

12. Algebraic Identities

EXERCISE 12(A)

Question 1.

Use direct method to evaluate the following products :

(i) $(x + 8)(x + 3)$

(ii) $(y + 5)(y - 3)$

(iii) $(a - 8)(a + 2)$

(iv) $(b - 3)(b - 5)$

(v) $(3x - 2y)(2x + y)$

(vi) $(5a + 16)(3a - 7)$

(vii) $(8 - b)(3 + b)$

Solution:

$$\begin{aligned}(i) (x + 8)(x + 3) &= (x \times x) + (x \times 3) + (8 \times x) + (8 \times 3) \\ &= x^2 + 3x + 8x + 24 \\ &= x^2 + 11x + 24\end{aligned}$$

$$\begin{aligned}(ii) (y + 5)(y - 3) &= (y \times y) + (y \times -3) + (5 \times y) + (5 \times -3) \\ &= y^2 + (-3y) + (5y) - 15 \\ &= y^2 - 3y + 5y - 15 \\ &= y^2 + 2y - 15\end{aligned}$$

$$\begin{aligned}(iii) (a - 8)(a + 2) &= (a \times a) + (a \times 2) + (-8) \times a + (-8)(2) \\ &= a^2 + 2a - 8a - 16 \\ &= a^2 - 6a - 16\end{aligned}$$

$$\begin{aligned}(iv) (b - 3)(b - 5) &= (b \times b) + (b \times -5) \\ &\quad + (-3) \times b + (-3)(-5) \\ &= b^2 - 5b - 3b + 15 \\ &= b^2 - 8b + 15\end{aligned}$$

$$\begin{aligned}(v) (3x - 2y)(2x + y) &= (3x \times 2x) + (3x \times y) \\ &\quad + (-2y \times 2x) + (-2y \times y) \\ &= 6x^2 + 3xy - 4xy - 2y^2 \\ &= 6x^2 - xy - 2y^2\end{aligned}$$

$$\begin{aligned}(vi) (5a + 16)(3a - 7) &= (5a \times 3a) + (5a \times -7) \\ &\quad + (16 \times 3a) + 16 \times -7 \\ &= 15a^2 + (-35a) + 48a + (-112) \\ &= 15a^2 - 35a + 48a - 112 \\ &= 15a^2 + 13a - 112\end{aligned}$$

$$\begin{aligned}(vii) (8 - b)(3 + b) &= (8 \times 3) + (8 \times b) \\ &\quad + (-b \times 3) + (-b \times b) \\ &= 24 + 8b - 3b - b^2 \\ &= 24 + 5b - b^2\end{aligned}$$

Question 2.

Use direct method to evaluate :

- (i) $(x+1)(x-1)$ (ii) $(2+a)(2-a)$
(iii) $(3b-1)(3b+1)$ (iv) $(4+5x)(4-5x)$
(v) $(2a+3)(2a-3)$ (vi) $(xy+4)(xy-4)$
(vii) $(ab+x^2)(ab-x^2)$
(viii) $(3x^2+5y^2)(3x^2-5y^2)$

(ix) $\left(z - \frac{2}{3}\right)\left(z + \frac{2}{3}\right)$

(x) $\left(\frac{3}{5}a + \frac{1}{2}\right)\left(\frac{3}{5}a - \frac{1}{2}\right)$

(xi) $(0.5 - 2a)(0.5 + 2a)$

(xii) $\left(\frac{a}{2} - \frac{b}{3}\right)\left(\frac{a}{2} + \frac{b}{3}\right)$

Solution:

Note : $(a+b)(a-b) = a^2 - b^2$

- (i) $(x+1)(x-1) = (x)^2 - (1)^2$
 $= x^2 - 1$
(ii) $(2+a)(2-a) = (2)^2 - (a)^2$
 $= 4 - a^2$
(iii) $(3b-1)(3b+1) = (3b)^2 - (1)^2$
 $= 9b^2 - 1$
(iv) $(4+5x)(4-5x) = (4)^2 - (5x)^2$
 $= 16 - 25x^2$
(v) $(2a+3)(2a-3) = (2a)^2 - (3)^2$
 $= 4a^2 - 9$
(vi) $(xy+4)(xy-4) = (xy)^2 - (4)^2$
 $= x^2y^2 - 16$
(vii) $(ab+x^2)(ab-x^2) = (ab)^2 - (x^2)^2$
 $= a^2b^2 - x^4$
(viii) $(3x^2+5y^2)(3x^2-5y^2) = (3x^2)^2 - (5y^2)^2$
 $= 9x^4 - 25y^4$
(ix) $\left(z - \frac{2}{3}\right)\left(z + \frac{2}{3}\right) = (z)^2 - \left(\frac{2}{3}\right)^2$
 $= z^2 - \frac{4}{9}$

$$(x) \quad \left(\frac{3}{5}a + \frac{1}{2}\right)\left(\frac{3}{5}a - \frac{1}{2}\right)$$

$$= \left(\frac{3}{5}a\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{9}{25}a^2 - \frac{1}{4}$$

$$(xi) \quad (0.5-2a)(0.5+2a)$$

$$= (0.5)^2 - (2a)^2$$

$$= 0.25 - 4a^2$$

$$(xii) \quad \left(\frac{a}{2} - \frac{b}{3}\right)\left(\frac{a}{2} + \frac{b}{3}\right) = \left(\frac{a}{2}\right)^2 - \left(\frac{b}{3}\right)^2$$

$$= \frac{a^2}{4} - \frac{b^2}{9}$$

Question 3.

Evaluate :

$$(i) \quad (a+1)(a-1)(a^2+1)$$

$$(ii) \quad (a+b)(a-b)(a^2+b^2)$$

$$(iii) \quad (2a-b)(2a+b)(4a^2+b^2)$$

$$(iv) \quad (3-2x)(3+2x)(9+4x^2)$$

$$(v) \quad (3x-4y)(3x+4y)(9x^2+16y^2)$$

$$(i) \quad (a+1)(a-1)(a^2+1)$$

$$= [(a)^2 - (1)^2](a^2+1)$$

$$= (a^2-1)(a^2+1)$$

$$= (a^2)^2 - (1)^2$$

$$= a^4 - 1$$

$$\begin{aligned}
(ii) \quad (a+b)(a-b)(a^2+b^2) &= (a^2-b^2)(a^2+b^2) \\
&= (a^2)^2 - (b^2)^2 \\
&= a^4 - b^4 \\
(iii) \quad (2a-b)(2a+b)(4a^2+b^2) &= [(2a)^2 - (b)^2](4a^2+b^2) \\
&= (4a^2-b^2)(4a^2+b^2) \\
&= (4a^2)^2 - (b^2)^2 \\
&= 16a^4 - b^4 \\
(iv) \quad (3-2x)(3+2x)(9+4x^2) &= [(3)^2 - (2x)^2](9+4x^2) \\
&= (9-4x^2)(9+4x^2) \\
&= (9)^2 - (4x^2)^2 \\
&= 81 - 16x^4 \\
(v) \quad (3x-4y)(3x+4y)(9x^2+16y^2) &= [(3x)^2 - (4y)^2](9x^2+16y^2) \\
&= (9x^2-16y^2)(9x^2+16y^2) \\
&= (9x^2)^2 - (16y^2)^2 \\
&= 81x^4 - 256y^4
\end{aligned}$$

Question 4.

Use the product $(a + b)(a - b) = a^2 - b^2$ to evaluate:

- (i) 21×19
- (ii) 33×27
- (iii) 103×97
- (iv) 9.8×10.2
- (v) 7.7×8.3
- (vi) 4.6×5.4

Solution:

$$\begin{aligned}
(i) \quad 21 \times 19 &= (20 + 1)(20 - 1) \\
&= (20)^2 - (1)^2 = 400 - 1 = 399 \\
(ii) \quad 33 \times 27 &= (30 + 3)(30 - 3) \\
&= (30)^2 - (3)^2 = 900 - 9 = 891 \\
(iii) \quad 103 \times 97 &= (100 + 3)(100 - 3) \\
&= (100)^2 - (3)^2 = 10000 - 9 = 9991 \\
(iv) \quad 9.8 \times 10.2 &= (10 - .2)(10 + .2) \\
&= (10)^2 - (.2)^2 = 100 - .04 = 99.96 \\
(v) \quad 7.7 \times 8.3 &= (8 - .3)(8 + .3) \\
&= (8)^2 - (.3)^2 = 64 - .09 = 63.91 \\
(vi) \quad 4.6 \times 5.4 &= (5 - .4)(5 + .4) \\
&= (5)^2 - (.4)^2 = 25 - .16 = 24.84
\end{aligned}$$

Question 5.

Evaluate :

(i) $(6 - xy)(6 + xy)$

(ii) $\left(7x + \frac{2}{3}y\right)\left(7x - \frac{2}{3}y\right)$

(iii) $\left(\frac{a}{2b} + \frac{2b}{a}\right)\left(\frac{a}{2b} - \frac{2b}{a}\right)$

(iv) $\left(3x - \frac{1}{2y}\right)\left(3x + \frac{1}{2y}\right)$

(v) $(2a + 3)(2a - 3)(4a^2 + 9)$

(vi) $(a + bc)(a - bc)(a^2 + b^2c^2)$

(vii) $(5x + 8y)(3x + 5y)$

(viii) $(7x + 15y)(5x - 4y)$

(ix) $(2a - 3b)(3a + 4b)$

(x) $(9a - 7b)(3a - b)$

Solution:

$$\begin{aligned}(6 - xy)(6 + xy) &= 6(6 + xy) - xy(6 + xy) \\ &= 36 + 6xy - 6xy + (xy)^2 = 36 - x^2y^2\end{aligned}$$

(ii) $\left(7x + \frac{2}{3}y\right)\left(7x - \frac{2}{3}y\right)$

$$= 7x\left(7x - \frac{2}{3}y\right) + \frac{2}{3}y\left(7x - \frac{2}{3}y\right)$$

$$= 49x^2 - \frac{14}{3}xy + \frac{14}{3}xy - \frac{4}{9}y^2 = 49x^2 - \frac{4}{9}y^2$$

(iii) $\left(\frac{a}{2b} + \frac{2b}{a}\right)\left(\frac{a}{2b} - \frac{2b}{a}\right)$

$$= \frac{a}{2b}\left(\frac{a}{2b} - \frac{2b}{a}\right) + \frac{2b}{a}\left(\frac{a}{2b} - \frac{2b}{a}\right)$$

$$= \frac{a^2}{4b^2} - 1 + 1 - \frac{4b^2}{a^2} = \frac{a^2}{4b^2} - \frac{4b^2}{a^2}$$

(iv) $\left(3x - \frac{1}{2y}\right)\left(3x + \frac{1}{2y}\right)$

$$= 3x\left(3x + \frac{1}{2y}\right) - \frac{1}{2y}\left(3x + \frac{1}{2y}\right)$$

$$= 9x^2 + \frac{3x}{2y} - \frac{3x}{2y} - \frac{1}{4y^2} = 9x^2 - \frac{1}{4y^2}$$

$$(v) (2a + 3) (2a - 3) (4a^2 + 9)$$

$$= [(2a)^2 - (3)^2] (4a^2 + 9)$$

$$[(a + b) (a - b) = a^2 - b^2]$$

$$= (4a^2 - 9) (4a^2 + 9)$$

$$= (4a^2)^2 - (9)^2 \quad [(a + b) (a - b) = a^2 - b^2]$$

$$= 16a^4 - 81$$

$$(vi) (a + bc) (a - bc) (a^2 + b^2c^2)$$

$$= [(a)^2 - (bc)^2] (a^2 + b^2c^2)$$

$$[(a + b) (a - b) = a^2 - b^2]$$

$$= (a^2 - b^2c^2) (a^2 + b^2c^2)$$

$$= (a^2)^2 - (b^2c^2)^2 \quad [\because (a + b) (c - b) = a^2 - b^2]$$

$$= a^4 - b^4c^4$$

$$(vii) (5x + 8y) (3x + 5y)$$

$$= 5x (3x + 5y) + 8y (3x + 5y)$$

$$= 15x^2 + 25xy + 24xy + 40y^2$$

$$= 15x^2 + 49xy + 40y^2$$

$$(viii) (7x + 15y) (5x - 4y)$$

$$= 7x (5x - 4y) + 15y (5x - 4y)$$

$$= 35x^2 - 28xy + 75xy - 60y^2$$

$$= 35x^2 + 47xy - 60y^2$$

$$(ix) (2a - 3b) (3a + 4b)$$

$$= 2a (3a + 4b) - 3b (3a + 4b)$$

$$= 6a^2 + 8ab - 9ab - 12b^2$$

$$= 6a^2 - ab - 12b^2$$

$$(x) (9a - 7b) (3a - b)$$

$$= 9a (3a - b) - 7b (3a - b)$$

$$= 27a^2 - 9ab - 21ab + 7b^2$$

$$= 27a^2 - 30ab + 7b^2$$

EXERCISE 12(B)

Question 1.

Expand :

(i) $(2a + b)^2$

(ii) $(a - 2b)^2$

(iii) $\left(a + \frac{1}{2a}\right)^2$

(iv) $\left(2a - \frac{1}{a}\right)^2$

(v) $(a+b-c)^2$

(vi) $(a-b+c)^2$

(vii) $\left(3x + \frac{1}{3x}\right)^2$

(viii) $\left(2x - \frac{1}{2x}\right)^2$

Solution:

$$\begin{aligned}(i) \quad (2a+b)^2 &= (2a)^2 + (b)^2 + 2 \times 2a \times b \\ & \quad [(a+b)^2 = a^2 + b^2 + 2ab] \\ &= 4a^2 + b^2 + 4ab\end{aligned}$$

$$\begin{aligned}(ii) \quad (a-2b)^2 &= (a)^2 + (2b)^2 - 2 \times a \times 2b \\ & \quad [(a-b)^2 = a^2 + b^2 - 2ab] \\ &= a^2 + 4b^2 - 4ab\end{aligned}$$

$$\begin{aligned}(iii) \quad \left(a + \frac{1}{2a}\right)^2 &= (a)^2 + \left(\frac{1}{2a}\right)^2 + 2 \times a \times \frac{1}{2a} \\ &= a^2 + \frac{1}{4a^2} + \frac{2a}{2a} \\ &= a^2 + \frac{1}{4a^2} + 1\end{aligned}$$

$$\begin{aligned}(iv) \quad \left(2a - \frac{1}{a}\right)^2 &= (2a)^2 + \left(\frac{1}{a}\right)^2 - 2 \times 2a \times \frac{1}{a} \\ &= 4a^2 + \frac{1}{a^2} - 4\end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad (a+b-c)^2 &= (a)^2 + (b)^2 + (-c)^2 \\
 &\quad + 2 \times a \times b + 2 \times b \times (-c) + 2 \times (-c) \times (a) \\
 &= a^2 + b^2 + c^2 + 2ab - 2bc - 2ca
 \end{aligned}$$

(Note : $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$)

$$\begin{aligned}
 \text{(vi)} \quad (a-b+c)^2 &= (a)^2 + (-b)^2 + (c)^2 + 2 \times a \times -b \\
 &\quad + 2(-b)(c) + 2 \times c \times a \\
 &= a^2 + b^2 + c^2 - 2ab - 2bc + 2ca
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii)} \quad \left(3x + \frac{1}{3x}\right)^2 &= (3x)^2 + \left(\frac{1}{3x}\right)^2 + 2 \times 3x \times \frac{1}{3x} \\
 &= 9x^2 + \frac{1}{9x^2} + 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(viii)} \quad \left(2x - \frac{1}{2x}\right)^2 &= (2x)^2 + \left(\frac{1}{2x}\right)^2 - 2 \times 2x \times \frac{1}{2x} \\
 &= 4x^2 + \frac{1}{4x^2} - 2
 \end{aligned}$$

Question 2.

Find the square of :

(i) $x+3y$

(ii) $2x-5y$

(iii) $a + \frac{1}{5a}$

(iv) $2a - \frac{1}{a}$

(v) $x-2y+1$

(vi) $3a-2b-5c$

(vii) $2x + \frac{1}{x} + 1$

(viii) $5-x + \frac{2}{x}$

(ix) $2x-3y+z$

(x) $x + \frac{1}{x} - 1$

Solution:

$$(i) \quad (x+3y)^2 = (x)^2 + (3y)^2 + 2 \times x \times 3y \\ = x^2 + 9y^2 + 6xy$$

$$(ii) \quad (2x-5y)^2 = (2x)^2 + (5y)^2 - 2 \times 2x \times 5y \\ = 4x^2 + 25y^2 - 20xy$$

$$(iii) \quad \left(a + \frac{1}{5a}\right)^2 = (a)^2 + \left(\frac{1}{5a}\right)^2 + 2 \times a \times \frac{1}{5a} \\ = a^2 + \frac{1}{25a^2} + \frac{2}{5}$$

$$(iv) \quad \left(2a - \frac{1}{a}\right)^2 = (2a)^2 + \left(\frac{1}{a}\right)^2 - 2 \times 2a \times \frac{1}{a} \\ = 4a^2 + \frac{1}{a^2} - 4$$

$$(v) \quad (x-2y+1)^2 = (x)^2 + (-2y)^2 + (1)^2 + 2 \times x \\ \times (-2y) + 2 \times (-2y) \times 1 + 2 \times 1 \times x \\ = x^2 + 4y^2 + 1 - 4xy - 4y + 2x$$

$$(vi) \quad (3a-2b-5c)^2 = (3a)^2 + (-2b)^2 + (-5c)^2 \\ + 2 \times 3a \times (-2b) + 2 \times (-2b) \times (-5c) \\ + 2 \times (-5c) \times 3a \\ = 9a^2 + 4b^2 + 25c^2 - 12ab \\ + 20bc - 30ca$$

$$(vii) \quad \left(2x + \frac{1}{x} + 1\right)^2 = (2x)^2 + \left(\frac{1}{x}\right)^2 + (1)^2 + 2 \times$$

$$2x \times \frac{1}{x} + 2 \times \frac{1}{x} \times 1 + 2 \times 1 \times 2x$$

$$= 4x^2 + \frac{1}{x^2} + 1 + 4 + \frac{2}{x} + 4x$$

$$= 4x^2 + \frac{1}{x^2} + 5 + \frac{2}{x} + 4x$$

$$(viii) \left(5 - x + \frac{2}{x}\right)^2 = (5)^2 + (-x)^2 + \left(\frac{2}{x}\right)^2$$

$$+ 2 \times 5 \times (-x) + 2(-x) \times \frac{2}{x} + 2 \times \frac{2}{x} \times 5$$

$$= 25 + x^2 + \frac{4}{x^2} - 10x - 4 + \frac{20}{x}$$

$$= 21 + x^2 + \frac{4}{x^2} - 10x + \frac{20}{x}$$

$$(ix) (2x - 3y + z)^2 = (2x)^2 + (-3y)^2 + (z)^2 + 2 \times 2x \times$$

$$-3y + 2(-3y) \times z + 2 \times z \times 2x$$

$$= 4x^2 + 9y^2 + z^2 - 12xy - 6yz + 4zx$$

$$(x) \left(x + \frac{1}{x} - 1\right)^2 = (x)^2 + \left(\frac{1}{x}\right)^2 + (-1)^2$$

$$+ 2 \times x \times \frac{1}{x} + 2 \times \frac{1}{x} \times (-1) + 2(-1) \times x$$

$$= x^2 + \frac{1}{x^2} + 1 + 2 - \frac{2}{x} - 2x$$

$$= x^2 + \frac{1}{x^2} + 3 - \frac{2}{x} - 2x$$

Question 3.

Evaluate:

Using expansion of $(a + b)^2$ or $(a - b)^2$

(i) $(208)^2$

(ii) $(92)^2$

(iii) $(415)^2$

(iv) $(188)^2$

(v) $(9.4)^2$

(vi) $(20.7)^2$

Solution:

$$\begin{aligned} \text{(i)} \quad (208)^2 &= (200 + 8)^2 \\ &= (200)^2 + (8)^2 + 2(200)(8) = 40000 + 64 + 3200 = 43264 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (92)^2 &= (100 - 8)^2 = (100)^2 + (8)^2 - 2(100)(8) \\ &= 10000 + 64 - 1600 = 10064 - 1600 = 8464 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad (415)^2 &= (400 + 15)^2 \\ &= (400)^2 + (15)^2 + 2(400)(15) = 160000 + 225 + 12000 = 172225 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad (188)^2 &= (200 - 12)^2 \\ &= (200)^2 + (12)^2 - 2(200)(12) = 40000 + 144 - 4800 \\ &= 40144 - 4800 = 35344 \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad (9.4)^2 &= (10 - .6)^2 \\ &= (10)^2 + (.6)^2 - 2(10)(.6) = 100 + .36 - 12 \\ &= 88 + .36 = 88.36 \end{aligned}$$

$$\begin{aligned} \text{(vi)} \quad (20.7)^2 &= (20 + .7)^2 = (20)^2 + (.7)^2 + 2(20)(.7) \\ &= 400 + .49 + 28 = 428 + .49 = 428.49 \end{aligned}$$

Question 4.

Expand :

$$\text{(i)} \quad (2a+b)^3$$

$$\text{(ii)} \quad (a-2b)^3$$

$$\text{(iii)} \quad (3x-2y)^3$$

$$\text{(iv)} \quad (x+5y)^3$$

$$\text{(v)} \quad \left(a + \frac{1}{a}\right)^3$$

$$\text{(vi)} \quad \left(2a - \frac{1}{2a}\right)^3$$

Solution:

$$(i) (2a+b)^3 = (2a)^3 + (b)^3 + 3 \times 2a \times b(2a+b)$$

$$[(a+b)^3 = a^3 + b^3 + 3ab(a+b)]$$

$$= 8a^3 + b^3 + 6ab(2a+b)$$

$$= 8a^3 + b^3 + 12a^2b + 6ab^2$$

$$(ii) (a-2b)^3 = (a)^3 - (2b)^3 - 3 \times a \times 2b(a-2b)$$

$$[(a-b)^3 = a^3 - b^3 - 3ab(a-b)]$$

$$= a^3 - 8b^3 - 6ab(a-2b)$$

$$= a^3 - 8b^3 - 6a^2b + 12ab^2$$

$$(iii) (3x-2y)^3 = (3x)^3 - (2y)^3 - 3 \times 3x \times 2y(3x-2y)$$

$$= 27x^3 - 8y^3 - 18xy(3x-2y)$$

$$= 27x^3 - 8y^3 - 54x^2y + 36xy^2$$

$$(iv) (x+5y)^3 = (x)^3 + (5y)^3 + 3 \times x \times 5y(x+5y)$$

$$= x^3 + 125y^3 + 15xy(x+5y)$$

$$= x^3 + 125y^3 + 15x^2y + 75y^2$$

$$(v) \left(a + \frac{1}{a}\right)^3$$

$$= a^3 + \left(\frac{1}{a}\right)^3 + 3 \times a \times \frac{1}{a} \times \left(a + \frac{1}{a}\right)$$

$$= a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$$

$$= a^3 + \frac{1}{a^3} + 3a + \frac{3}{a}$$

$$(vi) \left(2a - \frac{1}{2a}\right)^3 = (2a)^3 - \left(\frac{1}{2a}\right)^3 - 3 \times 2a$$

$$\times \frac{1}{2a} \left(2a - \frac{1}{2a}\right)$$

$$= 8a^3 - \frac{1}{8a^3} - 3\left(2a - \frac{1}{2a}\right)$$

$$= 8a^3 - \frac{1}{8a^3} - 6a + \frac{3}{2a}$$

Question 5.

Find the cube of :

- (i) $a+2$ (ii) $2a-1$
 (iii) $2a+3b$ (iv) $3b-2a$
 (v) $2x+\frac{1}{x}$ (v) $x-\frac{1}{2}$

Solution:

$$\begin{aligned} \text{(i)} \quad (a+2)^3 &= (a)^3 + (2)^3 + 3 \times a \times 2(a+2) \\ &= a^3 + 8 + 6a(a+2) \\ &= a^3 + 8 + 6a^2 + 12a \\ &= a^3 + 6a^2 + 12a + 8 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (2a-1)^3 &= (2a)^3 - (1)^3 - 3 \times 2a \times 1(2a-1) \\ &= 8a^3 - 1 - 6a(2a-1) \\ &= 8a^3 - 1 - 12a^2 + 6a \\ &= 8a^3 - 12a^2 + 6a - 1 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad (2a+3b)^3 &= (2a)^3 + (3b)^3 + 3 \times 2a \times 3b \\ &\quad (2a+3b) \\ &= 8a^3 + 27b^3 + 18ab(2a+3b) \\ &= 8a^3 + 27b^3 + 36a^2b + 54ab^2 \\ &= 8a^3 + 36a^2b + 54ab^2 + 27b^3 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad (3b-2a)^3 &= (3b)^3 - (2a)^3 - 3 \times 3b \times 2a(3b-2a) \\ &= 27b^3 - 8a^3 - 18ab(3b-2a) \end{aligned}$$

$$= 27b^3 - 8a^3 - 54ab^2 + 36a^2b$$

$$= 27b^3 - 54b^2a + 36ba^2 - 8a^3$$

$$(v) \quad \left(2x + \frac{1}{x}\right)^3$$

$$= (2x)^3 + \left(\frac{1}{x}\right)^3 + 3 \times 2x \times \frac{1}{x} \left(2x + \frac{1}{x}\right)$$

$$= 8x^3 + \frac{1}{x^3} + 6 \left(2x + \frac{1}{x}\right)$$

$$= 8x^3 + \frac{1}{x^3} + 12x + \frac{6}{x}$$

$$= 8x^3 + 12x + \frac{6}{x} + \frac{1}{x^3}$$

$$(vi) \quad \left(x - \frac{1}{2}\right)^3$$

$$= (x)^3 - \left(\frac{1}{2}\right)^3 - 3 \times x \times \frac{1}{2} \left(x - \frac{1}{2}\right)$$

$$= x^3 - \frac{1}{8} - \frac{3x}{2} \left(x - \frac{1}{2}\right)$$

$$= x^3 - \frac{1}{8} - \frac{3x^2}{2} + \frac{3x}{4}$$

$$= x^3 - \frac{3x^2}{2} + \frac{3x}{4} - \frac{1}{8}$$

EXERCISE 12(C)

Question 1.

If $a+b=5$ and $ab = 6$; find $a^2 + b^2$

Solution:

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$\Rightarrow (5)^2 = a^2 + b^2 + 2 \times 6$$

$$\Rightarrow 25 = a^2 + b^2 + 12$$

$$\Rightarrow 25 - 12 = a^2 + b^2$$

$$\Rightarrow 13 = a^2 + b^2$$

$$\therefore a^2 + b^2 = 13$$

Question 2.

If $a - b = 6$ and $ab = 16$; find $a^2 + b^2$

Solution:

$$\begin{aligned}
 (a-b)^2 &= a^2 + b^2 - 2ab \\
 \Rightarrow (6)^2 &= a^2 + b^2 - 2 \times 16 \\
 \Rightarrow 36 &= a^2 + b^2 - 32 \\
 \Rightarrow 36 + 32 &= a^2 + b^2 \\
 \Rightarrow 68 &= a^2 + b^2 \\
 \therefore a^2 + b^2 &= 68
 \end{aligned}$$

Question 3.

If $a^2 + b^2 = 29$ and $ab = 10$; find :

(i) $a + b$

(ii) $a - b$

Solution:

$$\begin{aligned}
 (i) \quad (a+b)^2 &= a^2 + b^2 + 2ab \\
 \Rightarrow (a+b)^2 &= 29 + 2 \times 10 \\
 \Rightarrow (a+b)^2 &= 29 + 20 \\
 \Rightarrow (a+b)^2 &= 49 \\
 \Rightarrow a+b &= \sqrt{49} \\
 \Rightarrow a+b &= 7 \\
 (ii) \quad (a-b)^2 &= a^2 + b^2 - 2ab \\
 \Rightarrow (a-b)^2 &= 29 - 2 \times 10 \\
 \Rightarrow (a-b)^2 &= 29 - 20 \\
 \Rightarrow (a-b)^2 &= 9 \\
 \Rightarrow a-b &= \sqrt{9} \\
 \Rightarrow a-b &= 3
 \end{aligned}$$

Question 4.

If $a^2 + b^2 = 10$ and $ab = 3$; find :

(i) $a - b$

(ii) $a + b$

Solution:

$$\begin{aligned}
 (i) \quad (a-b)^2 &= a^2 + b^2 - 2ab \\
 \Rightarrow (a-b)^2 &= 10 - 2 \times 3 \\
 \Rightarrow (a-b)^2 &= 10 - 6 \\
 \Rightarrow (a-b)^2 &= 4 \\
 \Rightarrow (a-b) &= \sqrt{4} \\
 \Rightarrow a-b &= 2 \\
 (ii) \quad (a+b)^2 &= a^2 + b^2 + 2ab \\
 \Rightarrow (a+b)^2 &= 10 + 2 \times 3 \\
 \Rightarrow (a+b)^2 &= 10 + 6 \\
 \Rightarrow (a+b)^2 &= 16 \\
 \Rightarrow (a+b) &= \sqrt{16} \\
 \Rightarrow (a+b) &= 4
 \end{aligned}$$

Question 5.

If $a + \frac{1}{a} = 3$; find $a^2 + \frac{1}{a^2}$

Solution:

$$\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow (3)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow 9 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow 9 - 2 = a^2 + \frac{1}{a^2}$$

$$\Rightarrow 7 = a^2 + \frac{1}{a^2}$$

$$\therefore a^2 + \frac{1}{a^2} = 7$$

Alternative Method :

$$a + \frac{1}{a} = 3$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = (3)^2$$

$$\Rightarrow a^2 + \frac{1}{a^2} + 2 = 9$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 9 - 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 7$$

Question 6.

If $a - \frac{1}{a} = 4$; find $a^2 + \frac{1}{a^2}$

Solution:

$$\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow (4)^2 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow 16 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow 16 + 2 = a^2 + \frac{1}{a^2}$$

$$\Rightarrow 18 = a^2 + \frac{1}{a^2}$$

$$\therefore a^2 + \frac{1}{a^2} = 18$$

Alternative Method :

$$a - \frac{1}{a} = 4$$

$$\Rightarrow \left(a - \frac{1}{a}\right)^2 = (4)^2$$

$$\Rightarrow a^2 + \frac{1}{a^2} - 2 = 16$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 16 + 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 18$$

Question 7.

If $a^2 + \frac{1}{a^2} = 23$; find $a + \frac{1}{a}$

Solution:

$$\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = 23 + 2$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = 25$$

$$\Rightarrow a + \frac{1}{a} = \sqrt{25}$$

$$\Rightarrow a + \frac{1}{a} = 5$$

Question 8.

If $a^2 + \frac{1}{a^2} = 11$; find $a - \frac{1}{a}$

Solution:

$$\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow \left(a - \frac{1}{a}\right)^2 = 11 - 2$$

$$\Rightarrow \left(a - \frac{1}{a}\right)^2 = 9$$

$$\Rightarrow a - \frac{1}{a} = \sqrt{9}$$

$$\Rightarrow a - \frac{1}{a} = 3$$

Question 9.

If $a + b + c = 10$ and $a^2 + b^2 + c^2 = 38$; find $ab + bc + ca$

Solution:

$$\begin{aligned}a+b+c &= 10 \\ \Rightarrow (a+b+c)^2 &= (10)^2 \\ \Rightarrow a^2+b^2+c^2+2ab+2bc+2ca &= 100 \\ \Rightarrow 38+2(ab+bc+ca) &= 100 \\ \Rightarrow 2(ab+bc+ca) &= 100-38 \\ \Rightarrow 2(ab+bc+ca) &= 62\end{aligned}$$

$$\Rightarrow (ab+bc+ca) = \frac{62}{2}$$

$$\Rightarrow ab+bc+ca = 31$$

Alternative Method :

$$\begin{aligned}(a+b+c)^2 &= a^2+b^2+c^2+2ab+2bc+2ca \\ \Rightarrow (10)^2 &= 38+2(ab+bc+ca) \\ \Rightarrow 100 &= 38+2(ab+bc+ca) \\ \Rightarrow 100-38 &= 2(ab+bc+ca) \\ \Rightarrow 62 &= 2(ab+bc+ca) \\ \Rightarrow \frac{62}{2} &= ab+bc+ca \\ \Rightarrow 31 &= ab+bc+ca \\ \therefore ab+bc+ca &= 31\end{aligned}$$

Question 10.

Find $a^2 + b^2 + c^2$; if $a + b + c = 9$ and $ab + bc + ca = 24$

Solution:

$$\begin{aligned}a+b+c &= 9 \\ \Rightarrow (a+b+c)^2 &= (9)^2 \\ \Rightarrow a^2+b^2+c^2+2ab+2bc+2ca &= 81 \\ \Rightarrow a^2+b^2+c^2+2(ab+bc+ca) &= 81 \\ \Rightarrow a^2+b^2+c^2+2 \times 24 &= 81 \\ \Rightarrow a^2+b^2+c^2+48 &= 81 \\ \Rightarrow a^2+b^2+c^2 &= 81-48 \\ \Rightarrow a^2+b^2+c^2 &= 33\end{aligned}$$

Question 11.

Find $a + b + c$; if $a^2 + b^2 + c^2 = 83$ and $ab + bc + ca = 71$

Solution:

$$\begin{aligned}(a+b+c)^2 &= a^2+b^2+c^2+2ab+2bc+2ca \\ \Rightarrow (a+b+c)^2 &= 83+2(ab+bc+ca) \\ \Rightarrow (a+b+c)^2 &= 83+2 \times 71 \\ \Rightarrow (a+b+c)^2 &= 83+142 \\ \Rightarrow (a+b+c)^2 &= 225 \\ \Rightarrow a+b+c &= \sqrt{225} \\ \Rightarrow a+b+c &= 15\end{aligned}$$

Question 12.

If $a + b = 6$ and $ab=8$; find $a^3 + b^3$

Solution:

$$\begin{aligned}a+b &= 6 \\ \Rightarrow (a+b)^3 &= (6)^3 \\ \Rightarrow a^3+b^3+3ab(a+b) &= 216 \\ \Rightarrow a^3+b^3+3 \times 8(6) &= 216 \\ \Rightarrow a^3+b^3+144 &= 216 \\ \Rightarrow a^3+b^3 &= 216 - 144 \\ \Rightarrow a^3+b^3 &= 72\end{aligned}$$

Alternative Method :

$$\begin{aligned}(a+b)^3 &= a^3+b^3+3ab(a+b) \\ \Rightarrow (6)^3 &= a^3+b^3+3 \times 8(6) \\ \Rightarrow 216 &= a^3+b^3+144 \\ \Rightarrow 216-144 &= a^3+b^3 \\ \Rightarrow 72 &= a^3+b^3 \\ \Rightarrow a^3+b^3 &= 72\end{aligned}$$

Question 13.

If $a - b=3$ and $ab = 10$; find $a^3 - b^3$

Solution:

$$a-b = 3$$

$$\Rightarrow (a-b)^3 = (3)^3$$

$$\Rightarrow a^3 - b^3 - 3ab(a-b) = 27$$

$$\Rightarrow a^3 - b^3 - 3 \times 10(3) = 27$$

$$\Rightarrow a^3 - b^3 - 90 = 27$$

$$\Rightarrow a^3 - b^3 = 27 + 90$$

$$\Rightarrow a^3 - b^3 = 117$$

Alternative Method :

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$\Rightarrow (3)^3 = a^3 - b^3 - 3 \times 10(3)$$

$$\Rightarrow 27 = a^3 - b^3 - 90$$

$$\Rightarrow 27 + 90 = a^3 - b^3$$

$$\Rightarrow 117 = a^3 - b^3$$

$$\Rightarrow a^3 - b^3 = 117$$

Question 14.

Find $a^3 + \frac{1}{a^3}$ if $a + \frac{1}{a} = 5$

Solution:

$$a + \frac{1}{a} = 5$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^3 = (5)^3$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3a \times \frac{1}{a} \left(a + \frac{1}{a}\right) = 125$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3(5) = 125 \quad \left[a + \frac{1}{a} = 5\right]$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 15 = 125$$

$$\Rightarrow a^3 + \frac{1}{a^3} = 125 - 15$$

$$\Rightarrow a^3 + \frac{1}{a^3} = 110$$

Question 15.

Find $a^3 - \frac{1}{a^3}$ if $a - \frac{1}{a} = 4$.

Solution:

$$a - \frac{1}{a} = 4$$

$$\Rightarrow \left(a - \frac{1}{a}\right)^3 = (4)^3$$

$$\Rightarrow a^3 - \frac{1}{a^3} - 3a \times \frac{1}{a} \left(a - \frac{1}{a}\right) = 64$$

$$\Rightarrow a^3 - \frac{1}{a^3} - 3(4) = 64 \quad [\because a - \frac{1}{a} = 4]$$

$$\Rightarrow a^3 - \frac{1}{a^3} - 12 = 64$$

$$\Rightarrow a^3 - \frac{1}{a^3} = 64 + 12$$

$$\Rightarrow a^3 - \frac{1}{a^3} = 76$$

Question 16.

If $2x - \frac{1}{2x} = 4$; find : (i) $4x^2 + \frac{1}{4x^2}$

(ii) $8x^3 - \frac{1}{8x^3}$

Solution:

$$(i) \quad 2x - \frac{1}{2x} = 4$$

$$\Rightarrow \left(2x - \frac{1}{2x}\right)^2 = (4)^2$$

$$\Rightarrow (2x)^2 + \left(\frac{1}{2x}\right)^2 - 2 \times 2x \times \frac{1}{2x} = 16$$

$$\Rightarrow 4x^2 + \frac{1}{4x^2} - 2 = 16$$

$$\Rightarrow 4x^2 + \frac{1}{4x^2} = 16 + 2$$

$$\Rightarrow 4x^2 + \frac{1}{4x^2} = 18$$

$$(ii) \quad 2x - \frac{1}{2x} = 4$$

$$\Rightarrow \left(2x - \frac{1}{2x}\right)^3 = (4)^3$$

$$\Rightarrow (2x)^3 - \left(\frac{1}{2x}\right)^3 - 3 \times 2x \times \frac{1}{2x} \left(2x - \frac{1}{2x}\right) = 64$$

$$\Rightarrow 8x^3 - \frac{1}{8x^3} - 3(4) = 64$$

$$\Rightarrow 8x^3 - \frac{1}{8x^3} - 12 = 64$$

$$\Rightarrow 8x^3 - \frac{1}{8x^3} = 64 + 12 \Rightarrow 8x^3 - \frac{1}{8x^3} = 76$$

Question 17.

If $3x + \frac{1}{3x} = 3$; find : (i) $9x^2 + \frac{1}{9x^2}$

(ii) $27x^3 + \frac{1}{27x^3}$

Solution:

(i) $3x + \frac{1}{3x} = 3$

$$\Rightarrow \left(3x + \frac{1}{3x}\right)^2 = (3)^2$$

$$\Rightarrow (3x)^2 + \left(\frac{1}{3x}\right)^2 + 2 \times 3x \times \frac{1}{3x} = 9$$

$$\Rightarrow 9x^2 + \frac{1}{9x^2} + 2 = 9 \Rightarrow 9x^2 + \frac{1}{9x^2} = 9 - 2$$

$$\Rightarrow 9x^2 + \frac{1}{9x^2} = 7$$

(ii) $3x + \frac{1}{3x} = 3$

$$\Rightarrow \left(3x + \frac{1}{3x}\right)^3 = (3)^3$$

$$\Rightarrow (3x)^3 + \left(\frac{1}{3x}\right)^3 + 3 \times 3x \times \frac{1}{3x} \left(3x + \frac{1}{3x}\right) = 27$$

$$\Rightarrow 27x^3 + \frac{1}{27x^3} + 3 \left(3x + \frac{1}{3x}\right) = 27$$

$$\Rightarrow 27x^3 + \frac{1}{27x^3} + 3(3) = 27$$

$$\Rightarrow 27x^3 + \frac{1}{27x^3} + 9 = 27$$

$$\Rightarrow 27x^3 + \frac{1}{27x^3} = 27 - 9$$

$$\Rightarrow 27x^3 + \frac{1}{27x^3} = 18$$

Question 18.

The sum of the squares of two numbers is 13 and their product is 6. Find:

- (i) the sum of the two numbers.
 (ii) the difference between them.

Solution:

Let x and y be the two numbers, then

$$x^2 + y^2 = 13 \text{ and } xy = 6$$

$$\begin{aligned} \text{(i)} \quad (x + y)^2 &= x^2 + y^2 + 2xy \\ &= 13 + 2 \times 6 = 13 + 12 = 25 \end{aligned}$$

$$\therefore x + y = \pm \sqrt{25} = \pm 5$$

$$\text{(ii)} \quad (x - y)^2 = x^2 + y^2 - 2xy = 13 - 12 = 1$$

$$\therefore x - y = \pm 1$$

EXERCISE 12(D)**Question 1.**

$$\text{(i)} \quad \left(3x + \frac{1}{2}\right) \left(2x + \frac{1}{3}\right)$$

$$\text{(ii)} \quad (2a + 0.5)(7a - 0.3)$$

$$\text{(iii)} \quad (9 - y)(7 + y) \quad \text{(iv)} \quad (2 - z)(15 - z)$$

$$\text{(v)} \quad (a^2 + 5)(a^2 - 3) \quad \text{(vi)} \quad (4 - ab)(8 + ab)$$

$$\text{(vii)} \quad (5xy - 7)(7xy + 9) \quad \text{(viii)} \quad (3a^2 - 4b^2)(8a^2 - 3b^2)$$

Solution:

$$\text{(i)} \quad \left(3x + \frac{1}{2}\right) \left(2x + \frac{1}{3}\right)$$

$$= 3x \left(2x + \frac{1}{3}\right) + \frac{1}{2} \left(2x + \frac{1}{3}\right)$$

$$= 6x^2 + x + x + \frac{1}{6} = 6x^2 + 2x + \frac{1}{6}$$

$$\text{(ii)} \quad (2a + 0.5)(7a - 0.3)$$

$$= 2a(7a - 0.3) + 0.5(7a - 0.3)$$

$$= 14a^2 - 0.6a + 3.5a - 0.15$$

$$= 14a^2 + 2.9a - 0.15$$

$$\text{(iii)} \quad (9 - y)(7 + y) = 9(7 + y) - y(7 + y)$$

$$= 63 + 9y - 7y - y^2 = 63 + 2y - y^2$$

$$\text{(iv)} \quad (2 - z)(15 - z) = 2(15 - z) - z(15 - z)$$

$$= 30 - 2z - 15z + z^2 = 30 - 17z + z^2$$

$$\text{(v)} \quad (a^2 + 5)(a^2 - 3) = a^2(a^2 - 3) + 5(a^2 - 3)$$

$$= a^4 - 3a^2 + 5a^2 - 15 = a^4 + 2a^2 - 15$$

$$(vi) (4 - ab)(8 + ab) = 4(8 + ab) - ab(8 + ab)$$

$$= 32 + 4ab - 8ab - a^2b^2 = 32 - 4ab - a^2b^2$$

$$(vii) (5xy - 7)(7xy + 9) = 5xy(7xy + 9) - 7(7xy + 9)$$

$$= 35x^2y^2 + 45xy - 49xy - 63$$

$$= 35x^2y^2 - 4xy - 63$$

$$(viii) (3a^2 - 4b^2)(8a^2 - 3b^2)$$

$$= 3a^2(8a^2 - 3b^2) - 4b^2(8a^2 - 3b^2)$$

$$= 24a^4 - 9a^2b^2 - 32a^2b^2 + 12b^4$$

$$= 24a^4 - 41a^2b^2 + 12b^4$$

Question 2.

Evaluate:

$$(i) \left(2x - \frac{3}{5}\right)\left(2x + \frac{3}{5}\right) \quad (ii) \left(\frac{4}{7}a + \frac{3}{4}b\right)\left(\frac{4}{7}a - \frac{3}{4}b\right)$$

$$(iii) (6 - 5xy)(6 + 5xy)$$

$$(iv) \left(2a + \frac{1}{2a}\right)\left(2a - \frac{1}{2a}\right)$$

$$(v) (4x^2 - 5y^2)(4x^2 + 5y^2)$$

$$(vi) (1.6x + 0.7y)(1.6x - 0.7y)$$

$$(vii) (m + 3)(m - 3)(m^2 + 9)$$

$$(viii) (3x + 4y)(3x - 4y)(9x^2 + 16y^2)$$

$$(ix) (a + bc)(a - bc)(a^2 + b^2c^2)$$

$$(x) 203 \times 197$$

$$(xi) 20.8 \times 19.2$$

Solution:

$$(i) \left(2x - \frac{3}{5}\right)\left(2x + \frac{3}{5}\right)$$

$$= (2x)^2 - \left(\frac{3}{5}\right)^2 \quad [\because (a - b)(a + b) = a^2 - b^2]$$

$$= 4x^2 - \frac{9}{25}$$

$$(ii) \left(\frac{4}{7}a + \frac{3}{4}b\right)\left(\frac{4}{7}a - \frac{3}{4}b\right)$$

$$= \left(\frac{4}{7}a\right)^2 - \left(\frac{3}{4}b\right)^2 \quad [\because (a - b)(a + b) = a^2 - b^2]$$

$$= \frac{16}{49}a^2 - \frac{9}{16}b^2$$

$$\text{(iii)} (6 - 5xy)(6 + 5xy)$$

$$= (6)^2 - (5xy)^2$$

$$= 36 - 25x^2y^2 \quad [\because (a - b)(a + b) = a^2 - b^2]$$

$$\text{(iv)} \left(2a + \frac{1}{2a}\right)\left(2a - \frac{1}{2a}\right)$$

$$= (2a)^2 - \left(\frac{1}{2a}\right)^2 \quad [\because (a - b)(a + b) = a^2 - b^2]$$

$$= 4a^2 - \frac{1}{4a^2}$$

$$\text{(v)} (4x^2 - 5y^2)(4x^2 + 5y^2)$$

$$= (4x^2)^2 - (5y^2)^2$$

$$= 16x^4 - 25y^4 \quad [\because (a - b)(a + b) = a^2 - b^2]$$

$$\text{(vi)} (1.6x + 0.7y)(1.6x - 0.7y)$$

$$= (1.6x)^2 - (0.7y)^2 \quad [\because (a - b)(a + b) = a^2 - b^2]$$

$$= 2.56x^2 - 0.49y^2$$

$$\text{(vii)} (m + 3)(m - 3)(m^2 + 9)$$

$$= (m)^2 - (3)^2 (m^2 + 9) \quad [\because (a - b)(a + b) = a^2 - b^2]$$

$$= (m^2 - 9)(m^2 + 9)$$

$$= (m^2)^2 - 9^2 = m^4 - 81$$

$$\begin{aligned}
& \text{(viii)} (3x + 4y) (3x - 4y) (9x^2 + 16y^2) \\
&= [(3x)^2 - (4y)^2] (9x^2 + 16y^2) \\
&\quad [\because (a - b) (a + b) = a^2 - b^2] \\
&= (9x^2 - 16y^2) (9x^2 + 16y^2) \\
&= (9x^2)^2 - (16y^2)^2 \quad [\because (a - b) (a + b) = a^2 - b^2] \\
&= 81x^4 - 256y^4 \\
& \text{(ix)} (a + bc) (a - bc) (a^2 + b^2c^2) \\
&= [a^2 - (bc)^2] (a^2 + b^2c^2) \\
&\quad [\because (a - b) (a + b) = a^2 - b^2] \\
&= (a^2 - b^2c^2) (a^2 + b^2c^2) \\
&= (a^2)^2 - (b^2c^2)^2 \quad [\because (a - b) (a + b) = a^2 - b^2] \\
&= a^4 - b^4c^4 \\
& \text{(x)} 203 \times 197 \\
&= (200 + 3) (200 - 3) \\
&= (200)^2 - (3)^2 = 40000 - 9 \\
&\quad [\because (a - b) (a + b) = a^2 - b^2] \\
&= 39991 \\
& \text{(xi)} 20.8 \times 19.2 = (20 + .8) (20 - .8) \\
&= (20)^2 - (.8)^2 \quad [\because (a - b) (a + b) = a^2 - b^2] \\
&= 400 - .64 = 399.36
\end{aligned}$$

Question 3.

Find the square of :

(i) $3x + \frac{2}{y}$

(ii) $\frac{5a}{6b} - \frac{6b}{5a}$

(iii) $2m^2 - \frac{2}{3}n^2$

(iv) $5x + \frac{1}{5x}$

(v) $8x + \frac{3}{2}y$

(vi) 607

(vii) 391

(viii) 9.7

Solution:

$$(i) 3x + \frac{2}{y}$$

$$\left(3x + \frac{2}{y}\right)^2 = (3x)^2 + \left(\frac{2}{y}\right)^2 + 2(3x)\left(\frac{2}{y}\right)$$

$$= 9x^2 + \frac{4}{y^2} + \frac{12x}{y}$$

$$(ii) \left(\frac{5a}{6b} - \frac{6b}{5a}\right)^2 = \left(\frac{5a}{6b}\right)^2 + \left(\frac{6b}{5a}\right)^2 - 2 \times \frac{5a}{6b} \times \frac{6b}{5a}$$

$$= \frac{25a^2}{36b^2} - 2 + \frac{36b^2}{25a^2}$$

$$(iii) 2m^2 - \frac{2}{3}n^2$$

$$\left(2m^2 - \frac{2}{3}n^2\right)^2 = (2m^2)^2 + \left(\frac{2}{3}n^2\right)^2 - 2 \times 2m^2 \times \frac{2}{3}n^2$$

$$= 4m^4 + \frac{4}{9}n^4 - \frac{8}{3}m^2n^2$$

$$= 4m^4 - \frac{8}{3}m^2n^2 + \frac{4}{9}n^4$$

$$(iv) \left(5x + \frac{1}{5x}\right)^2 = (5x)^2 + \frac{1}{(5x)^2} + 2 \times 5x \times \frac{1}{5x}$$

$$= 25x^2 + \frac{1}{25x^2} + 2 = 25x^2 + 2 + \frac{1}{25x^2}$$

$$(v) \left(8x + \frac{3}{2}y\right)^2 = (8x)^2 + \left(\frac{3}{2}y\right)^2 + 2 \times 8x \times \frac{3}{2}y$$

$$= 64x^2 + \frac{9}{4}y^2 + 24xy = 64x^2 + 24xy + \frac{9}{4}y^2$$

$$(vi) (607)^2 = (600 + 7)^2 = (600)^2 + (7)^2 + 2(600)(7)$$

$$= 360000 + 49 + 8400 = 368449$$

$$(vii) (391)^2 = (400 - 9)^2 = (400)^2 + 9^2 - 2(400)(9)$$

$$= 160000 + 81 - 7200 = 152881$$

$$(viii) (9.7)^2 = (10 - .3)^2 = (10)^2 + (.3)^2 - 2(10)(.3)$$

$$= 100 + .09 - 6 = 100.09 - 6.00 = 94.09$$

Question 4.

If $a + \frac{1}{a} = 2$, find :

(i) $a^2 + \frac{1}{a^2}$ (ii) $a^4 + \frac{1}{a^4}$

Solution:

$$\begin{aligned} \text{(i) } a^2 + \frac{1}{a^2} &= \left(a + \frac{1}{a}\right)^2 - 2 \\ &= (2)^2 - 2 = 4 - 2 = 2 \end{aligned}$$

$$\begin{aligned} \text{(ii) } a^4 + \frac{1}{a^4} &= \left(a^2 + \frac{1}{a^2}\right)^2 - 2 \\ &= (2)^2 - 2 = 4 - 2 = 2 \end{aligned}$$

Question 5.

If $m - \frac{1}{m} = 5$, find :

(i) $m^2 + \frac{1}{m^2}$ (ii) $m^4 + \frac{1}{m^4}$

(iii) $m^2 - \frac{1}{m^2}$

Solution:

$$\begin{aligned} \text{(i)} \quad m^2 + \frac{1}{m^2} &= \left(m - \frac{1}{m}\right)^2 + 2 \\ &= (5)^2 + 2 = 25 + 2 = 27 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad m^4 + \frac{1}{m^4} &= \left(m^2 + \frac{1}{m^2}\right)^2 - 2 \\ &= (27)^2 - 2 = 729 - 2 = 727 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad m^2 - \frac{1}{m^2} &= \left(m + \frac{1}{m}\right)\left(m - \frac{1}{m}\right) \\ &= 5\left(m + \frac{1}{m}\right) \end{aligned}$$

$$\begin{aligned} \text{Now } \left(m + \frac{1}{m}\right)^2 &= \left(m - \frac{1}{m}\right)^2 + 4 \\ &= (5)^2 + 4 = 25 + 4 = 29 \end{aligned}$$

$$\therefore m + \frac{1}{m} = \sqrt{29}$$

$$\therefore m^2 - \frac{1}{m^2} = (5)(\sqrt{29}) = 5\sqrt{29}$$

Question 6.

If $a^2 + b^2 = 41$ and $ab = 4$, find :

(i) $a - b$

(ii) $a + b$

Solution:

$$\begin{aligned} \text{(i)} \quad (a - b)^2 &= a^2 + b^2 - 2ab = 41 - 2(4) = 41 - 8 = 33 \end{aligned}$$

$$\therefore a - b = \sqrt{33}$$

$$\begin{aligned} \text{(ii)} \quad (a + b)^2 &= a^2 + b^2 + 2ab = 41 + 2(4) = 41 + 8 = 49 \Rightarrow (a + b)^2 = 49 \end{aligned}$$

$$\therefore a + b = 7$$

Question 7.

If $2a + \frac{1}{2a} = 8$, find :

(i) $4a^2 + \frac{1}{4a^2}$ (ii) $16a^4 + \frac{1}{16a^4}$

Solution:

$$(i) \ 4a^2 + \frac{1}{4a^2} = \left(2a + \frac{1}{2a}\right)^2 - 2 \cdot 2a \cdot \frac{1}{2a}$$

$$\left(2a + \frac{1}{2a}\right)^2 - 2 = (8)^2 - 2 = 64 - 2 = 62$$

$$(ii) \ 16a^4 + \frac{1}{16a^4} = \left(4a^2 + \frac{1}{4a^2}\right)^2 - 2 \cdot 4a^2 \cdot \frac{1}{4a^2}$$
$$= (62)^2 - 2 = 3844 - 2 = 3842$$

Question 8.

If $3x - \frac{1}{3x} = 5$, find :

(i) $9x^2 + \frac{1}{9x^2}$ (ii) $81x^4 + \frac{1}{81x^4}$

Solution:

$$(i) \ 9x^2 + \frac{1}{9x^2} = \left(3x - \frac{1}{3x}\right)^2 + 2$$

$$= (5)^2 + 2 = 25 + 2 = 27$$

$$(ii) \ 81x^4 + \frac{1}{81x^4} = \left(9x^2 + \frac{1}{9x^2}\right)^2 - 2$$

$$= (27)^2 - 2 = 729 - 2 = 727$$

Question 9.

Expand :

(i) $(3x - 4y + 5z)^2$

(ii) $(2a - 5b - 4c)^2$

(iii) $(5x + 3y)^3$

(iv) $(6a - 7b)^3$

Solution:

(i) $(3x - 4y + 5z)^2$

$$\begin{aligned} & (3x)^2 + (-4y)^2 + (5z)^2 + 2(3x)(-4y) + 2(-4y)(5z) + 2(5z)(3x) \\ &= 9x^2 + 16y^2 + 25z^2 - 24xy - 40yz + 30zx \end{aligned}$$

$$\begin{aligned} \text{(ii) } (2a - 5b - 4c)^2 &= (2a)^2 + (-5b)^2 + (-4c)^2 \\ &+ 2(2a)(-5b) + 2(-5b)(-4c) + 2(-4c)(2a) \\ &= 4a^2 + 25b^2 + 16c^2 - 20ab + 40bc - 16ca \end{aligned}$$

$$\text{(iii) } (5x + 3y)^3 = (5x)^3 + (3y)^3 + 3(5x)(3y)(5x + 3y)$$

$$= 125x^3 + 27y^3 + 45xy(5x + 3y)$$

$$= 125x^3 + 27y^3 + 225x^2y + 135xy^2$$

$$\text{(iv) } (6a - 7b)^3 = (6a)^3 - (7b)^3 - 3(6a)(7b)(6a - 7b)$$

$$= 216a^3 - 343b^3 - 126ab(6a - 7b)$$

$$= 216a^3 - 343b^3 - 756a^2b + 882ab^2$$

$$= 216a^3 - 756a^2b + 882ab^2 - 343b^3$$

Question 10.

If $a + b + c = 9$ and $ab + bc + ca = 15$, find: $a^2 + b^2 + c^2$.

Solution:

$$\text{Since } (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$\therefore (9)^2 = a^2 + b^2 + c^2 + 2(15)$$

$$81 = a^2 + b^2 + c^2 + 30$$

$$\therefore a^2 + b^2 + c^2 = 81 - 30 = 51$$

Question 11.

If $a + b + c = 11$ and $a^2 + b^2 + c^2 = 81$, find $ab + bc + ca$.

Solution:

Since $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$$\therefore (11)^2 = 81 + 2(ab + bc + ca)$$

$$\therefore 2(ab + bc + ca) = 121 - 81 = 40$$

$$ab + bc + ca = \frac{40}{2}$$

$$\Rightarrow ab + bc + ca = 20$$

Question 12.

If $3x - 4y = 5$ and $xy = 3$, find : $27x^3 - 64y^3$.

Solution:

$$27x^3 - 64y^3 = (3x)^3 - (4y)^3$$

$$= (3x - 4y)^3 + 3(3x)(4y)(3x - 4y)$$

$$[\because a^3 - b^3 = (a - b)^3 + 3ab(a - b)]$$

$$= (5)^3 + 36(xy)(3x - 4y) = 125 + 36(3)(5)$$

$$= 125 + 540 = 665$$

Question 13.

If $a + b = 8$ and $ab = 15$, find : $a^3 + b^3$.

Solution:

$$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$$

$$= (8)^3 - 3(15)(8) = 512 - 360 = 152$$

Question 14.

If $3x + 2y = 9$ and $xy = 3$, find : $27x^3 + 8y^3$

Solution:

$$27x^3 + 8y^3 = (3x)^3 + (2y)^3 = (3x + 2y)^3 - 3 \cdot 3x \cdot 2y(3x + 2y)$$

$$= (3x + 2y)^3 - 18xy(3x + 2y)$$

$$= (9)^3 - 18(3)(9) = 729 - 486 = 243$$

Question 15.

If $5x - 4y = 7$ and $xy = 8$, find : $125x^3 - 64y^3$

Solution:

$$\begin{aligned}125x^3 - 64y^3 &= (5x)^3 - (4y)^3 = (5x - 4y)^3 + 3(5x)(4y)(5x - 4y) \\&= (5x - 4y)^3 + 60xy(5x - 4y) \\&= (7)^3 + 60(8)(7) = 343 + 3360 = 3703\end{aligned}$$

Question 16.

The difference between two numbers is 5 and their products is 14. Find the difference between their cubes.

Solution:

Let x and y be two numbers, then $x - y = 5$ and $xy = 14$

$$\begin{aligned}\therefore x^3 - y^3 &= (x - y)^3 + 3xy(x - y) \\&= (5)^3 + 3 \times 14 \times 5 \\&= 125 + 210 = 335\end{aligned}$$