α

∴ $\boldsymbol{x = \frac{n\pi}{3}\ or\ x = m\pi + \alpha\ where\ tan\ \alpha = \frac{1}{\sqrt{2}}\ and\ m\ , n\ \epsilon\ Z}$

5 B. Question

Solve the following equations :

tan x + tan 2x = tan 3x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(–1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

tan x + tan 2x - tan 3x = 0

In order to solve the equation we need to reduce the equation into factor form so that we can equate the ratios with 0 and can solve the equation easily

As if we expand tan 3x = tan (x + 2x) we will get tan x + tan 2x common.

∴ tan x + tan 2x - tan 3x = 0

⇒ tan x + tan 2x - tan (x + 2x) = 0

As, tan (A + B) = $\frac{\tan A+\tan B}{1-\tan A\tan B}$

∴ tan x + tan 2x - $\frac{\tan x+\tan 2x}{1-\tan x\tan 2x} = 0$

⇒ $(\tan x + \tan 2x)(1 - \frac{1}{1-\tan x\tan 2x}) = 0$

⇒ $(\tan x + \tan 2x)(\frac{-\tan x\tan 2x}{1-\tan x tan\, 2x}) = 0$

∴ tan x + tan 2x = 0 or – tan x tan 2x = 0

Using, tan 2x = $\frac{2\tan x}{1-\tan^2 x}$ we have,

⇒ tan x = tan (-2x) or $\frac{2\tan^2 x}{1-\tan^2 x} = 0$

⇒ tan x = tan(-2x) or tan x = 0 = tan 0

if tan x = tan y, implies x = nπ + y, where n ∈ Z

∴ x = nπ + (-2x) or x = mπ + 0

⇒ 3x = nπ or x = mπ

∴ $x = \frac{n\pi}{3}$ $or\ x = m\pi\ where\ m\ ,n\ \epsilon\ Z$ **....ans**

5 B. Question

Solve the following equations :

tan x + tan 2x = tan 3x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

tan x + tan 2x - tan 3x = 0

In order to solve the equation we need to reduce the equation into factor form so that we can equate the ratios with 0 and can solve the equation easily

As if we expand tan 3x = tan (x + 2x) we will get tan x + tan 2x common.

∴ tan x + tan 2x - tan 3x = 0

⇒ tan x + tan 2x - tan (x + 2x) = 0

As, tan (A + B) = $\frac{\tan A+\tan B}{1-\tan A\tan B}$

∴ tan x + tan 2x - $\frac{\tan x+\tan 2x}{1-\tan x\tan 2x} = 0$

⇒ $(\tan x + \tan 2x)(1 - \frac{1}{1-\tan x\tan 2x}) = 0$

⇒ $(\tan x + \tan 2x)(\frac{-\tan x\tan 2x}{1-\tan x tan\ 2x}) = 0$

∴ tan x + tan 2x = 0 or – tan x tan 2x = 0

Using, tan 2x = $\frac{2\tan x}{1-\tan^2 x}$ we have,

⇒ tan x = tan (-2x) or $\frac{2\tan^2 x}{1-\tan^2 x} = 0$

⇒ tan x = tan(-2x) or tan x = 0 = tan 0

if tan x = tan y, implies x = nπ + y, where n ∈ Z

∴ x = nπ + (-2x) or x = mπ + 0

⇒ 3x = nπ or x = mπ

∴ $x = \frac{n\pi}{3}$ $or\ x = m\pi\ where\ m\ , n\ \epsilon\ Z$ **....ans**

5 C. Question

Solve the following equations :

tan 3x + tan x = 2 tan 2x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

tan x + tan 3x = 2tan 2x

⇒ tan x + tan 3x = tan 2x + tan 2x

⇒ tan 3x – tan 2x = tan 2x – tan x

⇒ $\frac{(\tan 3x - \tan 2x)(1 + \tan 3x \tan 2x)}{1 + \tan 3x \tan 2x} = \frac{(\tan 2x - \tan x)(1 + \tan x \tan 2x)}{1 + \tan 2x \tan x}$

As, tan (A - B) = $\frac{\tan A - \tan B}{1 + \tan A \tan B}$

∴ $\tan(3x - 2x)(1 + \tan 3x \tan 2x) = \tan(2x - x)\ (1 + \tan x \tan 2x)$

⇒ $\tan x\{1 + \tan 3x \tan 2x - 1 - \tan 2x \tan x\} = 0$

⇒ $\tan x \tan 2x\ (\tan 3x\ - \tan x) = 0$

∴ tan x = 0 or tan 2x = 0 or tan 3x = tan x

if tan x = tan y, implies x = nπ + y, where n ∈ Z

∴ x = nπ or 2x = mπ or 3x = kπ + x

∴ $x = n\pi\ or\ x = \frac{m\pi}{2}\ or\ x = \frac{k\pi}{2}$

∴ $x = n\pi\ or\ x = \frac{m\pi}{2}\ where\ m, n\ \epsilon\ Z\ \ldots.ans$

5 C. Question

Solve the following equations :

tan 3x + tan x = 2 tan 2x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

tan x + tan 3x = 2tan 2x

⇒ tan x + tan 3x = tan 2x + tan 2x

⇒ tan 3x – tan 2x = tan 2x – tan x

$\Rightarrow \frac{(\tan 3x-\tan 2x)(1+\tan 3x \tan 2x)}{1+\tan 3x \tan 2x}=\frac{(\tan 2x-\tan x)(1+\tan x \tan 2x)}{1+\tan 2x \tan x}$

As, tan (A - B) = $\frac{\tan A-\tan B}{1+\tan A \tan B}$

$\therefore \tan(3x-2x)(1+\tan 3x \tan 2x)=\tan(2x-x)\,(1+\tan x \tan 2x)$

$\Rightarrow \tan x\{1+\tan 3x \tan 2x-1-\tan 2x \tan x\}=0$

$\Rightarrow \tan x \tan 2x\,(\tan 3x-\tan x)=0$

∴ tan x = 0 or tan 2x = 0 or tan 3x = tan x

if tan x = tan y, implies x = nπ + y, where n ∈ Z

∴ x = nπ or 2x = mπ or 3x = kπ + x

$\therefore x=n\pi \text{ or } x=\frac{m\pi}{2} \text{ or } x=\frac{k\pi}{2}$

$\therefore \boldsymbol{x=n\pi \text{ or } x=\frac{m\pi}{2} \text{ where } m,n \in Z \;\ldots\text{ans}}$

6 A. Question

Solve the following equations :

$\sin x+\cos x=\sqrt{2}$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

$\sin x+\cos x=\sqrt{2}$

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$\therefore \frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x=1$

$\Rightarrow \sin x \cos\frac{\pi}{4}+\sin\frac{\pi}{4}\cos x=1$ { $\because \sin\frac{\pi}{4}=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$ }

$\Rightarrow \sin\left(\frac{\pi}{4}+x\right)=1$ { ∵ sin A cos B + cos A sin B = sin (A +B)}

$\Rightarrow \sin\left(\frac{\pi}{4}+x\right)=\sin\frac{\pi}{2}$

NOTE: We can also make the ratio of cos instead of sin, the answer remains same but the form of answer may look different, when you put values of n you will get same values with both forms

If sin x = sin y, implies x = nπ + (– 1)^n y, where n ∈ Z

$\therefore \frac{\pi}{4}+x=n\pi+(-1)^n\frac{\pi}{2}$

$\therefore x = n\pi + (-1)^n \frac{\pi}{2} - \frac{\pi}{4}$ *where* $n \epsilon Z$ *.....ans*

6 A. Question

Solve the following equations :

$\sin x + \cos x = \sqrt{2}$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)ⁿy, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

$\sin x + \cos x = \sqrt{2}$

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$\therefore \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = 1$

$\Rightarrow \sin x \cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos x = 1$ { $\because \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ }

$\Rightarrow \sin\left(\frac{\pi}{4} + x\right) = 1$ { ∵ sin A cos B + cos A sin B = sin (A +B)}

$\Rightarrow \sin\left(\frac{\pi}{4} + x\right) = \sin\frac{\pi}{2}$

NOTE: We can also make the ratio of cos instead of sin, the answer remains same but the form of answer may look different, when you put values of n you will get same values with both forms

If sin x = sin y, implies x = nπ + (– 1)ⁿ y, where n ∈ Z

$\therefore \frac{\pi}{4} + x = n\pi + (-1)^n \frac{\pi}{2}$

$\therefore x = n\pi + (-1)^n \frac{\pi}{2} - \frac{\pi}{4}$ *where* $n \epsilon Z$ *.....ans*

6 B. Question

Solve the following equations :

$\sqrt{3}\cos x + \sin x = 1$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)ⁿy, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

$\sqrt{3}\cos x + \sin x = 1$

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$\therefore \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = \frac{1}{2}$

$\Rightarrow \sin x \sin\frac{\pi}{6} + \cos\frac{\pi}{6}\cos x = \frac{1}{2}$ $\{\because \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ *and* $\sin\frac{\pi}{6} = \frac{1}{2}\}$

$\Rightarrow \cos\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$ {∵ cos A cos B + sin A sin B = cos (A - B)}

$\Rightarrow \cos\left(x - \frac{\pi}{6}\right) = \cos\frac{\pi}{3}$

If cos x = cos y, implies x = 2nπ ± y, where n∈ Z

$\therefore x - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}$

$\therefore x = 2n\pi \pm \frac{\pi}{3} + \frac{\pi}{6}$ $where\ n \in Z$

$\therefore x = 2n\pi + \frac{\pi}{2}$ $or\ 2n\pi - \frac{\pi}{6}$ $where\ n \in Z\ \ldots..ans$

6 B. Question

Solve the following equations :

$\sqrt{3}\cos x + \sin x = 1$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

$\sqrt{3}\,cos\,x + sin\,x = 1$

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$\therefore \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = \frac{1}{2}$

$\Rightarrow \sin x \sin\frac{\pi}{6} + \cos\frac{\pi}{6}\cos x = \frac{1}{2}$ $\{\because \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ *and* $\sin\frac{\pi}{6} = \frac{1}{2}\}$

$\Rightarrow \cos\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$ {∵ cos A cos B + sin A sin B = cos (A - B)}

$\Rightarrow \cos\left(x - \frac{\pi}{6}\right) = \cos\frac{\pi}{3}$

If cos x = cos y, implies x = 2nπ ± y, where n∈ Z

$\therefore x - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}$

$\therefore x = 2n\pi \pm \frac{\pi}{3} + \frac{\pi}{6}$ $where\ n \in Z$

$\therefore x = 2n\pi + \frac{\pi}{2}$ $or\ 2n\pi - \frac{\pi}{6}$ $where\ n \in Z\ \ldots..ans$

6 C. Question

Solve the following equations :

sin x + cos x = 1

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

$\sin x + \cos x = 1$.

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$\therefore \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$ { dividing by √2 both sides}

⇒ $\sin x \sin\frac{\pi}{4} + \cos\frac{\pi}{4}\cos x = \cos\frac{\pi}{4}$ { ∵ $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ }

⇒ $\cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$ { ∵ cos A cos B + sin A sin B = cos (A - B)}

NOTE: We can also make the ratio of sin instead of cos , the answer remains same but the form of answer may look different, when you put values of n you will get same values with both forms

If cos x = cos y, implies x = 2nπ ± y, where n∈ Z

$\therefore s\, x - \frac{\pi}{4} = \left(2n\pi \pm \frac{\pi}{4}\right)$

$\therefore x = \left(2n\pi \pm \frac{\pi}{4}\right) + \frac{\pi}{4}$ where $n \in Z$.

$x = 2n\pi$ or $x = 2n\pi + \frac{\pi}{2}$ where $n \in Z$ …..ans.

6 C. Question

Solve the following equations :

sin x + cos x = 1

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

$\sin x + \cos x = 1$.

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$\therefore \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$ { dividing by √2 both sides}

$\Rightarrow \sin x \sin\frac{\pi}{4} + \cos\frac{\pi}{4}\cos x = \cos\frac{\pi}{4}$ { $\because \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ }

$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$ { ∵ cos A cos B + sin A sin B = cos (A - B)}

NOTE: We can also make the ratio of sin instead of cos , the answer remains same but the form of answer may look different, when you put values of n you will get same values with both forms

If cos x = cos y, implies x = 2nπ ± y, where n∈ Z

$\therefore s x - \frac{\pi}{4} = \left(2n\pi \pm \frac{\pi}{4}\right)$

$\therefore x = \left(2n\pi \pm \frac{\pi}{4}\right) + \frac{\pi}{4}$ where $n \in Z$.

$x = 2n\pi$ or $x = 2n\pi + \frac{\pi}{2}$ where $n \in Z$ .....ans.

6 D. Question

Solve the following equations :

cosec x = 1 + cot x

Answer

deas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

cosec x = 1 + cot x

$\Rightarrow \frac{1}{\sin x} = 1 + \frac{\cos x}{\sin x}$

$\Rightarrow \sin x + \cos x = 1$.

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$\therefore s\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$ { dividing by √2 both sides}

$\Rightarrow \sin x \sin\frac{\pi}{4} + \cos\frac{\pi}{4}\cos x = \cos\frac{\pi}{4}$. { $\because \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$. }

$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$. { ∵ cos A cos B + sin A sin B = cos (A - B)}

NE: We can also make the ratio of sin instead of cos , the answer remains same but the form of answer may look different, when you put values of n you will get same values with both forms

If cos x = cos y, impls x = 2nπ ± y, where n∈ Z

$$\therefore x - \frac{\pi}{4} = \left(2n\pi \pm \frac{\pi}{4}\right).$$

$$\therefore x = \left(2n\pi \pm \frac{\pi}{4}\right) + \frac{\pi}{4} \text{ where } n \in Z$$

$$x = 2n\pi \text{ or } x = 2n\pi + \frac{\pi}{2} \text{ where } n \in Z \text{.....ans}$$

6 D. Question

Solve the following equations :

cosec x = 1 + cot x

Answer

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The general solution of any trigonometric equation is given as –

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• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

cosec x = 1 + cot x

$$\Rightarrow \frac{1}{\sin x} = 1 + \frac{\cos x}{\sin x}$$

$$\Rightarrow \sin x + \cos x = 1.$$

In all such problems we try to reduce the equation in an equation involving single trigonometric expression.

$\therefore s\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$ { dividing by √2 both sides}

$\Rightarrow \sin x \sin\frac{\pi}{4} + \cos\frac{\pi}{4}\cos x = \cos\frac{\pi}{4}$. { $\because \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$. }

$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$. { ∵ cos A cos B + sin A sin B = cos (A - B)}

NE: We can also make the ratio of sin instead of cos , the answer remains same but the form of answer may look different, when you put values of n you will get same values with both forms

If cos x = cos y, impls x = 2nπ ± y, where n∈ Z

$$\therefore x - \frac{\pi}{4} = \left(2n\pi \pm \frac{\pi}{4}\right).$$

$$\therefore x = \left(2n\pi \pm \frac{\pi}{4}\right) + \frac{\pi}{4} \text{ where } n \in Z$$

$$x = 2n\pi \text{ or } x = 2n\pi + \frac{\pi}{2} \text{ where } n \in Z \text{.....ans}$$

6 E. Question

Solve the following equations :

$$\left(\sqrt{3}-1\right)\cos x + \left(\sqrt{3}+1\right)\sin x = 2$$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$$\left(\sqrt{3}-1\right)\cos x + \left(\sqrt{3}+1\right)\sin x = 2$$

Dividing both sides by 2√2 :

We have,

.

⇒ $\cos\alpha\cos x + \sin\alpha\sin x = \cos\frac{\pi}{4}$ where cos α = π /4

⇒ $\cos(x-\alpha) = \cos\frac{\pi}{4}$ { cos π/4 = 1/√2 }

If cos x = cos y, implies x = 2nπ ± y, where n∈ Z

$$\therefore\ x-\alpha = 2n\pi \pm \frac{\pi}{4}\frac{\left(\sqrt{3}-1\right)}{2\sqrt{2}}\cos x + \frac{\left(\sqrt{3}+1\right)}{2\sqrt{2}}\sin x = \frac{1}{\sqrt{2}}$$

$$x = 2n\pi \pm \frac{\pi}{4} + \alpha \text{ where } \cos\alpha = \frac{\left(\sqrt{3}-1\right)}{2\sqrt{2}} \text{ and } n \in Z$$

6 E. Question

Solve the following equations :

$$\left(\sqrt{3}-1\right)\cos x + \left(\sqrt{3}+1\right)\sin x = 2$$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$$\left(\sqrt{3}-1\right)\cos x+\left(\sqrt{3}+1\right)\sin x=2$$

Dividing both sides by 2√2 :

We have,

.

⇒ $\cos\alpha\cos x+\sin\alpha\sin x=\cos\frac{\pi}{4}$ where cos α = π /4

⇒ $\cos(x-\alpha)=\cos\frac{\pi}{4}$ { cos π/4 = 1/√2 }

If cos x = cos y, implies x = 2nπ ± y, where n∈ Z

$$\therefore\ x-\alpha=2n\pi\pm\frac{\pi}{4}\frac{\left(\sqrt{3}-1\right)}{2\sqrt{2}}\cos x+\frac{\left(\sqrt{3}+1\right)}{2\sqrt{2}}\sin x=\frac{1}{\sqrt{2}}$$

$$x=2n\pi\pm\frac{\pi}{4}+\alpha\text{ where }\cos\alpha=\frac{\left(\sqrt{3}-1\right)}{2\sqrt{2}}\text{ and }n\in Z$$

7 . Question

Solve the following equations :

cot x + tan x = 2

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

cot x + tan x = 2

$$\frac{1}{\tan x}+\tan x=2$$

⇒ $\tan^2 x-2\tan x+1=0$

⇒ $(\tan x - 1)^2 = 0$

∴ tan x = 1 ⇒ tan x = tan π/4

If tan x = tan y, implies x = nπ + y, where n ∈ Z.

∴ x = nπ + π/4 where n ϵ Z ....ans

7 . Question

Solve the following equations :

cot x + tan x = 2

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

cot x + tan x = 2

$$\frac{1}{\tan x} + \tan x = 2$$

⇒ $\tan^2 x - 2\tan x + 1 = 0$

⇒ $(\tan x - 1)^2 = 0$

∴ tan x = 1 ⇒ tan x = tan π/4

If tan x = tan y, implies x = nπ + y, where n ∈ Z.

∴ x = nπ + π/4 where n ϵ Z ….ans

7 B. Question

Solve the following equations :

$2 \sin^2 x = 3 \cos x$, $0 \le x \le 2\pi$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

$2 \sin^2 x = 3 \cos x$, $0 \le x \le 2\pi$

⇒ $2(1 - \cos^2 x) = 3 \cos x$

⇒ $2\cos^2 x + 3\cos x - 2 = 0$

⇒ $2\cos^2 x + 4\cos x - \cos x - 2 = 0$

⇒ 2 cos x(cos x + 2) – 1(cos x + 2) = 0

⇒ (2cos x – 1)(cos x + 2) = 0

∴ cos x = �

or cos x = -2 { as cos x lies between -1 and 1 so this value is rejected }

∴ cos x = � = cos π/3

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z

∴ x = 2nπ ± π/3

But, $0 \le x \le 2\pi$

$\therefore x = \pi/3$ and $x = 2\pi - \pi/3 = 5\pi/3$ ….ans

7 B. Question

Solve the following equations :

$2 \sin^2 x = 3 \cos x, 0 \le x \le 2\pi$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies $x = n\pi + (-1)^n y$, where $n \in Z$.

• cos x = cos y, implies $x = 2n\pi \pm y$, where $n \in Z$.

• tan x = tan y, implies $x = n\pi + y$, where $n \in Z$.

given,

$2 \sin^2 x = 3 \cos x , 0 \le x \le 2\pi$

$\Rightarrow 2 (1 - \cos^2 x) = 3 \cos x$

$\Rightarrow 2 \cos^2 x + 3\cos x - 2 = 0$

$\Rightarrow 2 \cos^2 x + 4 \cos x - \cos x - 2 = 0$

$\Rightarrow 2 \cos x(\cos x + 2) - 1(\cos x + 2) = 0$

$\Rightarrow (2\cos x - 1)(\cos x + 2) = 0$

$\therefore \cos x =$ �

or $\cos x = -2$ { as cos x lies between -1 and 1 so this value is rejected }

$\therefore \cos x =$ � $= \cos \pi/3$

If cos x = cos y, implies $x = 2n\pi \pm y$, where $n \in Z$

$\therefore x = 2n\pi \pm \pi/3$

But, $0 \le x \le 2\pi$

$\therefore x = \pi/3$ and $x = 2\pi - \pi/3 = 5\pi/3$ ….ans

7 C. Question

Solve the following equations :

$\sec x \cos 5x + 1 = 0, 0 < x < \pi/2$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies $x = n\pi + (-1)^n y$, where $n \in Z$.

• cos x = cos y, implies $x = 2n\pi \pm y$, where $n \in Z$.

• tan x = tan y, implies $x = n\pi + y$, where $n \in Z$.

given,

$\sec x \cos 5x + 1 = 0, 0 < x < \pi/2$

$\Rightarrow \sec x \cos 5x = -1$

⇒ cos 5x = - cos x

∵ - cos x = cos (π – x)

∴ cos 5x = cos (π – x)

If cos x = cos y, implies 2nπ ± y, where n ∈ Z.

∴ 5x = 2nπ ± (π – x)

⇒ 5x = 2nπ + (π – x) or 5x = 2nπ – (π – x)

⇒ 6x = (2n+1)π or 4x = (2n-1)π

$$\therefore \ \mathrm{x} = (2\mathrm{n}+1)\frac{\pi}{6} \text{ or } \mathrm{x} = (2\mathrm{n}-1)\frac{\pi}{4} \text{ where } \mathrm{n} \in \mathrm{Z}$$

But, 0 < x < π/2

$$\therefore \ \boldsymbol{x = \frac{\pi}{6} \ and \ x = \frac{\pi}{4} \ldots ans}$$

7 C. Question

Solve the following equations :

sec x cos 5x + 1 = 0, 0 < x < π/2

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

sec x cos 5x + 1 = 0, 0 < x < π/2

⇒ sec x cos 5x = -1

⇒ cos 5x = - cos x

∵ - cos x = cos (π – x)

∴ cos 5x = cos (π – x)

If cos x = cos y, implies 2nπ ± y, where n ∈ Z.

∴ 5x = 2nπ ± (π – x)

⇒ 5x = 2nπ + (π – x) or 5x = 2nπ – (π – x)

⇒ 6x = (2n+1)π or 4x = (2n-1)π

$$\therefore \ \mathrm{x} = (2\mathrm{n}+1)\frac{\pi}{6} \text{ or } \mathrm{x} = (2\mathrm{n}-1)\frac{\pi}{4} \text{ where } \mathrm{n} \in \mathrm{Z}$$

But, 0 < x < π/2

$$\therefore \ \boldsymbol{x = \frac{\pi}{6} \ and \ x = \frac{\pi}{4} \ldots ans}$$

7 D. Question

Solve the following equations :

$5\cos^2 x + 7\sin^2 x - 6 = 0$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• $\sin x = \sin y$, implies $x = n\pi + (-1)^n y$, where $n \in Z$.

• $\cos x = \cos y$, implies $x = 2n\pi \pm y$, where $n \in Z$.

• $\tan x = \tan y$, implies $x = n\pi + y$, where $n \in Z$.

given,

$5\cos^2 x + 7\sin^2 x - 6 = 0$

$\Rightarrow 5\cos^2 x + 5\sin^2 x + 2\sin^2 x - 6 = 0$

$\Rightarrow 2\sin^2 x - 6 + 5 = 0$ {$\because \sin^2 x + \cos^2 x = 1$}

$\Rightarrow 2\sin^2 x - 1 = 0$

$\Rightarrow \sin^2 x = (1/2)$

$\therefore \sin x = \pm(1/\sqrt{2})$

$\Rightarrow \sin x = \pm \sin \pi/4$

If $\sin x = \sin y$, implies $x = n\pi + (-1)^n y$, where $n \in Z$.

$\therefore x = n\pi + (-1)^n (\pm(\pi/4))$ where $n \in Z$

$\therefore x = n\pi \pm \frac{\pi}{4}$ where $n \in Z$ **....ans**

7 D. Question

Solve the following equations :

$5\cos^2 x + 7\sin^2 x - 6 = 0$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• $\sin x = \sin y$, implies $x = n\pi + (-1)^n y$, where $n \in Z$.

• $\cos x = \cos y$, implies $x = 2n\pi \pm y$, where $n \in Z$.

• $\tan x = \tan y$, implies $x = n\pi + y$, where $n \in Z$.

given,

$5\cos^2 x + 7\sin^2 x - 6 = 0$

$\Rightarrow 5\cos^2 x + 5\sin^2 x + 2\sin^2 x - 6 = 0$

$\Rightarrow 2\sin^2 x - 6 + 5 = 0$ {$\because \sin^2 x + \cos^2 x = 1$}

$\Rightarrow 2\sin^2 x - 1 = 0$

$\Rightarrow \sin^2 x = (1/2)$

$\therefore \sin x = \pm(1/\sqrt{2})$

⇒ sin x = ± sin π /4

If sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

∴ x = nπ + (-1)^n (±(π / 4)) where n ϵ Z

∴ $x = n\pi \pm \frac{\pi}{4}$ *where n ϵ Z* **....ans**

7 E. Question

Solve the following equations :

sin x – 3 sin 2x + sin 3x = cos x – 3 cos 2x + cos 3x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

sin x – 3 sin 2x + sin 3x = cos x – 3 cos 2x + cos 3x

⇒ (sin x + sin 3x) – 3sin 2x – (cos x + cos 3x) + 3 cox 2x = 0

∵ sin A + sin B = $2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ and $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\therefore 2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - 3\sin 2x - 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) + 3\cos 2x = 0$

⇒ 2 sin 2x cos x – 3 sin 2x – 2 cos 2x cos x + 3 cos 2x = 0

⇒ sin 2x (2cos x – 3) - cos 2x (2cos x – 3) = 0

⇒ (2cos x – 3)(sin 2x – cos 2x) = 0

∴ cos x = 3/2 = 1.5 (not accepted as cos x lies between – 1 and 1)

Or sin 2x = cos 2x

∴ tan 2x = 1 = tan π/4

If tan x = tan y, implies x = nπ + y, where n ∈ Z.

∴ 2x = nπ + π/4

$\therefore x = \frac{n\pi}{2} + \frac{\pi}{8}$ *where n ϵ Z.....ans*

7 E. Question

Solve the following equations :

sin x – 3 sin 2x + sin 3x = cos x – 3 cos 2x + cos 3x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

sin x – 3 sin 2x + sin 3x = cos x – 3 cos 2x + cos 3x

⇒ (sin x + sin 3x) – 3sin 2x – (cos x + cos 3x) + 3 cox 2x = 0

∵ sin A + sin B = $2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ and $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\therefore 2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{3x-x}{2}\right)-3\sin 2x-2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)+3\cos 2x=0$

⇒ 2 sin 2x cos x – 3 sin 2x – 2 cos 2x cos x + 3 cos 2x = 0

⇒ sin 2x (2cos x – 3) - cos 2x (2cos x – 3) = 0

⇒ (2cos x – 3)(sin 2x – cos 2x) = 0

∴ cos x = 3/2 = 1.5 (not accepted as cos x lies between – 1 and 1)

Or sin 2x = cos 2x

∴ tan 2x = 1 = tan π/4

If tan x = tan y, implies x = nπ + y, where n ∈ Z.

∴ 2x = nπ + π/4

$\therefore x = \frac{n\pi}{2} + \frac{\pi}{8}$ *where n ϵ Z…..ans*

7 F. Question

Solve the following equations :

4 sin x cos x + 2 sin x + 2 cos x + 1 = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

4 sin x cos x + 2 sin x + 2 cos x + 1 = 0

⇒ 2sin x (2cos x + 1) + 1(2cos x + 1) = 0

⇒ (2cos x + 1)(2sin x + 1) = 0

∴ cos x = -1/2 or sin x = -1/2

⇒ cos x = cos (π - π/3) or sin x = sin (- π/6)

⇒ cos x = cos 2π/3 or sin x = sin (-π/6)

If sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

And cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

∴ x = 2nπ ± 2π/3 or x = mπ + $(-1)^m$ (-π/6)

Hence,

$$x = 2n\pi \pm \frac{2\pi}{3} \ or \ x = m\pi + (-1)^m \left(-\frac{\pi}{6}\right) \ where \ m, n \in Z \ldots ans$$

7 F. Question

Solve the following equations :

4 sin x cos x + 2 sin x + 2 cos x + 1 = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

4 sin x cos x + 2 sin x + 2 cos x + 1 = 0

⇒ 2sin x (2cos x + 1) + 1(2cos x + 1) = 0

⇒ (2cos x + 1)(2sin x + 1) = 0

∴ cos x = -1/2 or sin x = -1/2

⇒ cos x = cos (π - π/3) or sin x = sin (- π/6)

⇒ cos x = cos 2π/3 or sin x = sin (-π/6)

If sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

And cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

∴ x = 2nπ ± 2π/3 or x = mπ + $(-1)^m$ (-π/6)

Hence,

$$x = 2n\pi \pm \frac{2\pi}{3} \ or \ x = m\pi + (-1)^m \left(-\frac{\pi}{6}\right) \ where \ m, n \in Z \ldots ans$$

7 G. Question

Solve the following equations :

cos x + sin x = cos 2x + sin 2x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

cos x + sin x = cos 2x + sin 2x

cos x – cos 2x = sin 2x – sin x

{∵ sin A - sin B = $2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ & $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ }

$\therefore -2\sin\left(\frac{x+2x}{2}\right)\sin\left(\frac{x-2x}{2}\right) = 2\cos(\frac{2x+x}{2})\sin\left(\frac{2x-x}{2}\right)$

⇒ $2\sin\frac{3x}{2}\sin\frac{x}{2} = 2\cos\frac{3x}{2}\sin\frac{x}{2}$.

$\therefore \sin\frac{x}{2}\,(\sin\frac{3x}{2} - \cos\frac{3x}{2}) = 0$.

Hence,

Either, $\sin\frac{x}{2} = 0$ or $\sin\frac{3x}{2} = \cos\frac{3x}{2}$

⇒ $\sin\frac{x}{2} = \sin m\pi$ or $\tan\frac{3x}{2} = 1 = \tan\frac{\pi}{4}$

If tan x = tan y, implies x = nπ + y, where n ∈ Z.

$\therefore \frac{x}{2} = m\pi$ or $\frac{3x}{2} = n\pi + \frac{\pi}{4}$

⇒ $x = 2m\pi$ or $x = \frac{2n\pi}{3} + \frac{\pi}{6}$ where m,n ϵ Z ....**ans**

7 G. Question

Solve the following equations :

cos x + sin x = cos 2x + sin 2x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

cos x + sin x = cos 2x + sin 2x

cos x – cos 2x = sin 2x – sin x

{∵ sin A - sin B = $2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ & $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ }

$\therefore -2\sin\left(\frac{x+2x}{2}\right)\sin\left(\frac{x-2x}{2}\right) = 2\cos(\frac{2x+x}{2})\sin\left(\frac{2x-x}{2}\right)$

$$\Rightarrow 2\sin\frac{3x}{2}\sin\frac{x}{2} = 2\cos\frac{3x}{2}\sin\frac{x}{2}.$$

$$\therefore \sin\frac{x}{2}\left(\sin\frac{3x}{2} - \cos\frac{3x}{2}\right) = 0.$$

Hence,

Either, $\sin\frac{x}{2} = 0$ or $\sin\frac{3x}{2} = \cos\frac{3x}{2}$

$$\Rightarrow \sin\frac{x}{2} = \sin m\pi \text{ or } \tan\frac{3x}{2} = 1 = \tan\frac{\pi}{4}$$

If tan x = tan y, implies x = nπ + y, where n ∈ Z.

$$\therefore \frac{x}{2} = m\pi \text{ or } \frac{3x}{2} = n\pi + \frac{\pi}{4}$$

$\Rightarrow x = 2m\pi \text{ or } x = \frac{2n\pi}{3} + \frac{\pi}{6}$ where m,n ϵ Z ….**ans**

7 H. Question

Solve the following equations :

sin x tan x – 1 = tan x – sin x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

sin x tan x – 1 = tan x – sin x

⇒ sin x tan x – tan x + sin x – 1 = 0

⇒ tan x(sin x – 1) + (sin x – 1) = 0

⇒ (sin x – 1)(tan x + 1) = 0

∴ sin x = 1 or tan x = -1

⇒ sin x = sin π/2 or tan x = tan (- π/4)

If sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

and tan x = tan y, implies x = mπ + y, where n ∈ Z.

∴ x = nπ + $(-1)^n$ (π /2) or x = mπ + (- π/4)

$$\therefore x = n\pi + (-1)^n\left(\frac{\pi}{2}\right) \text{ or } x = m\pi - \frac{\pi}{4} \text{ where } n, m \epsilon Z\text{….ans}$$

7 H. Question

Solve the following equations :

sin x tan x – 1 = tan x – sin x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

given,

sin x tan x – 1 = tan x – sin x

⇒ sin x tan x – tan x + sin x – 1 = 0

⇒ tan x(sin x – 1) + (sin x – 1) = 0

⇒ (sin x – 1)(tan x + 1) = 0

∴ sin x = 1 or tan x = -1

⇒ sin x = sin π/2 or tan x = tan (- π/4)

If sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

and tan x = tan y, implies x = nπ + y, where n ∈ Z.

∴ x = nπ + (-1)^n (π /2) or x = mπ + (- π/4)

$$\therefore\ x = n\pi + (-1)^n\left(\frac{\pi}{2}\right) \text{ or } x = m\pi - \frac{\pi}{4} \text{ where } n, m \in Z \ldots\text{ans}$$

7 I. Question

Solve the following equations :

3 tan x + cot x = 5 cosec x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + (– 1)^{n}y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

3tan x + cot x = 5cosec x

$$\Rightarrow 3\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{5}{\sin x}$$

$$\Rightarrow 3\frac{\sin x}{\cos x} = \frac{5}{\sin x} - \frac{\cos x}{\sin x}$$

$$\Rightarrow 3\sin^2 x = (5 - \cos x)\cos x$$

$$\Rightarrow 3\sin^2 x + \cos^2 x = 5\cos x$$

⇒ $2\sin^2 x - 5\cos x + 1 = 0$ {∵ $\sin^2 x + \cos^2 x = 1$}

∴ $2(1-\cos^2 x) - 5\cos x + 1 = 0$

⇒ $2\cos^2 x + 5\cos x - 3 = 0$

⇒ $2\cos^2 x + 6\cos x - \cos x - 3 = 0$

⇒ $2\cos x(\cos x + 3) - 1(\cos x + 3) = 0$

⇒ $(\cos x + 3)(2\cos x - 1) = 0$

∴ cos x = -3 (neglected as cos x lies between -1 and 1)

or cos x = � (accepted value)

∴ $\cos x = \cos\frac{\pi}{3}$

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

∴ $x = 2n\pi \pm \frac{\pi}{3}$ *where* $n \in Z$*….ans*

7 I. Question

Solve the following equations :

3 tan x + cot x = 5 cosec x

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

3tan x + cot x = 5cosec x

⇒ $3\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{5}{\sin x}$

⇒ $3\frac{\sin x}{\cos x} = \frac{5}{\sin x} - \frac{\cos x}{\sin x}$

⇒ $3\sin^2 x = (5 - \cos x)\cos x$

⇒ $3\sin^2 x + \cos^2 x = 5\cos x$

⇒ $2\sin^2 x - 5\cos x + 1 = 0$ {∵ $\sin^2 x + \cos^2 x = 1$}

∴ $2(1-\cos^2 x) - 5\cos x + 1 = 0$

⇒ $2\cos^2 x + 5\cos x - 3 = 0$

⇒ $2\cos^2 x + 6\cos x - \cos x - 3 = 0$

$\Rightarrow 2\cos x(\cos x+3)-1(\cos x+3)=0$

$\Rightarrow (\cos x+3)(2\cos x-1)=0$

∴ cos x = -3 (neglected as cos x lies between -1 and 1)

or cos x = � (accepted value)

$\therefore \cos x=\cos\frac{\pi}{3}$

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

$\therefore\ x=2n\pi\pm\frac{\pi}{3}\ where\ n\epsilon Z\ldots.ans$

8. Question

Solve : 3 – 2 cos x – 4 sin x – cos 2x + sin 2x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

3 – 2 cos x – 4 sin x – cos 2x + sin 2x = 0

As, cos 2x = 1 – $2\sin^2$ x and sin 2x = 2sin x cos x

∴ 3 – 2cos x – 4sin x – (1 – $2\sin^2$ x) + 2sin x cos x = 0

⇒ $2\sin^2$ x – 4sin x + 2 – 2cos x + 2sin x cos x = 0

⇒ 2($\sin^2$ x – 2sin x + 1) + 2cos x(sin x – 1) = 0

⇒ 2$(\sin x - 1)^2$ + 2cos x(sin x – 1) = 0

⇒ (sin x – 1)(2cos x + 2sin x – 2) = 0

∴ sin x = 1 or sin x + cos x = 1

When, sin x = 1

We have,

sin x = sin π/2

If sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z

$\therefore\ x=n\pi+(-1)^n\left(\frac{\pi}{2}\right)\text{where}\, n\,\epsilon\, Z$

When, sin x + cos x = 1

$\therefore\ \frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x=\frac{1}{\sqrt{2}}$ { dividing by √2 both sides}

$\Rightarrow \sin x \sin\frac{\pi}{4} + \cos\frac{\pi}{4}\cos x = \cos\frac{\pi}{4}$ { $\because \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ }

$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$ { ∵ cos A cos B + sin A sin B = cos (A - B)}

If cos x = cos y, implies x = 2mπ ± y, where m ∈ Z

$\therefore x - \frac{\pi}{4} = \left(2m\pi \pm \frac{\pi}{4}\right)$

$\therefore x = \left(2m\pi \pm \frac{\pi}{4}\right) + \frac{\pi}{4}$ where $m \epsilon Z$

$\Rightarrow x = 2m\pi$ or $x = 2m\pi + \frac{\pi}{2}$ where $m \epsilon Z$

Hence,

$$x = n\pi + (-1)^n\left(\frac{\pi}{2}\right) \text{ or } x = 2m\pi \text{ or } x = 2m\pi + \frac{\pi}{2} \text{ where } m, n \epsilon Z$$

8. Question

Solve : 3 – 2 cos x – 4 sin x – cos 2x + sin 2x = 0

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

3 – 2 cos x – 4 sin x – cos 2x + sin 2x = 0

As, cos 2x = 1 – $2\sin^2$ x and sin 2x = 2sin x cos x

∴ 3 – 2cos x – 4sin x – (1 – $2\sin^2$ x) + 2sin x cos x = 0

⇒ $2\sin^2$ x – 4sin x + 2 – 2cos x + 2sin x cos x = 0

⇒ 2($\sin^2$ x – 2sin x + 1) + 2cos x(sin x – 1) = 0

⇒ 2$(\sin x - 1)^2$ + 2cos x(sin x – 1) = 0

⇒ (sin x – 1)(2cos x + 2sin x – 2) = 0

∴ sin x = 1 or sin x + cos x = 1

When, sin x = 1

We have,

sin x = sin π/2

If sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z

$\therefore x = n\pi + (-1)^n \left(\frac{\pi}{2}\right)$ where $n \in Z$

When, sin x + cos x = 1

$\therefore \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$ { dividing by √2 both sides}

⇒ $\sin x \sin\frac{\pi}{4} + \cos\frac{\pi}{4}\cos x = \cos\frac{\pi}{4}$ { $\because \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ }

⇒ $\cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$ { ∵ cos A cos B + sin A sin B = cos (A - B)}

If cos x = cos y, implies x = 2mπ ± y, where m∈ Z

$\therefore x - \frac{\pi}{4} = \left(2m\pi \pm \frac{\pi}{4}\right)$

$\therefore x = \left(2m\pi \pm \frac{\pi}{4}\right) + \frac{\pi}{4}$ where $m \in Z$

⇒ $x = 2m\pi$ or $x = 2m\pi + \frac{\pi}{2}$ where $m \in Z$

Hence,

$$x = n\pi + (-1)^n\left(\frac{\pi}{2}\right) \text{ or } x = 2m\pi \text{ or } x = 2m\pi + \frac{\pi}{2} \text{ where } m, n \in Z$$

9. Question

$3\sin^2 x - 5 \sin x \cos x + 8 \cos^2 x = 2$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies x = nπ + $(-1)^n$y, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$3\sin^2 x - 5 \sin x \cos x + 8 \cos^2 x = 2$

⇒ $3\sin^2 x + 3 \cos^2 x - 5\sin x \cos x + 5 \cos^2 x = 2$

⇒ $3 - 5\sin x \cos x + 5 \cos^2 x = 2$ {∵ $\sin^2 x + \cos^2 x = 1$ }

⇒ $5\cos^2 x + 1 = 5\sin x \cos x$

Squaring both sides:

⇒ $(5\cos^2 x + 1)^2 = (5\sin x \cos x)^2$

⇒ $25\cos^4 x + 10\cos^2 x + 1 = 25 \sin^2 x \cos^2 x$

⇒ $25\cos^4 x + 10\cos^2 x + 1 = 25 (1 - \cos^2 x) \cos^2 x$

$\Rightarrow 50\cos^4 x - 15\cos^2 x + 1 = 0$

$\Rightarrow 50\cos^4 x - 10\cos^2 x - 5\cos^2 x + 1 = 0$

$\Rightarrow 10\cos^2 x\,(5\cos^2 x - 1) - (5\cos^2 x - 1) = 0$

$\Rightarrow (10\cos^2 x - 1)(5\cos^2 x - 1) = 0$

$\therefore \cos^2 x = 1/10$ or $\cos^2 x = 1/5$

Hence, when $\cos^2 x = 1/10$

We have, $\cos x = \pm\frac{1}{\sqrt{10}}$

If cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

let $\cos\alpha = 1/\sqrt{10}$

$\therefore \cos(\pi - \alpha) = -1/\sqrt{10}$

$\therefore x = 2n\pi \pm \alpha$ or $x = 2n\pi \pm (\pi - \alpha)$

$\therefore$ when, $\cos x = \pm\frac{1}{\sqrt{10}}$

$$x = 2n\pi \pm \alpha \text{ or } x = 2n\pi \pm (\pi - \alpha) \text{ where } n \in Z \text{ and } \cos\alpha = \frac{1}{\sqrt{10}}$$

When $\cos^2 x = 1/5$

We have, $\cos x = \pm\frac{1}{\sqrt{5}}$.

If cos x = cos y, implies x = 2mπ ± y, where n ∈ Z.

let $\cos\beta = 1/\sqrt{5}$

$\therefore \cos(\pi - \beta) = -1/\sqrt{5}$

$\therefore x = 2m\pi \pm \beta$ or $x = 2m\pi \pm (\pi - \beta)$

$\therefore$ when, $\cos x = \pm\frac{1}{\sqrt{5}}$.

$$x = 2m\pi \pm \beta \text{ or } x = 2m\pi \pm (\pi - \beta) \text{ where } m \in Z \text{ and } \cos\beta = \frac{1}{\sqrt{5}}$$

…**ans**

9. Question

$3\sin^2 x - 5\sin x\cos x + 8\cos^2 x = 2$

Answer

Ideas required to solve the problem:

The general solution of any trigonometric equation is given as –

• sin x = sin y, implies $x = n\pi + (-1)^n y$, where n ∈ Z.

• cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

• tan x = tan y, implies x = nπ + y, where n ∈ Z.

Given,

$3\sin^2 x – 5 \sin x \cos x + 8 \cos^2 x = 2$

$\Rightarrow 3\sin^2 x + 3 \cos^2 x – 5\sin x \cos x + 5 \cos^2 x = 2$

$\Rightarrow 3 - 5\sin x \cos x + 5 \cos^2 x = 2$ $\{\because \sin^2 x + \cos^2 x = 1 \}$

$\Rightarrow 5\cos^2 x + 1 = 5\sin x \cos x$

Squaring both sides:

$\Rightarrow (5\cos^2 x + 1)^2 = (5\sin x \cos x)^2$

$\Rightarrow 25\cos^4 x + 10\cos^2 x + 1 = 25 \sin^2 x \cos^2 x$

$\Rightarrow 25\cos^4 x + 10\cos^2 x + 1 = 25 (1 - \cos^2 x) \cos^2 x$

$\Rightarrow 50\cos^4 x – 15 \cos^2 x + 1 = 0$

$\Rightarrow 50\cos^4 x – 10 \cos^2 x – 5\cos^2 x + 1 = 0$

$\Rightarrow 10\cos^2 x (5\cos^2 x – 1) – (5\cos^2 x – 1) = 0$

$\Rightarrow (10\cos^2 x - 1)(5\cos^2 x – 1) = 0$

$\therefore \cos^2 x = 1/10$ or $\cos^2 x = 1/5$

Hence, when $\cos^2 x = 1/10$

We have, $\cos x = \pm \frac{1}{\sqrt{10}}$

If cos x = cos y, implies x = 2mπ ± y, where n ∈ Z.

let $\cos \alpha = 1/\sqrt{10}$

$\therefore \cos (\pi – \alpha) = -1/\sqrt{10}$

$\therefore x = 2n\pi \pm \alpha$ or $x = 2n\pi \pm (\pi – \alpha)$

$\therefore$ when, $\cos x = \pm \frac{1}{\sqrt{10}}$

$$x = 2n\pi \pm \alpha \text{ or } x = 2n\pi \pm (\pi - \alpha) \text{ where } n \in Z \text{ and } \cos\alpha = \frac{1}{\sqrt{10}}$$

When $\cos^2 x = 1/5$

We have, $\cos x = \pm \frac{1}{\sqrt{5}}$.

If cos x = cos y, implies x = 2mπ ± y, where n ∈ Z.

let $\cos \beta = 1/\sqrt{5}$

$\therefore \cos (\pi – \beta) = -1/\sqrt{5}$

$\therefore x = 2m\pi \pm \beta$ or $x = 2m\pi \pm (\pi – \beta)$

$\therefore$ when, $\cos x = \pm \frac{1}{\sqrt{5}}$.

$$x = 2m\pi \pm \beta \text{ or } x = 2m\pi \pm (\pi - \beta) \text{ where } m \in Z \text{ and } \cos\beta = \frac{1}{\sqrt{5}}$$

...**ans**

10. Question

Solve : $2^{\sin^2 x} + 2^{\cos^2 x} = 2\sqrt{2}$

Answer

Given,

⇒ $2^{\sin^2 x} + 2^{\cos^2 x} = 2^{\frac{1}{2}} + 2^{\frac{1}{2}}.$

On comparing both sides, we have

$\sin^2 x = \cos^2 x =$ �

Note: If we want to give solution using above two equations then task will become tedious as sin x can be positive at that time cos will be negative and similar 4-5 cases will arise. So inspite of combining all solutions at the end,we proceed as follows

combining both we can say that,

all the solutions of first 2 equations combined will satisy this single equation

$\tan^2 x = 1$

$$\tan x = \pm 1 = \tan\left(\pm\frac{\pi}{4}\right)$$

tan x = tan y, implies x = nπ + y, where n ∈ Z.

$$\therefore x = n\pi + \frac{\pi}{4} \; or \; x = m\pi - \frac{\pi}{4} \; where \; m, n \epsilon Z \ldots ans$$

10. Question

Solve : $2^{\sin^2 x} + 2^{\cos^2 x} = 2\sqrt{2}$

Answer

Given,

⇒ $2^{\sin^2 x} + 2^{\cos^2 x} = 2^{\frac{1}{2}} + 2^{\frac{1}{2}}.$

On comparing both sides, we have

$\sin^2 x = \cos^2 x =$ �

Note: If we want to give solution using above two equations then task will become tedious as sin x can be positive at that time cos will be negative and similar 4-5 cases will arise. So inspite of combining all solutions at the end,we proceed as follows

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$\tan^2 x = 1$

$$\tan x = \pm 1 = \tan\left(\pm\frac{\pi}{4}\right)$$

tan x = tan y, implies x = nπ + y, where n ∈ Z.

$\therefore$ x $= n\pi + \frac{\pi}{4}$ *or* $x = m\pi - \frac{\pi}{4}$ *where m, n*ϵ *Z…ans*

Very Short Answer

1. Question

Write the number of solutions of the equation tan x + sec x = 2 cos x in the interval [0, 2π].

Answer

$\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2 \times \cos x$

sin x + 1 = 2 × $(\cos x)^2$

sin x + 1 = 2 × (1 - $(\sin x)^2$)

sin x + 1 = 2 – 2$(\sin x)^2$

2$(\sin x)^2$ + sin x - 1 = 0

Consider a=sin x

So, the equation will be

$2a^2$+a-1=0

From the equation a=0.5 or -1

Which implies

Sin x=0.5 or sin x=(-1)

Therefore x=30° or 270°

But for x=270° our equation will not be defined as cos (270°)=0

So, the solution for x=30°

According to trigonometric equations

If sin x=sin a

Then x=nπ – na

Here sin x=sin30

So, x=nπ + $(-1)^n$ × 30

For n=0, x=30 and n=1,x=150° and for n=2,x=390

Hence between 0 to 2π there are only 2 possible solutions.

2. Question

Write the number of solutions of the equation 4 sin x – 3 cos x = 7.

Answer

4sin x – 3cos x = 7

4sin x – 7 = 3cos x

Squaring both sides

16$(\sin x)^2$ + 49 – 56sin x = 9$(\cos x)^2$

16$(\sin x)^2$ + 49 – 56sin x = 9($(\sin x)^2$-1)

16$(\sin x)^2$-9$(\sin x)^2$-56sin x+49+9=0

$7(\sin x)^2$-56sin x+58=0

Solving the quadratic equation

Sin x = 6.7774 or 1.2225

But we know that sinθ lies between [-1,1]

So there are no solutions for this given equation

3. Question

Write the general solution of $\tan^2 2x = 1$.

Answer

$$\frac{\sin^2 2x}{\cos^2 2x} = 1$$

$\sin^2 2x = \cos^2 2x$

$\sin^2 2x = 1 - \sin^2 2x$

$2\sin^2 2x = 1$

$$\sin 2x = \frac{1}{\sqrt{2}}$$

sin 2x=sin45

So

$$2x = n\pi + \frac{\pi}{4}$$

$$x = \frac{n\pi}{2} + \frac{\pi}{8}$$

4. Question

Write the set of values of a for which the equation √3 sin x – cos x = a has no solution.

Answer

$$\frac{\sqrt{3}}{2}\sin x - \frac{\cos x}{2} = a$$

cos30°sin x – sin30°cos x =a

sin (x-30)=a

As the range of sin function is from [-1,1]

So the value of a can be R-[-1,1]

i.e. a ∈ (-∞, -2) ∪ (2, ∞)

5. Question

If cos x = k has exactly one solution in [0, 2 π], then write the value(s) of k.

Answer

As cos x = cos θ

Then x=2nπ ± θ

And it is said that it has exactly one solution.

So θ=0 and

$$x = \frac{2n\pi}{2}$$

=nπ

In the given interval taking n=1,x=π {n=0 is not possible as cos 0 = 1 not -1 but cos π is -1}

6. Question

Write the number of points of intersection of the curves 2y = 1 and y = cos x, 0 ≤ x ≤ 2π.

Answer

2y=1

i.e. $y = \frac{1}{2}$

and y = cos x

so, to get the intersection points we must equate both the equations

i.e. $\cos x = \frac{1}{2}$

so, cos x = cos 60°

and we know if cos x = cos a

then x=2nπ ± a where a ϵ [0, π]

so here

$$x = 2n\pi \pm \frac{\pi}{3}$$

So the possible values which belong [0,2π] are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

There are a total of 2 points of intersection.

7. Question

Write the values of x in [0, π] for which $\sin 2x, \frac{1}{2}$ and cos 2x are in A.P.

Answer

a, a+r, a+2r

so $A_1+A_3=2A_2$

here sin2x + cos2x = 1

2sin x cos x + 1-2sin^{2}x =1

sin x cos x - sin^{2}x=0

sin x (cos x-sin x)=0

if sin x =0

then x =0, π

if sin x=cos x

then x = π/4

So, all possible values are $0, \frac{\pi}{4}, \pi$

8. Question

Write the number of points of intersection of the curves 2y = - 1 and y = cosec x.

Answer

Y=cosec x and $y = -\frac{1}{2}$

So

$$\frac{1}{\text{sinx}} = \frac{-1}{2}$$

Sin x = -2

Which is not possible

So

There are 0 points of intersection.

9. Question

Write the solution set of the equation (2 cos x + 1) (4 cos x + 5) = 0 in the interval (0, 2 π].

Answer

8 $\cos^2$x+10 cos x+4 cos x+5=0

8 $\cos^2$x+14 cos x+5=0

Solving the quadratic equation, we get,

cos x = -0.5

cos x = cos 120°

$$x = 2n\pi \pm \frac{2\pi}{3}$$

So $x = \frac{2\pi}{3}$ when $n = 0$,

And when n=1 $x = \frac{4\pi}{3}$

10. Question

Write the number of values of x in [0, 2 π] that satisfy the equation $\sin^2 x - \cos x = \frac{1}{4}$.

Answer

1-$\cos^2$x-cos x=0.25

cos x ^{2}x+cos x-0.75=0

Solving the quadratic equation we get

cos x = 0.5

cos x = cos60°

$$x = 2n\pi \pm \frac{\pi}{3}$$

x=60° when n=0

And x=300° when n=1

11. Question

If 3 tan (x - 15^o) = tan (x + 15^o), 0 ≤ x ≤ 90^o, find x.

Answer

Let tan (15°) = tan(45°-30°)

We know that

$$\tan(A-B) = \frac{(\tan A - \tan B)}{1 + \tan A \tan B}$$

$$\tan(45° - 30°) = \frac{(\tan 45° - \tan 30°)}{1 + \tan 45° \tan 30°}$$

$$\tan(45 - 30) = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$\tan 15 = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

We now

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \times \tan b}$$

And

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \times \tan b}$$

So, 3 tan (x - 15^{o}) = tan (x + 15^{o}) can be written as follows

$$3 \times \frac{\tan x - \tan 15}{1 + \tan x \times \tan 15} = \frac{\tan x + \tan 15}{1 - \tan x \times \tan 15}$$

(3 tan x – 3tan15)(1-tan x × tan15) = (1+tan x × tan15)(tan x + tan15)

3 tan x – 3 tan15-3 tan^{2}x tan(15-3) tan x tan^{2}15 = tan x + tan15 + tan^{2}x tan15 + tan x tan^{2}15

Solving the equation,

And putting

$$\tan 15 = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

We get tan x - 1 = 0

Therefore, tan x =1

So, x=45°

Or

$$x = \frac{\pi}{4}$$

12. Question

If 2 sin^2 x = 3 cos x, where 0 ≤ x ≤ 2 π, then find the value of x.

Answer

2sin^{2}x=3cos x

2-2cos^{2}x=3cos x

Solving the quadratic equation, we get

cos x= 1/2

Therefore x=60° and 300°

i.e.

$\theta = \frac{\pi}{3}$ and $\frac{5\pi}{3}$

13. Question

If sec x cos 5x + 1 = 0, where $0 < x \leq \frac{\pi}{2}$, find the value of x.

Answer

$\frac{\cos 5x}{\cos x} = -1$

cos 5x = -cos x

cos 5x+cos x = 0

We know

$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

Here

$\cos 5x + \cos x = 2\cos\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right)$

Now from the above equation it would be,

2cos 3x cos 2x=0

cos 3x cos 2x=0

cos 3x=0 or cos 2x=0

for cos3x=0

$3x = (2n+1)\left(\frac{\pi}{2}\right)$

$x = (2n+1)\left(\frac{\pi}{6}\right)$

for cos2x=0

$2x = (2n+1)\left(\frac{\pi}{2}\right)$

$x = (2n+1)\left(\frac{\pi}{4}\right)$

so the values of the x less than equal to 90° are $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}$

MCQ

1. Question

Mark the Correct alternative in the following:

The smallest value of x satisfying the equation √3 (cot x + tan x) = 4 is

A. 2 π /3

B. π /3

C. π /6

D. π /12

Answer

$$\sqrt{3}\left(\frac{1}{\tan x}+\tan x\right)=4$$

$$\sqrt{3}\left(\frac{1+\tan^2 x}{\tan x}\right)=4$$

$\sqrt{3}+\sqrt{3}\tan^2 x = 4\tan x$

$\sqrt{3}\tan^2 x-4\tan x+\sqrt{3}=0$

Therefore

$$\tan x=\sqrt{3} \text{ or } \tan x=\frac{1}{\sqrt{3}}$$

Therefore $x=\frac{\pi}{3}$ or $\frac{\pi}{6}$

But here the smallest angle is $\pi/6$

Option C

2. Question

Mark the Correct alternative in the following:

If $\cos x + \sqrt{3}\sin x = 2$, then x =

A. $\pi/3$

B. $2\pi/3$

C. $4\pi/3$

D. $5\pi/3$

Answer

$\cos^2 x = (2-\sqrt{3}\sin x)^2$

$1-\sin^2 x = 4+3\sin^2 x-4\sqrt{3}\sin x$

$4\sin^2 x-4\sqrt{3}\sin x+3=0$

$$\sin x=\frac{\sqrt{3}}{2}$$

$$x=\frac{\pi}{3}$$

Option A

3. Question

Mark the Correct alternative in the following:

If tan px – tan qx = 0, then the values of θ form a series in

A. AP

B. GP

C. HP

D. None of these

Answer

Tan x=tana

X=nπ+a

So tan px – tan qx = 0

tan px = tan qx

px = nπ + qx

(p-q)x=nπ

$$x = \frac{n\pi}{p - q}$$

$$x = \frac{\pi}{p - q}, \frac{2\pi}{p - q}, \frac{3\pi}{p - q}$$

Here in this series $a = r = \frac{\pi}{p-q}$

So, this is in AP.

Option A

4. Question

Mark the Correct alternative in the following:

If a is any real number, the number of roots of cot x – tan x = a in the first quadrant is (are).

A. 2

B. 0

C. 1

D. None of these

Answer

$$\frac{1}{\tan x} - \tan x = a$$

$$\frac{1 - \tan^2 x}{\tan x} = a$$

$1-\tan^2 x=a \tan x$

$\tan^2 x+a \tan x - 1 = 0$

$$\tan x = \frac{-a \pm \sqrt{a^2 - 4(-1)}}{2}$$

$$\tan x = \frac{-a \pm \sqrt{a^2 + 4}}{2}$$

As it is given a be any real number take a=0,

For a=0

$$\tan x = \frac{\pm\sqrt{0 + 4}}{2}$$

Tan x = +1 or -1

In first quadrant only tan(π/4)=1

So, there is only one root that lies in the first quadrant.

Option C

5. Question

Mark the Correct alternative in the following:

The general solution of the equation 7 $\cos^2 x$ + 3 $\sin^2 x$ = 4 is

A. $x = 2n\pi \pm \frac{\pi}{6}, n \in Z$

B. $x = 2n\pi \pm \frac{2\pi}{3}, n \in Z$

C. $x = 2n\pi \pm \frac{\pi}{3}, n \in Z$

D. none of these

Answer

$7\cos^2x+3(1-\cos^2x)=4$

$7\cos^2x + 3 - 3\cos^2x = 4$

$4\cos^2x - 1 = 0$

$\cos x = \frac{1}{2}$

cos x = cos60°

Then

$x = 2n\pi \pm \frac{\pi}{3}, n \in Z$

6. Question

Mark the Correct alternative in the following:

A solution of the equation $\cos^2 x + \sin x + 1 = 0$, lies in the interval

A. (- π/4, π /4)

B. (π /4, 3 π /4)

C. (3 π /4, 5 π /4)

D. (5 π/4, 7 π/4)

Answer

$1-\sin^2x+\sin x+1=0$

$\sin^2x-\sin x-2=0$

sin x=-1

$x = \frac{3\pi}{2}$

Option D

7. Question

Mark the Correct alternative in the following:

The number of solution in [0, π/2] of the equation cos 3x tan 5x = sin 7x is

A. 5

B. 7

C. 6

D. None of these

Answer

cos 3x tan 5x = sin 7x

$$\cos 3x\left(\frac{\sin 5x}{\cos 5x}\right) = \sin 7x$$

2 cos 3x sin 5x = 2 cos 5x sin 7x

sin 8x + sin2x = sin 12x + sin 2x

sin 8x = sin12x

sin 12x - sin8x = 0

2 sin 2x cos 10x = 0

If sin2x=0

Then, x=0

If cos10x = 0

Then $10x = \frac{\pi}{2}$

$$x = \frac{\pi}{20}, \frac{3\pi}{20}, \frac{5\pi}{20}, \frac{7\pi}{20}, \frac{9\pi}{20}$$

and x=0 (from above equation sin2x=0)

So, there are 6 possible solutions.

Option C

8. Question

Mark the Correct alternative in the following:

The general value of x satisfying the equation $\sqrt{3}\sin x + \cos x = \sqrt{3}$ is given by

A. $x = n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{3}, n \in Z$

B. $x = n\pi + (-1)^n \frac{\pi}{3} - \frac{\pi}{6}, n \in Z$

C. $x = n\pi \pm \frac{\pi}{6}, n \in Z$

D. $x = n\pi \pm \frac{\pi}{3}, n \in Z$

Answer

$Cos^2x=(\sqrt{3}-\sqrt{3}\sin x)^2$

$1-\sin^2x = 3+3\sin^2x-6\sin x$

$4\sin^2x-6\sin x+2=0$

$2\sin^2x-3\sin x+1=0$

sin x = 1 or 0.5

We know,

x = nπ + $(-1)^n\theta$

$$x = n\pi + (-1)^n\left(\frac{\pi}{2}\right) \text{ or } x = n\pi + (-1)^n\left(\frac{\pi}{6}\right)$$

Therefore, the values of x are

$$\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{13\pi}{6}$$

So, these values are obtained for different value of n from the equation

$$x = n\pi + (-1)^n\frac{\pi}{3} - \frac{\pi}{6}, n \in Z$$

So, Option B

9. Question

Mark the Correct alternative in the following:

The smallest positive angle which satisfies the equation $2\sin^2 x + \sqrt{3}\cos x + 1 = 0$ is

A. $\frac{5\pi}{6}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

Answer

$2(1-\cos^2 x)+\sqrt{3}\cos x + 1 = 0$

$2 - 2\cos^2 x + \sqrt{3}\cos x + 1 = 0$

$2\cos^2 x - \sqrt{3}\cos x - 3 = 0$

$$\cos x = \sqrt{3} \text{ or } \frac{-\sqrt{3}}{2}$$

$$x = \pi - \frac{\pi}{6}$$

$$x = \frac{5\pi}{6}$$

Option A

10. Question

Mark the Correct alternative in the following:

If $4\sin^2 x = 1$, then the values of x are

A. $2n\pi \pm \frac{\pi}{3}, n \in Z$

B. $n\pi \pm \frac{\pi}{3}, n \in Z$

C. $n\pi \pm \frac{\pi}{6}, n \in Z$

D. $2n\pi \pm \frac{\pi}{6}, n \in Z$

Answer

$\sin x = \frac{1}{2} \text{ or } \frac{-1}{2}$

sin x = sin a

Here a= 30° or -30°

X=nπ +(-1)^n a

So, the values of x are

$n\pi \pm \frac{\pi}{6}, n \in Z$

Option C

11. Question

Mark the Correct alternative in the following:

If cot x – tan x = sec x, then x is equal to

A. $2n\pi + \frac{3\pi}{2}, n \in Z$

B. $n\pi + (-1)^n \frac{\pi}{6}, n \in Z$

C. $n\pi + \frac{\pi}{2}, n \in Z$

D. None of these

Answer

$\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \frac{1}{\cos x}$

$\frac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \frac{1}{\cos x}$

$\frac{1 - \sin^2 x - \sin^2 x}{\sin x} = 1$

1-2sin^{2}x=sin x

2sin^{2}x+sin x-1=0

Sin x = 0.5 or -1

But the equation is invalid for sin x=-1

So, sin x = 0.5 = sin(π/6)

Hence x=$n\pi+\left(-1\right)^{n}\frac{\pi}{6}, n\in Z$

Option B

12. Question

Mark the Correct alternative in the following:

A value of x satisfying is

A. $\frac{5\pi}{3}$

B. $\frac{4\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{\pi}{3}$

Answer

$\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x=1$

cos 60 cos x + sin 60 sin x = 1

cos (60-x)=1

cos (60-x)=cos 0°

x=60°

Option D

13. Question

Mark the Correct alternative in the following:

In (0, π), the number of solutions of the equation tan x + tan 2x + tan 3x = tan x tan 2x tan 3x is

A. 7

B. 5

C. 4

D. 2

Answer

tan x+tan2x+tan3x -tan xtan2xtan3x=0

tan x + tan2x + tan3x (1-tan xtan2x) = 0

tan x + tan2x = -tan3x (1-tan xtan2x)

$\frac{\tan x+\tan 2x}{1-\tan x\tan 2x}=-\tan 3x$

tan 3x=-tan3x

2 tan3x=0

tan 3x=0

3x=2nπ

$$X = \frac{2n\pi}{3}$$

For

n=0, x=0

n=1,

$$x = \frac{2\pi}{3}$$

$n = 2,$

$$x = \frac{4\pi}{3} > \pi$$

so, there are only two possible solutions

Option D

14. Question

Mark the Correct alternative in the following:

The number of values of x in [0, 2π] that satisfy the equation $\sin^2 x - \cos x = \frac{1}{4}$

A. 1

B. 2

C. 3

D. 4

Answer

1-$\cos^2$x-cos x-0.25=0

$$\cos^2 x + \cos x - \frac{3}{4} = 0$$

Solving the quadratic equation, we get

Cos x = 0.5

So x= 60° or 300°

Hence there are 2 values

Option B

15. Question

Mark the Correct alternative in the following:

If $e^{\sin x} - e^{-\sin x} - 4 = 0$, then x =

A. 0

B. $\sin^{-1}\{\log_e(2 - \sqrt{5})\}$

C. 1

D. None of these

Answer

$Log_e\ (e^{\sin x}-e^{-\sin x}) = \log_e(4)$

$$\frac{\log_e e^{\sin x}}{\log_e e^{-\sin x}} = \log_e 4$$

$$\frac{\sin x}{-\sin x} = \log_e 4$$

$$-1 = \log_e 4$$

But the above equation is not true so there are no possible values of x for this given equation

Option D

16. Question

Mark the Correct alternative in the following:

The equation 3 cos x + 4 sin x = 6 has .... Solution

A. finite

B. infinite

C. one

D. no

Answer

$4\sin x = 6-3\cos x$

$16\sin^2 x = 36+9\cos^2 x-36\cos x$

$16-16\cos^2 x = 36+9\cos^2 x-36\cos x$

$25\cos^2 x-36\cos x+20=0$

As both the roots are imaginary there exists no value of x satisfying this given equation.

No Solution

Option D

17. Question

Mark the Correct alternative in the following:

If $\sqrt{3}\cos x + \sin x = \sqrt{2},$ then general value of θ is

A. $n\pi + (-1)^n \frac{\pi}{4}, n \in Z$

B. $(-1)^n \frac{\pi}{4} - \frac{\pi}{3}, n \in Z$

C. $n\pi + \frac{\pi}{4} - \frac{\pi}{3}, n \in Z$

D. $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{3}, n \in Z$

Answer

$3\cos^2 x=(\sqrt{2} - \sin x)^2$

$3 - 3\sin^2 x=2 +\sin^2 x - 2\sqrt{2}\sin x$

$4\sin^2x - 2\sqrt{2}\sin x - 1 = 0$

$$\sin x = \frac{\sqrt{3}+1}{2\sqrt{2}} \text{ or } \frac{-\sqrt{3}+1}{2\sqrt{2}}$$

So, x=15° or 345°

And these values are obtained by the following equation

$$n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{3}, n \in Z$$

Option D

18. Question

Mark the Correct alternative in the following:

General solution of tan 5x = cot 2x is

A. $\frac{n\pi}{7} + \frac{\pi}{2}, n \in Z$

B. $x = \frac{n\pi}{7} + \frac{\pi}{3}, n \in Z$

C. $x = \frac{n\pi}{7} + \frac{\pi}{14}, n \in Z$

D. $x = \frac{n\pi}{7} - \frac{\pi}{14}, n \in Z$

Answer

$$\tan 5x = \tan(\frac{\pi}{2} - 2x)$$

$$\tan 5x - \tan(\frac{\pi}{2} - 2x) = 0$$

$$\frac{\sin 5x}{\cos 5x} - \frac{\sin\left(\frac{\pi}{2} - 2x\right)}{\cos\left(\frac{\pi}{2} - 2x\right)} = 0$$

$$\frac{\sin 5x\cos\left(\frac{\pi}{2} - 2x\right) - \sin\left(\frac{\pi}{2} - 2x\right)\cos 5x}{\cos 5x\cos\left(\frac{\pi}{2} - 2x\right)} = 0$$

$$\frac{\sin(5x - \frac{\pi}{2} + 2x)}{\cos 5x\cos\left(\frac{\pi}{2} - 2x\right)} = 0$$

This implies $\sin\left(7x - \frac{\pi}{2}\right) = 0$

But $\cos 5x\cos\left(\frac{\pi}{2} - 2x\right) \neq 0$

So $\sin\left(7x - \frac{\pi}{2}\right) = 0$

$$\sin\left(7x - \frac{\pi}{2}\right) = \sin 0$$

$$7x - \frac{\pi}{2} = n\pi$$

$x = \frac{n\pi}{7} + \frac{\pi}{14}$

Option C

19. Question

Mark the Correct alternative in the following:

The solution of the equation $\cos^2 x + \sin x + 1 = 0$ lies in the interval

A. (- π/4, π/4)

B. (- π/3, π/4)

C. (3π/4, 5π/4)

D. (5π/4, 7π/4)

Answer

$1-\sin^2x+\sin x+1=0$

$\sin^2x-\sin x-2=0$

sin x = -1

so, x= 3π/2

and it lies between (5π/4, 7π/4)

Option D

20. Question

Mark the Correct alternative in the following:

If and 0 < x < 2 π, then the solution are

A. $x = \frac{\pi}{3}, \frac{4\pi}{3}$

B. $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

C. $x = \frac{2\pi}{3}, \frac{7\pi}{6}$

D. $\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$

Answer

We know if cos x=cos a

Then

x = 2nπ ± a

here $\cos x = \cos\left(\frac{2\pi}{3}\right)$

when n=0,

$x = \frac{2\pi}{3}$

when n=1,

$$x = 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

Option B

21. Question

Mark the Correct alternative in the following:

The number of values of x in the interval [0. 5 π] satisfying the equation 3 $\sin^2$ x – 7 sin x + 2 = 0 is

A. 0

B. 5

C. 6

D. 10

Answer

$3\sin^2 x - 7\sin x + 2 = 0$

Solving the equation, we get

$$\sin x = \frac{1}{3}$$

$$a = \sin^{-1}\left(\frac{1}{3}\right)$$

=19.47122

x= nπ + $(-1)^n$ a

For

n=0, x=a

n=1, x = π – a

n=2, x = 2π + a

n=3, x = 3π – a

n=4, x = 4π + a

n=5, x = 5π + a

So, there are 6 values less then 5π.

Option C.