

13. Factorisation

EXERCISE 13(A)

Factorise :

Question 1.

$$15x + 5$$

Solution:

$$15x + 5 = 5(3x + 1)$$

Question 2.

$$a^3 - a^2 + a$$

Solution:

$$a^3 - a^2 + a = a(a^2 - a + 1)$$

Question 3.

$$3x^2 + 6x^3$$

Solution:

$$3x^2 + 6x^3 = 3x^2(1 + 2x)$$

Question 4.

$$4a^2 - 8ab$$

Solution:

$$4a^2 - 8ab = 4a(a - 2b)$$

Question 5.

$$2x^3b^2 - 4x^5b^4$$

Solution:

$$2x^3b^2 - 4x^5b^4 = 2x^3b^2(1 - 2x^2b^2)$$

Question 6.

$$15x^4y^3 - 20x^3y$$

Solution:

$$15x^4y^3 - 20x^3y = 5x^3y(3xy^2 - 4)$$

Question 7.

$$a^3b - a^2b^2 - b^3$$

Solution:

$$a^3b - a^2b^2 - b^3 = b(a^3 - a^2b - b^2)$$

Question 8.

$$6x^2y + 9xy^2 + 4y^3$$

Solution:

$$6x^2y + 9xy^2 + 4y^3 = y(6x^2 + 9xy + 4y^2).$$

Question 9.

$$17a^6b^8 - 34a^4b^6 + 51a^2b^4$$

Solution:

$$\begin{aligned} & 17a^6b^8 - 34a^4b^6 + 51a^2b^4 \\ &= 17a^2b^4 (a^4b^4 - 2a^2b^2 + 3) \end{aligned}$$

Question 10.

$$3x^5y - 27x^4y^2 + 12x^3y^3$$

Solution:

$$\begin{aligned} & 3x^5y - 27x^4y^2 + 12x^3y^3 \\ &= 3x^3y(x^2 - 9xy + 4y^2) \end{aligned}$$

Question 11.

$$x^2(a-b) - y^2(a-b) + z^2(a-b)$$

Solution:

$$\begin{aligned} & x^2(a-b) - y^2(a-b) + z^2(a-b) \\ &= (a-b)(x^2 - y^2 + z^2) \end{aligned}$$

Question 12.

$$(x+y)(a+b) + (x-y)(a+b)$$

Solution:

$$\begin{aligned} & (x+y)(a+b) + (x-y)(a+b) \\ &= (a+b)(x+y+x-y) \\ &= (a+b)(2x) \\ &= 2x(a+b) \end{aligned}$$

Question 13.

$$2b(2a+b) - 3c(2a+b)$$

Solution:

$$\begin{aligned} 2b(2a+b) - 3c(2a+b) \\ = (2a+b)(2b-3c) \end{aligned}$$

Question 14.

$$12abc - 6a^2b^2c^2 + 3a^3b^3c^3$$

Solution:

$$\begin{aligned} 12abc - 6a^2b^2c^2 + 3a^3b^3c^3 \\ = 3abc(4 - 2abc + a^2b^2c^2) \end{aligned}$$

Question 15.

$$4x(3x - 2y) - 2y(3x - 2y)$$

Solution:

$$\begin{aligned} 4x(3x - 2y) - 2y(3x - 2y) \\ = (3x - 2y)(4x - 2y) = (3x - 2y) \times 2(2x - y) \\ = 2(3x - 2y)(2x - y) \end{aligned}$$

Question 16.

$$(a + 2b)(3a + b) - (a + b)(a + 2b) + (a + 2b)^2$$

Solution:

$$\begin{aligned} (a + 2b)(3a + b) - (a + b)(a + 2b) + (a + 2b)^2 \\ = (a + 2b)(3a + b - a - b + a + 2b) \\ = (a + 2b)(3a + 2b) \end{aligned}$$

Question 17.

$$6xy(a^2 + b^2) + 8yz(a^2 + b^2) - 10xz(a^2 + b^2)$$

Solution:

$$\begin{aligned} 6xy(a^2 + b^2) + 8yz(a^2 + b^2) - 10xz(a^2 + b^2) \\ \text{H.C.F. of 6, 8, 10} = 2, \text{ then} \\ 2(a^2 + b^2)(3xy + 4yz - 5xz) \end{aligned}$$

EXERCISE 13(B)

Question 1.

Factorise : $a^2 + ax + ab + bx$

Solution:

$$\begin{aligned}a^2 + ax + ab + bx \\&= (a^2 + ax) + (ab + bx) \\&= a(a+x) + b(a+x) \\&= (a+x)(a+b)\end{aligned}$$

Question 2.

Factorise : $a^2 - ab - ca + bc$

Solution:

$$\begin{aligned}a^2 - ab - ca + bc \\&= a(a-b) - c(a-b) \\&= (a-b)(a-c)\end{aligned}$$

Question 3.

Factorise : $ab - 2b + a^2 - 2a$

Solution:

$$\begin{aligned}ab - 2b + a^2 - 2a \\&= b(a-2) + a(a-2) \\&= (a-2)(b+a)\end{aligned}$$

Question 4.

Factorise : $a^3 - a^2 + a - 1$

Solution:

$$\begin{aligned}a^3 - a^2 + a - 1 \\&= a^2(a-1) + 1(a-1) \\&= (a-1)(a^2+1)\end{aligned}$$

Question 5.

Factorise : $2a - 4b - xa + 2bx$

Solution:

$$\begin{aligned}2a - 4b - xa + 2bx \\&= 2(a-2b) - x(a-2b) \\&= (a-2b)(2-x)\end{aligned}$$

Question 6.

Factorise : $xy - ay - ax + a^2 + bx - ab$

Solution:

$$\begin{aligned} xy - ay - ax + a^2 + bx - ab \\ &= y(x-a) - a(x-a) + b(x-a) \\ &= (x-a)(y-a+b) \end{aligned}$$

Question 7.

Factorise : $3x^5 - 6x^4 - 2x^3 + 4x^2 + x - 2$

Solution:

$$\begin{aligned} 3x^5 - 6x^4 - 2x^3 + 4x^2 + x - 2 \\ &= 3x^4(x-2) - 2x^2(x-2) + 1(x-2) \\ &= (x-2)(3x^4 - 2x^2 + 1) \end{aligned}$$

Question 8.

Factorise : $-x^2y - x + 3xy + 3$

Solution:

$$\begin{aligned} -x^2y - x + 3xy + 3 \\ &= 3 - x + 3xy - x^2y \quad (\text{By grouping}) \\ &= 1(3 - x) + xy(3 - x) \\ &= (3 - x)(1 + xy) = (xy + 1)(3 - x) \end{aligned}$$

Question 9.

Factorise : $6a^2 - 3a^2b - bc^2 + 2c^2$

Solution:

$$\begin{aligned} 6a^2 - 3a^2b - bc^2 + 2c^2 \\ &= 6a^2 - 3a^2b + 2c^2 - bc^2 \quad (\text{By grouping}) \\ &= 3a^2(2 - b) + c^2(2 - b) \\ &= (2 - b)(3a^2 + c^2) \end{aligned}$$

Question 10.

Factorise : $3a^2b - 12a^2 - 9b + 36$

Solution:

$$\begin{aligned} & 3a^2b - 12a^2 - 9b + 36 \\ &= 3a^2(b - 4) - 9(b - 4) \quad (\text{By grouping}) \\ &= (b - 4)(3a^2 - 9) = (b - 4)3(a^2 - 3) \\ &= 3(b - 4)(a^2 - 3) \end{aligned}$$

Question 11.

Factorise : $x^2 - (a-3)x - 3a$

Solution:

$$\begin{aligned} & x^2 - (a-3)x - 3a \\ &= x^2 - ax + 3x - 3a \\ &= x(x-a) + 3(x-a) \\ &= (x-a)(x+3) \end{aligned}$$

Question 12.

Factorise : $x^2 - (b-2)x - 2b$

Solution:

$$\begin{aligned} & x^2 - (b-2)x - 2b \\ &= x^2 - bx + 2x - 2b \\ &= x(x-b) + 2(x-b) \\ &= (x-b)(x+2) \end{aligned}$$

Question 13.

Factorise : $a(b-c) - d(c-b)$

Solution:

$$\begin{aligned} & a(b-c) - d(c-b) \\ &= a(b-c) + d(b-c) \\ &= (b-c)(a+d) \end{aligned}$$

Question 14.

Factorise : $ab^2 - (a-c)b - c$

Solution:

$$\begin{aligned} & ab^2 - (a - c)b - c \\ &= ab^2 - ab + bc - c \\ &= ab(b - 1) + c(b - 1) \\ &= (b - 1)(ab + c) \end{aligned}$$

Question 15.

Factorise : $(a^2 - b^2)c + (b^2 - c^2)a$

Solution:

$$\begin{aligned} & (a^2 - b^2)c + (b^2 - c^2)a \\ &= a^2c - b^2c + ab^2 - ac^2 \\ &= a^2c - ac^2 + ab^2 - b^2c \\ &= ac(a - c) + b^2(a - c) \\ &= (a - c)(ac + b^2) \end{aligned}$$

Question 16.

Factorise : $a^3 - a^2 - ab + a + b - 1$

Solution:

$$\begin{aligned} & a^3 - a^2 - ab + a + b - 1 \\ &= a^3 - a^2 - ab + b + a - 1 \\ &= a^2(a - 1) - b(a - 1) + 1(a - 1) \\ &= (a - 1)(a^2 - b + 1) \end{aligned}$$

Question 17.

Factorise : $ab(c^2 + d^2) - a^2cd - b^2cd$

Solution:

$$\begin{aligned} & ab(c^2 + d^2) - a^2cd - b^2cd \\ &= abc^2 + abd^2 - a^2cd - b^2cd \\ &= abc^2 - a^2cd - b^2cd + abd^2 \\ &= ac(bc - ad) - bd(bc - ad) \\ &= (bc - ad)(ac - bd) \end{aligned}$$

Question 18.

Factorise : $2ab^2 - aby + 2cby - cy^2$

Solution:

$$\begin{aligned} &= 2ab^2 + 2cby - aby - cy^2 \\ &= 2b(ab + cy) - y(ab + cy) \\ &= (ab + cy)(2b - y) \end{aligned}$$

Question 19.

Factorise : $ax + 2bx + 3cx - 3a - 6b - 9c$

Solution:

$$\begin{aligned} &ax + 2bx + 3cx - 3a - 6b - 9c \\ &= x(a + 2b + 3c) - 3(a + 2b + 3c) \quad \text{(By grouping)} \\ &= (a + 2b + 3c)(x - 3) \end{aligned}$$

Question 20.

Factorise : $2ab^2c - 2a + 3b^3c - 3b - 4b^2c^2 + 4c$

Solution:

$$\begin{aligned} &2ab^2c - 2a + 3b^3c - 3b - 4b^2c^2 + 4c \\ &= 2a(b^2c - 1) + 3b(b^2c - 1) - 4c(b^2c - 1) \\ &= (b^2c - 1)(2a + 3b - 4c) \end{aligned}$$

EXERCISE 13(C)

Note : $a^2 - b^2 = (a + b)(a - b)$

Question 1.

Factorise : $16 - 9x^2$

Solution:

$$16 - 9x^2 = (4)^2 - (3x)^2 = (4 + 3x)(4 - 3x)$$

Question 2.

Factorise : $1 - 100a^2$

Solution:

$$1 - 100a^2 = (1)^2 - (10a)^2 = (1 + 10a)(1 - 10a)$$

Question 3.

Factorise : $4x^2 - 81y^2$

Solution:

$$4x^2 - 81y^2 = (2x)^2 - (9y)^2 = (2x + 9y)(2x - 9y)$$

Question 4.

Factorise : $\frac{4}{25} - 25b^2$

Solution:

$$\begin{aligned}\frac{4}{25} - 25b^2 &= \left(\frac{2}{5}\right)^2 - (5b)^2 \\ &= \left(\frac{2}{5} + 5b\right)\left(\frac{2}{5} - 5b\right)\end{aligned}$$

Question 5.

Factorise : $(a+2b)^2 - a^2$

Solution:

$$\begin{aligned}(a+2b)^2 - a^2 &= (a+2b)^2 - (a)^2 \\ &= (a+2b+a)(a+2b-a) \\ &= (2a+2b)(2b) \\ &= 2(a+b)(2b) \\ &= 2 \times 2b(a+b) \\ &= 4b(a+b)\end{aligned}$$

Question 6.

Factorise : $(5a-3b)^2 - 16b^2$

Solution:

$$\begin{aligned}(5a-3b)^2 - 16b^2 &= (5a-3b)^2 - (4b)^2 \\ &= (5a-3b+4b)(5a-3b-4b) \\ &= (5a+b)(5a-7b)\end{aligned}$$

Question 7.

Factorise : $a^4 - (a^2 - 3b^2)^2$

Solution:

$$\begin{aligned} a^4 - (a^2 - 3b^2)^2 &= (a^2)^2 - (a^2 - 3b^2)^2 \\ &= (a^2 + a^2 - 3b^2) (a^2 - a^2 + 3b^2) \\ &= (2a^2 - 3b^2) (3b^2) \\ &= 3b^2 (2a^2 - 3b^2) \end{aligned}$$

Question 8.

Factorise : $(5a-2b)^2 - (2a-b)^2$

Solution:

$$\begin{aligned} (5a-2b)^2 - (2a-b)^2 &= (5a-2b+2a-b) (5a-2b-2a+b) \\ &= (7a-3b) (3a-b) \end{aligned}$$

Question 9.

Factorise : $1-25(a+b)^2$

Solution:

$$\begin{aligned} 1-25(a+b)^2 &= (1)^2 - [5(a+b)]^2 \\ &= [1+5(a+b)] [1-5(a+b)] \\ &= (1+5a+5b) (1-5a-5b) \end{aligned}$$

Question 10.

Factorise : $4(2a+b)^2 - (a-b)^2$

Solution:

$$\begin{aligned} 4(2a+b)^2 - (a-b)^2 &= [2(2a+b)]^2 - (a-b)^2 \\ &= [2(2a+b)+a-b][2(2a+b)-a+b] \\ &= (4a+2b+a-b) (4a+2b-a+b) \\ &= (5a+b) (3a+3b) \\ &= (5a+b) 3(a+b) \\ &= 3(5a+b) (a+b) \end{aligned}$$

Question 11.

Factorise : $25(2x+y)^2 - 16(x-y)^2$

Solution:

$$\begin{aligned}25(2x+y)^2 - 16(x-y)^2 \\&= [5(2x+y)]^2 - [4(x-y)]^2 \\&= (10x+5y)^2 - (4x-4y)^2 \\&= (10x+5y+4x-4y)(10x+5y \\&\quad - 4x+4y) \\&= (14x+y)(6x+9y) \\&= (14x+y) 3(2x+3y) \\&= 3(14x+y)(2x+3y)\end{aligned}$$

Question 12.

Factorise : $49(x-y)^2 - 9(2x+y)^2$

Solution:

$$\begin{aligned}&= [7(x-y)]^2 - [3(2x+y)]^2 \\&= (7x-7y)^2 - (6x+3y)^2 \\&= (7x-7y+6x+3y)(7x-7y-6x-3y) \\&= (13x-4y)(x-10y)\end{aligned}$$

Question 13.

Evaluate : $\left(6\frac{2}{3}\right)^2 - \left(2\frac{1}{3}\right)^2$

Solution:

$$\begin{aligned}\left(6\frac{2}{3}\right)^2 - \left(2\frac{1}{3}\right)^2 &= \left(\frac{20}{3}\right)^2 - \left(\frac{7}{3}\right)^2 \\&= \left(\frac{20}{3} + \frac{7}{3}\right)\left(\frac{20}{3} - \frac{7}{3}\right) \\&= \left(\frac{27}{3}\right)\left(\frac{13}{3}\right) = 9 \times \frac{13}{3} = 39\end{aligned}$$

Question 14.

Evaluate :

$$\left(7\frac{3}{10}\right)^2 - \left(2\frac{1}{10}\right)^2$$

Solution:

$$\begin{aligned} & \left(7\frac{3}{10}\right)^2 - \left(2\frac{1}{10}\right)^2 \\ &= \left(\frac{73}{10}\right)^2 - \left(\frac{21}{10}\right)^2 \\ &= \left(\frac{73}{10} + \frac{21}{10}\right)\left(\frac{73}{10} - \frac{21}{10}\right) \\ &= \left(\frac{94}{10}\right)\left(\frac{52}{10}\right) \\ &= \left(\frac{47}{5}\right)\left(\frac{26}{5}\right) = \frac{1222}{25} = 48\frac{22}{25} \end{aligned}$$

Question 15.

Evaluate :

$$(0.7)^2 - (0.3)^2$$

Solution:

$$\begin{aligned} & (0.7)^2 - (0.3)^2 \\ &= (0.7 + 0.3)(0.7 - 0.3) \\ &= 1 \times 0.4 = 0.4 \end{aligned}$$

Question 16.

Evaluate :

$$(4.5)^2 - (1.5)^2$$

Solution:

$$\begin{aligned} & (4.5)^2 - (1.5)^2 \\ &= (4.5 + 1.5)(4.5 - 1.5) \\ &= 6 \times 3 = 18 \end{aligned}$$

Question 17.

Factorise : $75(x + y)^2 - 48(x - y)^2$

Solution:

$$\begin{aligned} & 75(x+y)^2 - 48(x-y)^2 \\ &= 3[25(x+y)^2 - 16(x-y)^2] \\ &= 3[\{5(x+y)\}^2 - \{4(x-y)\}^2] \\ &\text{Using } a^2 - b^2 = (a+b)(a-b) \\ &= 3[5(x+y) + 4(x-y)][5(x+y) - 4(x-y)] \\ &= 3[5x + 5y + 4x - 4y][5x + 5y - 4x + 4y] \\ &= 3(9x + y)(x + 9y) \end{aligned}$$

Question 18.

Factorise : $a^2 + 4a + 4 - b^2$

Solution:

$$\begin{aligned} & a^2 + 4a + 4 - b^2 \\ & \left\{ \begin{array}{l} \because (a+b)^2 = a^2 + 2ab + b^2 \\ a^2 - b^2 = (a+b)(a-b) \end{array} \right\} \\ &= (a)^2 + 2 \times a \times 2 + (2)^2 - (b)^2 \\ &= (a+2)^2 - (b)^2 \\ &= (a+2+b)(a+2-b) \\ &= (a+b+2)(a-b+2) \end{aligned}$$

Question 19.

Factorise : $a^2 - b^2 - 2b - 1$

Solution:

$$\begin{aligned} & a^2 - b^2 - 2b - 1 \\ &= a^2 - (b^2 + 2b + 1) \\ & \left\{ \begin{array}{l} \because (a+b)^2 = a^2 + b^2 + 2ab \\ a^2 - b^2 = (a+b)(a-b) \end{array} \right\} \\ &= (a)^2 - (b+1)^2 \\ &= (a+b+1)(a-b-1) \end{aligned}$$

Question 20.

Factorise : $x^2 + 6x + 9 - 4y^2$

Solution:

$$x^2 + 6x + 9 - 4y^2$$

$$= (x)^2 + 2 \times x \times 3 + (3)^2 - (2y)^2$$

$$\left\{ \begin{array}{l} \because (a+b)^2 = a^2 + 2ab + b^2 \\ a^2 - b^2 = (a+b)(a-b) \end{array} \right\}$$

$$= (x+3)^2 - (2y)^2$$

$$= (x+3+2y)(x+3-2y)$$

$$= (x+2y+3)(x-2y+3)$$

EXERCISE 13(D)

Question 1.

Factorise : $x^2 + 6x + 8$

Solution:

$$x^2 + 6x + 8 = x^2 + 4x + 2x + 8$$

$$= x(x+4) + 2(x+4)$$

$$= (x+4)(x+2)$$

Question 2.

Factorise : $x^2 + 4x + 3$

Solution:

$$x^2 + 4x + 3 = x^2 + 3x + x + 3$$

$$= x(x+3) + 1(x+3)$$

$$= (x+3)(x+1)$$

Question 3.

Factorise : $a^2 + 5a + 6$

Solution:

$$a^2 + 5a + 6 = a^2 + 3a + 2a + 6$$

$$= a(a+3) + 2(a+3)$$

$$= (a+3)(a+2)$$

Question 4.

Factorise : $a^2 - 5a + 6$

Solution:

$$\begin{aligned}a^2-5a+6 &= a^2-3a-2a+6 \\&= a(a-3)-2(a-3) \\&= (a-3)(a-2)\end{aligned}$$

Question 5.

Factorise : a^2+5a-6

Solution:

$$\begin{aligned}a^2+5a-6 &= a^2+6a-a-6 \\&= a(a+6)-1(a+6) \\&= (a+6)(a-1)\end{aligned}$$

Question 6.

Factorise : $x^2+5xy+4y^2$

Solution:

$$\begin{aligned}x^2+5xy+4y^2 &= x^2+4xy+xy+4y^2 \\&= x(x+4y)+y(x+4y) \\&= (x+4y)(x+y)\end{aligned}$$

Question 7.

Factorise : $a^2-3a-40$

Solution:

$$\begin{aligned}a^2-3a-40 &= a^2-8a+5a-40 \\&= a(a-8)+5(a-8) \\&= (a-8)(a+5)\end{aligned}$$

Question 8.

Factorise : x^2-x-72

Solution:

$$\begin{aligned}x^2-x-72 &= x^2-9x+8x-72 \\&= x(x-9)+8(x-9) \\&= (x-9)(x+8)\end{aligned}$$

Question 9.

Factorise : $x^2-10xy+24y^2$

Solution:

$$\begin{aligned}x^2 - 10xy + 24y^2 &= x^2 - 6xy - 4xy + 24y^2 \\&= x(x - 6y) - 4y(x - 6y) \\&= (x - 6y)(x - 4y)\end{aligned}$$

Question 10.

Factorise : $2a^2 + 7a + 6$

Solution:

$$\begin{aligned}2a^2 + 7a + 6 &= 2a^2 + 4a + 3a + 6 \\&= 2a(a + 2) + 3(a + 2) \\&= (a + 2)(2a + 3)\end{aligned}$$

Question 11.

Factorise : $3a^2 - 5a + 2$

Solution:

$$\begin{aligned}3a^2 - 5a + 2 &= 3a^2 - 3a - 2a + 2 \\&= 3a(a - 1) - 2(a - 1) \\&= (a - 1)(3a - 2)\end{aligned}$$

Question 12.

Factorise : $7b^2 - 8b + 1$

Solution:

$$\begin{aligned}7b^2 - 8b + 1 &= 7b^2 - 7b - b + 1 \\&= 7b(b - 1) - 1(b - 1) \\&= (b - 1)(7b - 1)\end{aligned}$$

Question 13.

Factorise : $2a^2 - 17ab + 26b^2$

Solution:

$$\begin{aligned}2a^2 - 17ab + 26b^2 &= 2a^2 - 13ab - 4ab + 26b^2 \\&= a(2a - 13b) - 2b(2a - 13b) \\&= (2a - 13b)(a - 2b)\end{aligned}$$

Question 14.

Factorise : $2x^2 + xy - 6y^2$

Solution:

$$\begin{aligned}2x^2 + xy - 6y^2 &= 2x^2 + 4xy - 3xy - 6y^2 \\&= 2x(x + 2y) - 3y(x + 2y) \\&= (x + 2y)(2x - 3y)\end{aligned}$$

Question 15.

Factorise : $4c^2 + 3c - 10$

Solution:

$$\begin{aligned}4c^2 + 3c - 10 &= 4c^2 + 8c - 5c - 10 \\&= 4c(c + 2) - 5(c + 2) \\&= (c + 2)(4c - 5)\end{aligned}$$

Question 16.

Factorise : $14x^2 + x - 3$

Solution:

$$\begin{aligned}14x^2 + x - 3 &= 14x^2 + 7x - 6x - 3 \\&= 7x(2x + 1) - 3(2x + 1) \\&= (2x + 1)(7x - 3)\end{aligned}$$

Question 17.

Factorise : $6 + 7b - 3b^2$

Solution:

$$\begin{aligned}6 + 7b - 3b^2 &= 6 + 9b - 2b - 3b^2 \\&= 3(2 + 3b) - b(2 + 3b) \\&= (2 + 3b)(3 - b)\end{aligned}$$

Question 18.

Factorise : $5 + 7x - 6x^2$

Solution:

$$\begin{aligned}5 + 7x - 6x^2 &= 5 + 10x - 3x - 6x^2 \\&= 5(1 + 2x) - 3x(1 + 2x) \\&= (1 + 2x)(5 - 3x)\end{aligned}$$

Question 19.

Factorise : $4 + y - 14y^2$

Solution:

$$\begin{aligned}4+y-14y^2 &= 4+8y-7y-14y^2 \\&= 4(1+2y)-7y(1+2y) \\&= (1+2y)(4-7y)\end{aligned}$$

Question 20.

Factorise : $5+3a-14a^2$

Solution:

$$\begin{aligned}5+3a-14a^2 &= 5+10a-7a-14a^2 \\&= 5(1+2a)-7a(1+2a) \\&= (1+2a)(5-7a)\end{aligned}$$

Question 21.

Factorise : $(2a+b)^2+5(2a+b)+6$

Solution:

$$\begin{aligned}\text{Let } (2a+b) &= x \\(2a+b)^2 &= x^2 \\(2a+b)^2+5(2a+b)+6 &= x^2+5x+6 \\&= x^2+3x+2x+6 \\&= x(x+3)+2(x+3) \\&= (x+3)(x+2) \\&= (2a+b+3)(2a+b+2) \\&\quad (\text{Substituting the value of } x)\end{aligned}$$

Question 22.

Factorise : $1-(2x+3y)-6(2x+3y)^2$

Solution:

$$\begin{aligned}\text{Let } (2x+3y) &= a \\ \therefore (2x+3y)^2 &= a^2 \\ \therefore 1-(2x+3y)-6(2x+3y)^2 \\ &= 1-a-6a^2 \\ &= 1-3a+2a-6a^2 \\ &= 1(1-3a)+2a(1-3a) \\ &= (1-3a)(1+2a) \\ &= [1-3(2x+3y)][1+2(2x+3y)] \\ &\quad \text{(Substituting the value of } a) \\ &= (1-6x-9y)(1+4x+6y)\end{aligned}$$

Question 23.

Factorise : $(x-2y)^2-12(x-2y)+32$

Solution:

$$\begin{aligned}\text{Let } (x-2y) &= a \\ \therefore (x-2y)^2 &= a^2 \\ \therefore (x-2y)^2-12(x-2y)+32 \\ &= a^2-12a+32 \\ &= a^2-8a-4a+32 \\ &= a(a-8)-4(a-8) \\ &= (a-8)(a-4) \\ &= (x-2y-8)(x-2y-4) \\ &\quad \text{(Substituting the value of } a)\end{aligned}$$

Question 24.

Factorise : $8 + 6(a+b) - 5(a+b)^2$

Solution:

Let $a+b = x$

$$(a+b)^2 = x^2$$

$$\begin{aligned} 8+6(a+b)-5(a+b)^2 &= 8+6x-5x^2 \\ &= 8+10x-4x-5x^2 \\ &= 2(4+5x)-x(4+5x) \\ &= (4+5x)(2-x) \\ &= [4+5(a+b)][2-(a+b)] \end{aligned}$$

(Substituting the value of x)

$$= [4+5a+5b][2-a-b]$$

Question 25.

Factorise : $2(x+2y)^2 - 5(x+2y) + 2$

Solution:

$$2(x+2y)^2 - 5(x+2y) + 2$$

Let $x+2y = a$, then

$$2a^2 - 5a + 2$$

$$\Rightarrow 2a^2 - a - 4a + 2$$

$$= a(2a-1) - 2(2a-1)$$

$$= (2a-1)(a-2)$$

$$= \{2(x+2y-1)\} \{(x+2y)-2\}$$

$$= (2x+4y-2)(x+2y-2)$$

$$\left\{ \begin{array}{l} \because 2 \times 2 = 4 \\ -5 = -1 - 4 \\ 4 = (-1) \times (-4) \end{array} \right\}$$

EXERCISE 13(E)

Question 1.

In each case find whether the trinomial is a perfect square or not:

(i) $x^2+14x+49$

(ii) $a^2-10a+25$

(iii) $4x^2+4x+1$

(iv) $9b^2+12b+16$

(v) $16x^2-16xy+y^2$

(vi) $x^2-4x+16$

Solution:

$$\begin{aligned}(i) \quad x^2 + 14x + 49 &= (x)^2 + 2 \times x \times 7 + (7)^2 \\ &= (x+7)^2 \\ &[\because a^2 + 2ab + b^2 = (a+b)^2]\end{aligned}$$

$\therefore$ The given trinomial $x^2 + 14x + 49$ is a perfect square.

$$\begin{aligned}(ii) \quad a^2 - 10a + 25 &= (a)^2 - 2 \times a \times 5 + (5)^2 \\ &= (a-5)^2 \\ &[\because a^2 - 2ab + b^2 = (a-b)^2]\end{aligned}$$

$\therefore$ The given trinomial $a^2 - 10a + 25$ is a perfect square.

$$\begin{aligned}(iii) \quad 4x^2 + 4x + 1 &= (2x)^2 + 2 \times 2x \times 1 + (1)^2 \\ &= (2x+1)^2 \\ &[\because a^2 + 2ab + b^2 = (a+b)^2]\end{aligned}$$

$\therefore$ The given trinomial $4x^2 + 4x + 1$ is a perfect square.

$$\begin{aligned}(iv) \quad 9b^2 + 12b + 16 &= (3b)^2 + 3b \times 4 + (4)^2 \\ &= x^2 + xy + y^2 \\ &[\text{Taking } 3b=x, \text{ and } 4=y]\end{aligned}$$

$\therefore$ The given trinomial cannot be expressed as $x^2 + 2xy + y^2$. Hence, it is not a perfect square.

$$\begin{aligned}(v) \quad 16x^2 - 16xy + y^2 &= (4x)^2 - 4 \times 4x \times y + (y)^2 \\ &= a^2 - 4ab + b^2 \\ &[\text{Taking } 4x=a, \text{ and } y=b]\end{aligned}$$

$\therefore$ The given trinomial cannot be expressed as $a^2 - 2ab + b^2$.

$\therefore$ It is not a perfect square.

$$\begin{aligned}(vi) \quad x^2 - 4x + 16 &= (x)^2 - x \times 4 + (4)^2 \\ &= a^2 - ab + b^2 \\ &[\text{Taking } x=a, \text{ and } 4=b]\end{aligned}$$

$\therefore$ The given trinomial cannot be expressed as $a^2 - 2ab + b^2$.

Hence, it is not a perfect square.

Question 2.

Factorise completely $2 - 8x^2$.

Solution:

$$\begin{aligned}2 - 8x^2 &= 2(1 - 4x^2) \\&= 2[(1)^2 - (2x)^2] \\&= 2(1 + 2x)(1 - 2x) \\&\quad \text{Note : } a^2 - b^2 = (a + b)(a - b)\end{aligned}$$

Question 3.

Factorise completely : $8x^2y - 18y^3$

Solution:

$$\begin{aligned}8x^2y - 18y^3 &= 2y(4x^2 - 9y^2) \\&= 2y[(2x)^2 - (3y)^2] \\&= 2y(2x + 3y)(2x - 3y)\end{aligned}$$

Question 4.

Factorise completely : $ax^2 - ay^2$

Solution:

$$\begin{aligned}ax^2 - ay^2 &= a(x^2 - y^2) \\&= a(x + y)(x - y)\end{aligned}$$

Question 5.

Factorise completely : $25x^3 - x$

Solution:

$$\begin{aligned}25x^3 - x &= x(25x^2 - 1) \\&= x[(5x)^2 - (1)^2] \\&= x(5x + 1)(5x - 1)\end{aligned}$$

Question 6.

Factorise completely : $a^4 - b^4$

Solution:

$$\begin{aligned}a^4 - b^4 &= (a^2)^2 - (b^2)^2 \\&= (a^2 + b^2)(a^2 - b^2) \\&= (a^2 + b^2)(a + b)(a - b)\end{aligned}$$

Question 7.

Factorise completely : $16x^4 - 81y^4$

Solution:

$$\begin{aligned}16x^4 - 81y^4 &= (4x^2)^2 - (9y^2)^2 \\&= (4x^2 + 9y^2)(4x^2 - 9y^2) \\&= (4x^2 + 9y^2)[(2x)^2 - (3y)^2] \\&= (4x^2 + 9y^2)(2x + 3y)(2x - 3y)\end{aligned}$$

Question 8.

Factorise completely : $625 - x^4$

Solution:

$$\begin{aligned}625 - x^4 &= (25)^2 - (x^2)^2 \\&= (25 + x^2)(25 - x^2) \\&= (25 + x^2)[(5)^2 - (x)^2] \\&= (25 + x^2)(5 + x)(5 - x)\end{aligned}$$

Question 9.

Factorise completely : $x^2 - y^2 - 3x - 3y$

Solution:

$$\begin{aligned}x^2 - y^2 - 3x - 3y &= (x^2 - y^2) - 3(x + y) \\&= (x + y)(x - y) - 3(x + y) \\&= (x + y)(x - y - 3)\end{aligned}$$

Question 10.

Factorise completely : $x^2 - y^2 - 2x + 2y$

Solution:

$$\begin{aligned}x^2 - y^2 - 2x + 2y &= (x^2 - y^2) - 2(x - y) \\&= (x + y)(x - y) - 2(x - y) \\&= (x - y)(x + y - 2)\end{aligned}$$

Question 11.

Factorise completely : $3x^2 + 15x - 72$

Solution:

$$\begin{aligned}3x^2 + 15x - 72 &= 3(x^2 + 5x - 24) \\&= 3[x^2 + 8x - 3x - 24] \\&= 3[x(x + 8) - 3(x + 8)] \\&= 3[(x + 8)(x - 3)] \\&= 3(x + 8)(x - 3)\end{aligned}$$

Question 12.

Factorise completely : $2a^2 - 8a - 64$

Solution:

$$\begin{aligned}2a^2 - 8a - 64 &= 2[a^2 - 4a - 32] \\&= 2[a^2 - 8a + 4a - 32] \\&= 2[a(a-8) + 4(a-8)] \\&= 2[(a-8)(a+4)] \\&= 2(a-8)(a+4)\end{aligned}$$

Question 13.

Factorise completely : $5b^2 + 45b + 90$

Solution:

$$\begin{aligned}5b^2 + 45b + 90 &= 5[b^2 + 9b + 18] \\&= 5[b^2 + 6b + 3b + 18] \\&= 5[b(b+6) + 3(b+6)] \\&= 5[(b+6)(b+3)] \\&= 5(b+6)(b+3)\end{aligned}$$

Question 14.

Factorise completely : $3x^2y + 11xy + 6y$

Solution:

$$\begin{aligned}3x^2y + 11xy + 6y &= y(3x^2 + 11x + 6) \\&= y[(3x^2 + 9x + 2x + 6)] \\&= y[3x(x+3) + 2(x+3)] \\&= y[(x+3)(3x+2)] \\&= y(x+3)(3x+2)\end{aligned}$$

Question 15.

Factorise completely : $5ap^2 + 11ap + 2a$

Solution:

$$\begin{aligned}5ap^2 + 11ap + 2a &= a[5p^2 + 11p + 2] \\&= a[5p^2 + 10p + p + 2] \\&= a[5p(p+2) + 1(p+2)] \\&= a[(p+2)(5p+1)] \\&= a(p+2)(5p+1)\end{aligned}$$

Question 16.

Factorise completely : $a^2 + 2ab + b^2 - c^2$

Solution:

$$\begin{aligned} a^2 + 2ab + b^2 - c^2 &= (a^2 + 2ab + b^2) - c^2 \\ &= (a+b)^2 - (c)^2 \\ &= (a+b+c)(a+b-c) \end{aligned}$$

Question 17.

Factorise completely : $x^2 + 6xy + 9y^2 + x + 3y$

Solution:

$$\begin{aligned} x^2 + 6xy + 9y^2 + x + 3y &= [(x)^2 + 2 \times x \times 3y + (3y)^2] + (x+3y) \\ &= [x+3y]^2 + (x+3y) \\ &= (x+3y)(x+3y) + (x+3y) \\ &= (x+3y)(x+3y+1) \end{aligned}$$

Question 18.

Factorise completely : $4a^2 - 12ab + 9b^2 + 4a - 6b$

Solution:

$$\begin{aligned} [4a^2 - 12ab + 9b^2] + (4a - 6b) &= [(2a)^2 - 2 \times 2a \times 3b + (3b)^2] + 2(2a - 3b) \\ &= (2a - 3b)^2 + 2(2a - 3b) \\ &= (2a - 3b)(2a - 3b + 2) \end{aligned}$$

Question 19.

Factorise completely : $2a^2b^2 - 98b^4$

Solution:

$$\begin{aligned} 2a^2b^2 - 98b^4 &= 2b^2(a^2 - 49b^2) \\ &= 2b^2[(a)^2 - (7b)^2] \\ &= 2b^2(a+7b)(a-7b) \end{aligned}$$

Question 20.

Factorise completely : $a^2 - 16b^2 - 2a - 8b$

Solution:

$$\begin{aligned} (a^2 - 16b^2) - 2a - 8b &= [(a)^2 - (4b)^2] - 2(a+4b) \\ &= (a+4b)(a-4b) - 2(a+4b) \\ &= (a+4b)(a-4b-2) \end{aligned}$$

EXERCISE 13(F)

Question 1.

Factorise :

(i) $6x^3 - 8x^2$ (ii) $35a^3b^2c + 42ab^2c^2$

(iii) $36x^2y^2 - 30x^3y^3 + 48x^3y^2$

(iv) $8(2a + 3b)^3 - 12(2a + 3b)^2$

(v) $9a(x - 2y)^4 - 12a(x - 2y)^3$

Solution:

(i) $6x^3 - 8x^2 = 2x^2(3x - 4)$

(ii) $35a^3b^2c + 42ab^2c^2 = 7ab^2c(5a^2 + 6c)$

(iii) $36x^2y^2 - 30x^3y^3 + 48x^3y^2 = 6x^2y^2(6 - 5xy + 8xy)$

(iv) $8(2a + 3b)^3 - 12(2a + 3b)^2$

$= 4(2a + 3b)^2 [2(2a + 3b) - 3]$

$= 4(2a + 3b)^2 [4a + 6b - 3]$

(v) $9a(x - 2y)^4 - 12a(x - 2y)^3$

$= 3a(x - 2y)^3 (3(x - 2y) - 4)$

$= 3a(x - 2y)^3 (3x - 6y - 4)$

Question 2.

Factorise :

(i) $a^2 - ab - 3a + 3b$ (ii) $x^2y - xy^2 + 5x - 5y$

(iii) $a^2 - ab(1 - b) - b^3$

(iv) $xy^2 + (x - 1)y - 1$

(v) $(ax + by)^2 + (bx - ay)^2$

(vi) $ab(x^2 + y^2) - xy(a^2 + b^2)$

(vii) $m - 1 - (m - 1)^2 + am - a$

Solution:

$$\begin{aligned}
\text{(i)} \quad a^2 - ab - 3a + 3b &= a(a - b) - 3(a - b) \\
&= (a - b)(a - 3) \\
\text{(ii)} \quad x^2y - xy^2 + 5x - 5y &= xy(x - y) + 5(x - y) \\
&= (x - y)(xy + 5) \\
\text{(iii)} \quad a^2 - ab(1 - b) - b^3 &= a^2 - ab + ab^2 - b^3 \\
&= a(a - b) + b^2(a - b) = (a - b)(a + b^2) \\
\text{(iv)} \quad xy^2 + (x - 1)y - 1 &= xy^2 + xy - y - 1 \\
&= xy(y + 1) - 1(y + 1) = (xy - 1)(y + 1) \\
\text{(v)} \quad (ax + by)^2 + (bx - ay)^2 &= a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 - 2abxy \\
&= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2 = a^2x^2 + a^2y^2 + b^2x^2 + b^2y^2 \\
&= a^2(x^2 + y^2) + b^2(x^2 + y^2) = (x^2 + y^2)(a^2 + b^2) \\
\text{(vi)} \quad ab(x^2 + y^2) - xy(a^2 + b^2) &= abx^2 + aby^2 - a^2xy - b^2xy = abx^2 - a^2xy + aby^2 - b^2xy \\
&= abx^2 - a^2xy - b^2xy + aby^2 \\
&= ax(bx - ay) - by(bx - ay) \\
&= (bx - ay)(ax - by) \\
\text{(vii)} \quad m - 1 - (m - 1)^2 + am - a &= (m - 1) - (m - 1)^2 + a(m - 1) \\
&= (m - 1)(1 - (m - 1) + a) \\
&= (m - 1)(1 - m + 1 + a) \\
&= (m - 1)(2 - m + a)
\end{aligned}$$

Question 3.

Factorise :

$$\text{(i)} \quad a^2 - (b - c)^2 \quad \text{(ii)} \quad 25(2x - y)^2 - 16(x - 2y)^2$$

$$\text{(iii)} \quad 16(5x + 4)^2 - 9(3x - 2)^2 \quad \text{(iv)} \quad 9x^2 - \frac{1}{16}$$

$$\text{(v)} \quad 25(x - 2y)^2 - 4$$

Solution:

$$\begin{aligned}
& \text{(i) } a^2 - (b - c)^2 \\
&= (a - (b - c))(a + b - c) \\
&\quad [a^2 - b^2 = (a - b)(a + b)] \\
&= (a - b + c)(a + b - c) \\
& \text{(ii) } 25(2x - y)^2 - 16(x - 2y)^2 \\
&= (5(2x - y))^2 - (4(x - 2y))^2 \\
&= [5(2x - y) - 4(x - 2y)][5(2x - y) + 4(x - 2y)] \\
&= [10x - 5y - 4x + 8y][10x - 5y + 4x - 8y] \\
&\quad [a^2 - b^2 = (a - b)(a + b)] \\
&= (6x + 3y)(14x - 13y) = 3(2x + y)(14x - 13y) \\
& \text{(iii) } 16(5x + 4)^2 - 9(3x - 2)^2 \\
&= (4(5x + 4))^2 - (3(3x - 2))^2 \\
&= [4(5x + 4) - 3(3x - 2)][4(5x + 4) + 3(3x - 2)] \\
&\quad [a^2 - b^2 = (a - b)(a + b)] \\
&= (20x + 16 - 9x + 6)[20x + 16 + 9x - 6] \\
&= [11x + 22](29x + 10) = 11(x + 2)(29x + 10) \\
& \text{(iv) } 9x^2 - \frac{1}{16} = (3x)^2 - \left(\frac{1}{4}\right)^2 \\
&= \left(3x - \frac{1}{4}\right)\left(3x + \frac{1}{4}\right) \quad [(a^2 - b^2) = (a - b)(a + b)] \\
& \text{(v) } 25(x - 2y)^2 - 4 = (5(x - 2y))^2 - 2^2 \\
&= [5(x - 2y) - 2][5(x - 2y) + 2] \\
&\quad [a^2 - b^2 = (a - b)(a + b)] \\
&= (5x - 10y - 2)(5x - 10y + 2)
\end{aligned}$$

Question 4.

Factorise :

- | | |
|--|---------------------------|
| (i) $a^2 - 23a + 42$ | (ii) $a^2 - 23a - 108$ |
| (iii) $1 - 18x - 63x^2$ | (iv) $5x^2 - 4xy - 12y^2$ |
| (v) $x(3x + 14) + 8$ | (vi) $5 - 4x(1 + 3x)$ |
| (vii) $x^2y^2 - 3xy - 40$ | |
| (viii) $(3x - 2y)^2 - 5(3x - 2y) - 24$ | |
| (ix) $12(a + b)^2 - (a + b) - 35$ | |

Solution:

(i) $a^2 - 23a + 42$

$[42 = 21 \times 2 \text{ and } 21 + 2 = 23]$

$$= a^2 - 21a - 2a + 42 = a(a - 21) - 2(a - 21)$$

$$= (a - 21)(a - 2)$$

(ii) $a^2 - 23a - 108$

$$= a^2 - 27a + 4a - 108$$

$[27 \times 4 = 108 \text{ and } 27 - 4 = 23]$

$$= a(a - 27) + 4(a - 27)$$

$$= (a - 27)(a + 4)$$

(iii) $1 - 18x - 63x^2 = 1 - 21x + 3x - 63x^2$

$$= 1(1 - 21x) + 3x(1 - 21x)$$

$$= (1 - 21x)(1 + 3x)$$

(iv) $5x^2 - 4xy - 12y^2 = 5x^2 - 10xy + 6xy - 12y^2$

$$= 5x(x - 2y) + 6y(x - 2y) = (x - 2y)(5x + 6y)$$

(v) $x(3x + 14) + 8 = 3x^2 + 14x + 8$

$$= 3x^2 + 12x + 2x + 8 = 3x(x + 4) + 2(x + 4)$$

$$= (x + 4)(3x + 2)$$

(vi) $5 - 4x(1 + 3x) = 5 - 4x - 12x^2$

$$= 5 - 10x + 6x - 12x^2 = 5(1 - 2x) + 6x(1 - 2x)$$

$$= (1 - 2x)(5 + 6x)$$

(vii) $x^2 y^2 - 3xy - 40 = x^2 y^2 - 8xy + 5xy - 40$

$$= xy(xy - 8) + 5(xy - 8) = (xy - 8)(xy + 5)$$

(viii) $(3x - 2y)^2 - 5(3x - 2y) - 24$

$$= (3x - 2y)^2 - 8(3x - 2y) + 3(3x - 2y) - 24$$

$$= (3x - 2y)(3x - 2y - 8) + 3(3x - 2y - 8)$$

$$= (3x - 2y - 8)(3x - 2y + 3)$$

(ix) $12(a + b)^2 - (a + b) - 35$

$$= 12(a + b)^2 - 21(a + b) + 20(a + b) - 35$$

$$= 3(a + b)[4(a + b) - 7] + 5[4(a + b) - 7]$$

$$= (4a + 4b - 7)(3a + 3b + 5)$$

Question 5.

Factorise :

(i) $15(5x - 4)^3 - 10(5x - 4)$

(ii) $3a^2x - bx + 3a^2 - b$

(iii) $b(c - d)^2 + a(d - c) + 3(c - d)$

(iv) $ax^2 + b^2y - ab^2 - x^2y$

(v) $1 - 3x - 3y - 4(x + y)^2$

Solution:

(i) $15(5x - 4)^3 - 10(5x - 4) = 5(5x - 4)[3(5x - 4)^2 - 2]$

$= 5(5x - 4)[3(25x^2 - 40x + 16) - 2]$

$= 5(5x - 4)(75x^2 - 120x + 46)$

(ii) $3a^2x - bx + 3a^2 - b$

$= x(3a^2 - b) + 1(3a^2 - b) = (x + 1)(3a^2 - b)$

(iii) $b(c - d)^2 + a(d - c) + 3(c - d)$

$= b(c - d)^2 - a(c - d) + 3(c - d)$

$= (c - d)[b(c - d) - a + 3]$

$= (c - d)(bc - bd - a + 3)$

(iv) $ax^2 + b^2y - ab^2 - x^2y$

$= ax^2 - ab^2 + b^2y - x^2y = a(x^2 - b^2) + y(b^2 - x^2)$

$= a(x^2 - b^2) - y(x^2 - b^2) = (x^2 - b^2)(a - y)$

$= (x - b)(x + b)(a - y)$

(v) $1 - 3x - 3y - 4(x + y)^2$

$= 1 - 3(x + y) - 4(x + y)^2$

$= 1 - 4(x + y) + (x + y) - 4(x + y)^2$

$= 1[1 - 4(x + y)] + (x + y)[1 - 4(x + y)]$

$= [1 - 4x - 4y](1 + x + y)$

Question 6.

Factorise :

(i) $2a^3 - 50a$

(ii) $54a^2b^2 - 6$

(iii) $64a^2b - 144b^3$

(iv) $(2x - y)^3 - (2x - y)$

(v) $x^2 - 2xy + y^2 - z^2$

(vi) $x^2 - y^2 - 2yz - z^2$

$$(vii) 7a^5 - 567a \quad (viii) 5x^2 - \frac{20x^4}{9}$$

Solution:

$$(i) 2a^3 - 50a = 2a(a^2 - 25) = 2a(a^2 - 5^2) \\ = 2a(a - 5)(a + 5)$$

$$(ii) 54a^2b^2 - 6 = 6(9a^2b^2 - 1) = 6[(3ab)^2 - (1)^2] \\ = 6(3ab - 1)(3ab + 1)$$

$$(iii) 64a^2b - 144b^3 \\ = 16b(4a^2 - 9b^2) = 16b[(2a)^2 - (3b)^2] \\ = 16b(2a + 3b)(2a - 3b)$$

$$(iv) (2x - y)^3 - (2x - y) = (2x - y)[(2x - y)^2 - 1] \\ = (2x - y)(2x - y - 1)(2x - y + 1)$$

$$(v) x^2 - 2xy + y^2 - z^2 \\ = (x^2 - 2xy + y^2) - z^2 = (x - y)^2 - (z)^2 \\ = (x - y - z)(x - y + z)$$

$$(vi) x^2 - y^2 - 2yz - z^2 = x^2 - (y^2 + 2yz + z^2) \\ = x^2 - (y + z)^2 = (x - y - z)(x + y + z)$$

$$(vii) 7a^5 - 567a = 7a(a^4 - 81) \\ = 7a(a^2)^2 - (9)^2 \\ = 7a(a^2 + 9)(a^2 - 9) \\ = 7a(a^2 + 9)((a)^2 - (3)^2) \\ = 7a(a^2 + 9)(a + 3)(a - 3)$$

$$(viii) 5x^2 - \frac{20x^4}{9} = 5x^2 \left[1 - \frac{4x^2}{9} \right] \\ = 5x^2 \left[1 - \frac{2x}{3} \right] \left[1 + \frac{2x}{3} \right]$$

Question 7.

Factorise $xy^2 - xz^2$, Hence, find the value of:

$$(i) 9 \times 8^2 - 9 \times 2^2 \quad (ii) 40 \times 5.5^2 - 40 \times 4.5^2$$

Solution:

$$xy^2 - xz^2 = x(y^2 - z^2) = x(y - z)(y + z)$$

$$(i) 9 \times 8^2 - 9 \times 2^2 = 9(8^2 - 2^2)$$

$$= 9(8 - 2)(8 + 2) = 9(6)(10) = 540$$

$$(ii) 40 \times 5.5^2 - 40 \times 4.5^2 = 40(5.5^2 - 4.5^2)$$

$$= 40(5.5 - 4.5)(5.5 + 4.5)$$

$$= 40(1)(10) = 400$$

Question 8.

Factorise :

$$(i) (a - 3b)^2 - 36b^2$$

$$(ii) 25(a - 5b)^2 - 4(a - 3b)^2$$

$$(iii) a^2 - 0.36b^2$$

$$(iv) a^4 - 625$$

$$(v) x^4 - 5x^2 - 36$$

$$(vi) 15(2x - y)^2 - 16(2x - y) - 15$$

Solution:

$$(i) (a - 3b)^2 - 36b^2$$

$$= (a - 3b)^2 - (6b)^2$$

$$= (a - 3b + 6b)(a - 3b - 6b)$$

$$= (a + 3b)(a - 9b) \quad \left\{ \begin{array}{l} a^2 - b^2 \\ = (a + b)(a - b) \end{array} \right\}$$

$$(ii) 25(a - 5b)^2 - 4(a - 3b)^2$$

$$= [5(a - 5b)]^2 - [2(a - 3b)]^2$$

$$= (5a - 25b)^2 - (2a - 6b)^2$$

$$= (5a - 25b + 2a - 6b)(5a - 25b - 2a + 6b)$$

$$\{a^2 - b^2 = (a + b)(a - b)\}$$

$$= (7a - 31b)(3a - 19b)$$

$$\begin{aligned}
 \text{(iii)} \quad & a^2 - 0.36 b^2 \\
 &= (a)^2 - (0.6b)^2 \\
 &= (a + 0.6b) (a - 0.6b) \\
 &\quad \{a^2 - b^2 = (a + b) (a - b)\}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & a^4 - 625 = (a^2)^2 - (25)^2 \\
 &= (a^2 + 25) (a^2 - 25) \\
 &\quad \{a^2 - b^2 = (a + b) (a - b)\} \\
 &= (a^2 + 25) \{(a)^2 - (5)^2\} \\
 &= (a^2 + 25) (a + 5) (a - 5)
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad & x^4 - 5x^2 - 36 \\
 &= (x^2)^2 - 5x^2 - 36 \\
 &= (x^2)^2 - 9x^2 + 4x^2 - 36 \quad \left\{ \begin{array}{l} \because -36 = -9 \times 4 \\ -5 = -9 + 4 \end{array} \right\} \\
 &= x^2 (x^2 - 9) + 4 (x^2 - 9) \\
 &= (x^2 - 9) (x^2 + 4) \\
 &= \{x^2 - (3)^2\} \{x^2 + 4\} \\
 &= (x + 3) (x - 3) (x^2 + 4) \\
 &= (x^2 + 4) (x + 3) (x - 3)
 \end{aligned}$$

$$\text{(vi)} \quad 15 (2x - y)^2 - 16 (2x - y) - 15$$

Let $2x - y = a$, then

$$\begin{aligned}
 & 15a^2 - 16a - 15 \\
 &= 15a^2 - 25a + 9a - 15
 \end{aligned}$$

$$\left\{ \begin{array}{l} \because 15 \times (-15) = -225 \\ \therefore -225 = -25 \times 9 \\ -16 = -25 + 9 \end{array} \right\}$$

$$\begin{aligned}
 &= 5a (3a - 5) + 3 (3a - 5) \\
 &= (3a - 5) (5a + 3) \\
 &= [3 (2x - y) - 5] [5 (2x - y) + 3] \\
 &= (6x - 3y - 5) (10x - 5y + 3)
 \end{aligned}$$

Question 9.

Factorise $a^2b - b^3$ Using this result, find the value of $101^2 \times 100 - 100^3$.

Solution:

$$a^2b - b^3$$

$$b(a^2 - b^2)$$

$$b(a + b)(a - b)$$

Now,

$$101^2 \times 100 - 100^3$$

$$= 100(101^2 - 100^2)$$

$$= 100(101 + 100)(101 - 100)$$

$$= 100(201)(1)$$

$$= 20100$$

Question 10.

Evaluate (using factors): $301^2 \times 300 - 300^3$.

Solution:

$$301^2 \times 300 - 300^3$$

$$= 300(301^2 - 300^2)$$

$$= 300(301 + 300)(301 - 300)$$

$$= 300(601)(1)$$

$$= 180300$$