

14. Linear Equations in one Variable

(With Problems Based on Linear equations)

EXERCISE 14(A)

Solve the following equations:

Question 1.

$$20 = 6 + 2x$$

Solution:

$$20 = 6 + 2x$$

$$20 - 6 = 2x$$

$$14 = 2x$$

$$7 = x$$

$$x = 7$$

Question 2.

$$15 + x = 5x + 3$$

Solution:

$$15 - 3 = 5x - x$$

$$12 = 4x$$

$$3 = x$$

$$x = 3$$

Question 3.

$$\frac{3x+2}{x-6} = -7$$

Solution:

$$3x + 2 = -7(x - 6) \text{ (by cross multiplying)}$$

$$3x + 2 = -7x + 42$$

$$3x + 7x = 42 - 2$$

$$10x = 40$$

$$x = 4$$

Question 4.

$$3a - 4 = 2(4 - a)$$

Solution:

$$3a - 4 = 8 - 2a$$

$$3a + 2a = 8 + 4$$

$$5a = 12$$

$$a = 2.4$$

Question 5.

$$3(b - 4) = 2(4 - b)$$

Solution:

$$\begin{aligned}\Rightarrow 3b-12 &= 8-2b \\ \Rightarrow 3b+2b &= 8+12 \\ \Rightarrow 5b &= 20 \\ \Rightarrow b &= \frac{20}{5} \\ \Rightarrow b &= 4\end{aligned}$$

Question 6.

$$\frac{x+2}{9} = \frac{x+4}{11}$$

Solution:

$$\begin{aligned}\Rightarrow 11(x+2) &= 9(x+4) \\ &\quad \text{(by cross multiplying)} \\ \Rightarrow 11x+22 &= 9x+36 \\ \Rightarrow 11x-9x &= 36-22 \\ \Rightarrow 2x &= 14 \\ \Rightarrow x &= \frac{14}{2} \\ \Rightarrow x &= 7\end{aligned}$$

Question 7.

$$\frac{x-8}{5} = \frac{x-12}{9}$$

Solution:

$$\begin{aligned}\Rightarrow 9(x-8) &= 5(x-12) \\ &\quad \text{(by cross multiplying)} \\ \Rightarrow 9x-72 &= 5x-60 \\ \Rightarrow 9x-5x &= -60+72 \\ \Rightarrow 4x &= 12 \\ \Rightarrow x &= \frac{12}{4} \\ \Rightarrow x &= 3\end{aligned}$$

Question 8.

$$5(8x + 3) = 9(4x + 7)$$

Solution:

$$\Rightarrow 40x + 15 = 36x + 63$$

$$\Rightarrow 40x - 36x = 63 - 15$$

$$\Rightarrow 4x = 48$$

$$\Rightarrow x = \frac{48}{4}$$

$$\Rightarrow x = 12$$

Question 9.

$$3(x + 1) = 12 + 4(x - 1)$$

Solution:

$$3(x + 1) = 12 + 4(x - 1)$$

$$3x + 3 = 12 + 4x - 4$$

$$3x - 4x = 12 - 4 - 3$$

$$-x = 5$$

$$x = -5$$

Question 10.

$$\frac{3x}{4} - \frac{1}{4}(x - 20) = \frac{x}{4} + 32$$

Solution:

$$\Rightarrow \frac{3x}{4} - \frac{x}{4} + 5 = \frac{x}{4} + 32$$

$$\Rightarrow \frac{3x}{4} - \frac{x}{4} - \frac{x}{4} = 32 - 5$$

$$\Rightarrow \frac{3x - x - x}{4} = 27$$

$$\Rightarrow \frac{x}{4} = 27$$

$$\Rightarrow x = 27 \times 4$$

$$\Rightarrow x = 108$$

Question 11.

$$3a - \frac{1}{5} = \frac{a}{5} + 5\frac{2}{5}$$

Solution:

$$\Rightarrow 3a - \frac{a}{5} = 5\frac{2}{5} + \frac{1}{5}$$

$$\Rightarrow 3a - \frac{a}{5} = \frac{27}{5} + \frac{1}{5}$$

$$\Rightarrow 3a \times 5 - \frac{a}{5} \times 5 = \frac{27}{5} \times 5 + \frac{1}{5} \times 5$$

(Multiplying each term by 5)

$$\Rightarrow 15a - a = 27 + 1$$

$$\Rightarrow 14a = 28$$

$$\Rightarrow a = \frac{28}{14}$$

$$\Rightarrow a = 2$$

Question 12.

$$\frac{x}{3} - 2\frac{1}{2} = \frac{4x}{9} - \frac{2x}{3}$$

Solution:

$$\Rightarrow \frac{x}{3} - \frac{5}{2} = \frac{4x}{9} - \frac{2x}{3}$$

Since, L.C.M. of denominators 3, 2, 9 and 3 = 18

$$\Rightarrow \frac{x}{3} \times 18 - \frac{5}{2} \times 18 = \frac{4x}{9} \times 18 - \frac{2x}{3} \times 18$$

[Multiplying each term by 18]

$$\Rightarrow 6x - 45 = 8x - 12x$$

$$\Rightarrow 6x + 12x - 8x = 45$$

$$\Rightarrow 18x - 8x = 45$$

$$\Rightarrow 10x = 45$$

$$\Rightarrow x = \frac{45}{10}$$

$$\Rightarrow x = 4.5$$

Question 13.

$$\frac{4(y+2)}{5} = 7 + \frac{5y}{13}$$

Solution:

$$\Rightarrow \frac{4y+8}{5} = 7 + \frac{5y}{13}$$

$$\Rightarrow \frac{4y+8}{5} = \frac{91+5y}{13}$$

(by cross multiplying)

$$\Rightarrow 13(4y+8) = 5(91+5y)$$

$$\Rightarrow 52y+104 = 455+25y$$

$$\Rightarrow 52y-25y = 455-104$$

$$\Rightarrow 27y = 351$$

$$\Rightarrow y = \frac{351}{27}$$

$$\Rightarrow y = 13$$

Question 14.

$$\frac{a+5}{6} - \frac{a+1}{9} = \frac{a+3}{4}$$

Solution:

Since, L.C.M. of denominators 6,9 and 4 = 36

$$\therefore \frac{a+5}{6} \times 36 - \frac{a+1}{9} \times 36 = \frac{a+3}{4} \times 36$$

(Multiplying each term by 36)

$$\Rightarrow 6(a+5) - 4(a+1) = 9(a+3)$$

$$\Rightarrow 6a+30-4a-4 = 9a+27$$

$$\Rightarrow 6a-4a-9a = 27-30+4$$

$$\Rightarrow 6a-13a = 1$$

$$\Rightarrow -7a = 1$$

$$\Rightarrow a = -\frac{1}{7}$$

Question 15.

$$\frac{2x-13}{5} - \frac{x-3}{11} = \frac{x-9}{5} + 1$$

Solution:

$$\Rightarrow \frac{2x-13}{5} - \frac{x-3}{11} = \frac{x-9}{5} + \frac{1}{1}$$

Since, L.C.M. of denominators 5, 11, 5 and 1 = 55

$$\therefore \frac{2x-13}{5} \times 55 - \frac{x-3}{11} \times 55 = \frac{x-9}{5} \times 55 + \frac{1}{1} \times 55$$

$$\Rightarrow 11(2x-13) - 5(x-3) = 11(x-9) + 55$$

$$\Rightarrow 22x - 143 - 5x + 15 = 11x - 99 + 55$$

$$\Rightarrow 22x - 5x - 11x = -99 + 55 + 143 - 15$$

$$\Rightarrow 6x = 198 - 114$$

$$\Rightarrow 6x = 84$$

$$\Rightarrow x = \frac{84}{6}$$

$$\Rightarrow x = 14$$

Question 16.

$$6(6x-5) - 5(7x-8) = 12(4-x) + 1$$

Solution:

$$6(6x-5) - 5(7x-8) = 12(4-x) + 1$$

$$36x - 30 - 35x + 40 = 48 - 12x + 1$$

$$\Rightarrow x + 12x = 49 - 10$$

$$\Rightarrow 13x = 39 \Rightarrow x = \frac{39}{13} \Rightarrow x = 3$$

Question 17.

$$(x-5)(x+3) = (x-7)(x+4)$$

Solution:

$$(x-5)(x+3) = (x-7)(x+4)$$

$$\Rightarrow x^2 + 3x - 5x - 15 = x^2 + 4x - 7x - 28$$

$$\Rightarrow -2x - 15 = -3x - 28$$

$$\Rightarrow 3x - 2x = 15 - 28 \Rightarrow x = -13$$

Question 18.

$$(x-5)^2 - (x+2)^2 = -2$$

Solution:

$$\begin{aligned}
& (x-5)^2 - (x+2)^2 = -2 \\
\Rightarrow & (x^2 - 10x + 25) - (x^2 + 4x + 4) = -2 \\
\Rightarrow & x^2 - 10x + 25 - x^2 - 4x - 4 = -2 \\
\Rightarrow & -10x - 4x + 25 - 4 = -2 \\
\Rightarrow & -14x = 4 - 2 - 25 = -23 \\
\Rightarrow & x = \frac{-23}{-14} = \frac{23}{14} = 1\frac{9}{14}
\end{aligned}$$

Question 19.

$$(x-1)(x+6) - (x-2)(x-3) = 3$$

Solution:

$$\begin{aligned}
& (x-1)(x+6) - (x-2)(x-3) = 3 \\
& x^2 - x + 6x - 6 - (x^2 - 3x - 2x + 6) = 3 \\
\Rightarrow & x^2 - x + 6x - 6 - x^2 + 3x + 2x - 6 = 3 \\
\Rightarrow & -x + 6x + 3x + 2x - 6 - 6 = 3 \\
\Rightarrow & -x + 11x - 6 - 6 = 3 \\
& 10x = 15 \\
& x = \frac{15}{10} = \frac{3}{2} = 1\frac{1}{2} \text{ Ans.}
\end{aligned}$$

Question 20.

$$\frac{3x}{x+6} - \frac{x}{x+5} = 2$$

Solution:

$$\begin{aligned}
& \frac{3x}{x+6} - \frac{x}{x+5} = 2 \\
\Rightarrow & \frac{3x(x+5) - x(x+6)}{(x+6)(x+5)} = 2 \\
\Rightarrow & \frac{3x^2 + 15x - x^2 - 6x}{x^2 + 5x + 6x + 30} = 2 \\
\Rightarrow & \frac{2x^2 + 9x}{x^2 + 11x + 30} = 2 \\
\Rightarrow & 2x^2 + 9x = 2(x^2 + 11x + 30) \\
\Rightarrow & 2x^2 + 9x = 2x^2 + 22x + 60 \\
\Rightarrow & 2x^2 - 2x^2 + 9x - 22x = 60 \Rightarrow -13x = 60 \\
& x = -\frac{60}{13} = -4\frac{8}{13}
\end{aligned}$$

Question 21.

$$\frac{1}{x-1} + \frac{2}{x-2} = \frac{3}{x-3}$$

Solution:

$$= \frac{1(x-2) + 2(x-1)}{(x-1)(x-2)} = \frac{3}{x-3}$$

$$\Rightarrow \frac{x-2+2x-2}{x^2-2x-x+2} = \frac{3}{x-3}$$

$$\Rightarrow \frac{3x-4}{x^2-3x+2} = \frac{3}{x-3}$$

$$\Rightarrow (x-3)(3x-4) = 3(x^2-3x+2)$$

$$\Rightarrow 3x^2 - 4x - 9x + 12 = 3x^2 - 9x + 6$$

$$\Rightarrow 3x^2 - 13x - 3x^2 + 9x = 6 - 12$$

$$\Rightarrow -4x = -6$$

$$x = \frac{-6}{-4} = \frac{3}{2} = 1\frac{1}{2}$$

Question 22.

$$\frac{x-1}{7x-14} = \frac{x-3}{7x-26}$$

$$\frac{x-1}{7x-14} = \frac{x-3}{7x-26}$$

$$\Rightarrow (x-1)(7x-26) = (7x-14)(x-3)$$

$$\Rightarrow 7x^2 - 7x - 26x + 26 = 7x^2 - 14x - 21x + 42$$

$$\Rightarrow -33x + 26 = -35x + 42$$

$$\Rightarrow 35x - 33x = 42 - 26$$

$$\Rightarrow 2x = 16 \Rightarrow x = 8$$

Question 23.

$$\frac{1}{x-1} - \frac{1}{x} = \frac{1}{x+3} - \frac{1}{x+4}$$

Solution:

$$\frac{1}{x-1} - \frac{1}{x} = \frac{1}{x+3} - \frac{1}{x+4}$$

$$\Rightarrow \frac{x-(x-1)}{(x-1)x} = \frac{(x+4)-(x+3)}{(x+3)(x+4)}$$

$$= \frac{1}{(x-1)x} = \frac{1}{(x+3)(x+4)}$$

$$= (x+3)(x+4) = x(x-1)$$

$$\Rightarrow x^2 + 4x + 3x + 12 = x^2 - x$$

$$\Rightarrow x^2 + 7x - x^2 + x = -12$$

$$8x = -12$$

$$x = -\frac{12}{8} = -\frac{3}{2} = -1\frac{1}{2}$$

Question 24.

$$\text{Solve: } \frac{2x}{3} - \frac{x-1}{6} + \frac{7x-1}{4} = 2\frac{1}{6}$$

Hence, find the value of 'a', if $\frac{1}{a} + 5x = 8$.**Solution:**

$$\frac{2x}{3} - \frac{x-1}{6} + \frac{7x-1}{4}$$

$$= 2\frac{1}{6}$$

$$\Rightarrow \frac{2x}{3} - \frac{x-1}{6} + \frac{7x-1}{4}$$

$$= \frac{13}{6}$$

$$\frac{8x - 2x + 2 + 21x - 3}{12} = 26$$

(L.C.M. of 3, 6, 4, 6 = 12)

$$\Rightarrow 27x - 1 = 26$$

$$\Rightarrow 27x = 26 + 1$$

$$\Rightarrow x = \frac{27}{27} = 1$$

$$\text{Now, } \frac{1}{a} + 5x = 8$$

$$\Rightarrow \frac{1}{a} + 5 \times 1 = 8$$

$$\Rightarrow \frac{1}{a} + 5 = 8$$

$$\Rightarrow \frac{1}{a} = 8 - 5 = 3$$

$$\therefore 3a = 1$$

$$\Rightarrow a = \frac{1}{3}$$

$$\therefore x = 1 \text{ and } a = \frac{1}{3}$$

Question 25.

$$\text{Solve: } \frac{4-3x}{5} + \frac{7-x}{3} + 4\frac{1}{3} = 0$$

Hence find the value of 'p' if $2p - 2x + 1 = 0$

Solution:

$$\Rightarrow \frac{4-3x}{5} + \frac{7-x}{3} + 4\frac{1}{3} = 0$$

$$\Rightarrow \frac{4-3x}{5} + \frac{7-x}{3} + \frac{13}{3} = 0$$

$$\frac{12-9x+35-5x+65}{15} = 0$$

(L.C.M. of 5, 3, 3 = 15)

$$-14x + 112 = 0$$

$$\Rightarrow -14x = -112$$

$$\Rightarrow x = \frac{-112}{-14}$$

$$= 8$$

Hence $x = 8$

Now, $3p - 2x + 1 = 0$

$$\Rightarrow 3p - 2 \times 8 + 1 = 0$$

$$\Rightarrow 3p - 16 + 1 = 0$$

$$\Rightarrow 3p - 15 = 0$$

$$\Rightarrow 3p = 15$$

$$\Rightarrow p = 5$$

Question 26.

Solve: $0.25 + \frac{1.95}{x} = 0.9$

Solution:

$$0.25 + \frac{1.95}{x} = 0.9$$

$$0.25x + 1.95 = 0.9x$$

$$\Rightarrow 0.9x - 0.25x = 1.95$$

$$\Rightarrow 0.65x = 1.95$$

$$\Rightarrow x = \frac{1.95}{0.65} = 3$$

$$\text{Hence, } x = 3$$

Question 27.

Solve: $5x - \left(4x + \frac{5x-4}{7}\right) = \frac{4x-14}{3}$

Solution:

$$5x - \left(4x + \frac{5x-4}{7}\right) = \frac{4x-14}{3}$$

$$5x - \left(\frac{28x+5x-4}{7}\right) = \frac{4x-14}{3}$$

$$\frac{35x-33x+4}{7} = \frac{4x-14}{3}$$

$$3 \times (2x + 4) = 7 \times (4x - 14)$$

$$6x + 12 = 28x - 98$$

$$22x = 98 + 12$$

$$x = \frac{110}{22} = 5$$

EXERCISE 14(B)

Question 1.

Fifteen less than 4 times a number is 9. Find the number.

Solution:

Let the required number be x

4 times the number = $4x$

15 less than 4 times the number = $4x - 15$

According to the statement :

$$4x - 15 = 9$$

$$\Rightarrow 4x = 9 + 15$$

$$\Rightarrow 4x = 24$$

$$\Rightarrow x = 6$$

Question 2.

If Megha's age is increased by three times her age, the result is 60 years. Find her age

Solution:

Let Megha's age = x years

Three times Megha's age = $3x$ years

According to the statement :

$$x + 3x = 60$$

$$\Rightarrow 4x = 60$$

$$\Rightarrow x = 15$$

Megha's age = 15 years

Question 3.

28 is 12 less than 4 times a number. Find the number.

Solution:

Let the required number be x

4 times the number = $4x$

12 less than 4 times the number = $4x - 12$

According to the statement

$$4x - 12 = 28$$

$$\Rightarrow 4x = 28 + 12$$

$$\Rightarrow 4x = 40$$

$$x = 10$$

Required number = 10

Question 4.

Five less than 3 times a number is -20. Find the number.

Solution:

Let the required number = x

3 times the number = $3x$

5 less than 3 times the number = $3x - 5$

According to statement :

$$3x - 5 = -20$$

$$\Rightarrow 3x = -20 + 5$$

$$\Rightarrow 3x = -15$$

$$\Rightarrow x = -5$$

Required number = -5

Question 5.

Fifteen more than 3 times Neetu's age is the same as 4 times her age. How old is she ?

Solution:

Let Neetu's age = x years

3 times Neetu's age = $3x$ years

Fifteen more than 3 times Neetu's age = $(3x + 15)$ years

4 times Neetu's age = $4x$

According to the statement :

$$4x = 3x + 15$$

$$\Rightarrow 4x - 3x = 15$$

$$\Rightarrow x = 15$$

Neetu's age = 15 years

Question 6.

A number decreased by 30 is the same as 14 decreased by 3 times the number; Find the number.

Solution:

Let the required number = x

The number decreased by 30 = $x - 30$

14 decreased by 3 times the number = $14 - 3x$

According to the statement :

$$x - 30 = 14 - 3x$$

$$\Rightarrow x + 3x = 14 + 30$$

$$\Rightarrow 4x = 44$$

$$x = 11$$

Required number = 11

Question 7.

A's salary is same as 4 times B's salary. If together they earn Rs.3,750 a month, find the salary of each.

Solution:

Let B's salary = Rs. x

A's salary = Rs. $4x$

According to the statement :

$$x + 4x = 3750$$

$$\Rightarrow 5x = 3750$$

$$\Rightarrow x = 750$$

$$4x = 750 \times 4 = 3000$$

A's salary = Rs. 3000

B's salary = Rs. 750

Question 8.

Separate 178 into two parts so that the first part is 8 less than twice the second part.

Solution:

Let first part = x

Second part = $178 - x$

According to the problem :

First Part = 8 less than twice the second part

$$x = 2(178 - x) - 8$$

$$\Rightarrow x = 356 - 2x - 8$$

$$\Rightarrow x + 2x = 356 - 8$$

$$\Rightarrow 3x = 348$$

$$\Rightarrow x = 116$$

First Part = 116

$$\Rightarrow \text{Second Part} = 178 - x = 178 - 116 = 62$$

First Part = 116

$$\Rightarrow \text{Second Part} = 62$$

Alternative Method :

Let Second part = x

First part = $2x - 8$

According to the problem :

$$x + 2x - 8 = 178$$

$$\Rightarrow x + 2x = 178 + 8$$

$$\Rightarrow 3x = 186$$

$$\Rightarrow x = 62$$

$$\text{First part} = 2x - 8 = 2 \times 62 - 8 = 124 - 8 = 116$$

Second part = 62

Question 9.

Six more than one-fourth of a number is two-fifth of the number. Find the number.

Solution:

Let the required number = x

$$\therefore \text{One-fourth of the number} = \frac{x}{4}$$

$$\text{Two-fifth of the number} = \frac{2x}{5}$$

According to the statement :

$$\frac{2x}{5} = 6 + \frac{x}{4}$$

$$\Rightarrow \frac{2x}{5} - \frac{x}{4} = \frac{6}{1}$$

$$\Rightarrow \frac{2x}{5} \times 20 - \frac{x}{4} \times 20 = 6 \times 20$$

[Multiplying each term by 20 because
L.C.M. of 5, 4 and 1 = 20]

$$\Rightarrow 8x - 5x = 120$$

$$\Rightarrow 3x = 120$$

$$\Rightarrow x = \frac{120}{3}$$

Required number = 40

$$x = 40$$

Question 10.

The length of a rectangle is twice its width. If its perimeter is 54 cm; find its length.

Solution:

Let width of the rectangle = x cm

Length of the rectangle = $2x$ cm

Perimeter of the rectangle = $2 [\text{Length} + \text{Width}] = 2 [2x + x] = 2 \times 3x = 6x$ cm

Given perimeter = 54 cm

$$6x = 54$$

$$\Rightarrow x = 9$$

$$\text{Length} = 2x = 2 \times 9 = 18 \text{ cm}$$

Question 11.

A rectangle's length is 5 cm less than twice its width. If the length is decreased by 5 cm and width is increased by 2 cm; the perimeter of the resulting rectangle will be 74 cm. Find the length and the width of the original rectangle.

Solution:

Let width of the original rectangle = x cm

Length of the original rectangle = $(2x - 5)$ cm

Now, new length of the rectangle = $2x - 5 - 5 = (2x - 10)$ cm

New width of the rectangle = $(x + 2)$ cm

New perimeter = $2[\text{Length} + \text{Width}] = 2[2x - 10 + x + 2] = 2[3x - 8] = (6x - 16)$ cm

Given; new perimeter = 74 cm

$$6x - 16 = 74$$

$$\Rightarrow 6x = 74 + 16$$

$$\Rightarrow 6x = 90$$

$$\Rightarrow x = 15$$

$$\text{Length of the original rectangle} = 2x - 5 = 2 \times 15 - 5 = 30 - 5 = 25 \text{ cm}$$

$$\text{Width of the original rectangle} = x = 15 \text{ cm}$$

Question 12.

The sum of three consecutive odd numbers is 57. Find the numbers.

Solution:

Let the three consecutive odd numbers be x , $x+2$, $x+4$.

According to the statement :

$$x + x + 2 + x + 4 = 57$$

$$\Rightarrow x + x + x = 57 - 2 - 4$$

$$\Rightarrow 3x = 51$$

$$\Rightarrow x = 17$$

Three consecutive odd numbers are 17, 19, 21

Question 13.

A man's age is three times that of his son, and in twelve years he will be twice as old as his son would be. What are their present ages.

Solution:

Let present age of the son = x years

present age of the man = $3x$ years

In 12 years :

Son's age will be = $(x + 12)$ years

The man's age will be = $(3x + 12)$ years

According to the statement :

$$3x + 12 = 2(x + 12)$$

$$\Rightarrow 3x + 12 = 2x + 24$$

$$\Rightarrow 3x - 2x = 24 - 12$$

$$\Rightarrow x = 12$$

$$3x = 3 \times 12 = 36$$

Hence, present age of the man = 36 years

Present age of the son = 12 years.

Question 14.

A man is 42 years old and his son is 12 years old. In how many years will the age of the son be half the age of the man at that time?

Solution:

Man's age = 42 years

Son's age = 12 years

Let after x years the age of the son will be half the age of the man.

Man's age after x years = $(42 + x)$ years

Son's age after x years = $(12 + x)$ years

According to the statement :

$$12 + x = \frac{42 + x}{2}$$

$$\Rightarrow 2(12 + x) = 42 + x$$

(by cross multiplying)

$$\Rightarrow 24 + 2x = 42 + x$$

$$\Rightarrow 2x - x = 42 - 24$$

$$\Rightarrow x = 18$$

Hence after 18 years, the age of the son will be half the age of the man

Question 15.

A man completed a trip of 136 km in 8 hours. Some part of the trip was covered at 15 km/hr and the remaining at 18 km/hr. Find the part of the trip covered at 18 km/hr.

Solution:

Total distance of the trip = 136 km.

Let part of the trip covered at 18 km/hr.

$$= x \text{ km.}$$

∴ Distance of the trip covered at 15 km/hr

$$= (136-x) \text{ km}$$

Time taken by the man to cover x km

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{x}{18} \text{ hours}$$

Time taken by the man to cover $(136-x)$ km

$$= \frac{136-x}{15} \text{ hours}$$

Total time taken by the man to cover a trip of 136 km = 8 hours

$$\therefore \frac{x}{18} + \frac{136-x}{15} = 8$$

$$\Rightarrow \frac{x}{18} \times 90 + \frac{136-x}{15} \times 90 = 8 \times 90$$

[Multiplying each term by 90 because
L.C.M. of denominators = 90]

$$\Rightarrow 5x + 6(136-x) = 720$$

$$\Rightarrow 5x + 816 - 6x = 720$$

$$\Rightarrow 5x - 6x = 720 - 816$$

$$\Rightarrow -x = -96$$

$$\Rightarrow x = 96$$

∴ Part of the trip covered at 18 km/hr

$$= 96 \text{ km}$$

Question 16.

The difference of two numbers is 3 and the difference of their squares is 69. Find the numbers.

Solution:

Let one number = x

Second number = $x + 3$ [Difference of two numbers is 3]

According to the statement :

$$\begin{aligned}
& (x+3)^2 - (x)^2 = 69 \\
\Rightarrow & (x)^2 + (3)^2 + 2 \times x \times 3 - x^2 = 69 \\
\Rightarrow & x^2 + 9 + 6x - x^2 = 69 \\
\Rightarrow & 6x = 69 - 9 \\
\Rightarrow & 6x = 60 \\
\Rightarrow & x = \frac{60}{6} \\
\Rightarrow & x = 10
\end{aligned}$$

One number = 10

Second number = $x + 3 = 10 + 3 = 13$

Question 17.

Two consecutive natural numbers are such that one-fourth of the smaller exceeds one-fifth of the greater by 1. Find the numbers.

Solution:

Let two consecutive natural numbers = $x, x+1$

$$\therefore \text{One-fourth of the smaller} = \frac{x}{4}$$

$$\text{One-fifth of the greater} = \frac{x+1}{5}$$

According to the statement :

$$\frac{x}{4} = \frac{x+1}{5} + 1 \Rightarrow \frac{x}{4} - \frac{x+1}{5} = 1$$

$$\Rightarrow \frac{5x - 4(x+1)}{20} = 1 \Rightarrow \frac{5x - 4x - 4}{20} = 1$$

$$\Rightarrow \frac{x - 4}{20} = 1$$

$$\Rightarrow x - 4 = 20$$

(Cross-multiplying)

$$\Rightarrow x = 20 + 4 \Rightarrow x = 24$$

$$\therefore x+1 = 24+1 = 25$$

Two consecutive numbers are 24 and 25

Question 18.

Three consecutive whole numbers are such that if they be divided by 5, 3 and 4 respectively; the sum of the quotients is 40. Find the numbers.

Solution:

Let the three consecutive whole numbers be x , $x + 1$ and $x + 2$
According to the statement:

$$\frac{x}{5} + \frac{x+1}{3} + \frac{x+2}{4} = 40$$

$$\Rightarrow \frac{x}{5} \times 60 + \frac{x+1}{3} \times 60 + \frac{x+2}{4} \times 60 = 40 \times 60$$

[Multiplying each term by 60 because
L.C.M. of denominators = 60]

$$\Rightarrow 12x + 20(x+1) + 15(x+2) = 2400$$

$$\Rightarrow 12x + 20x + 20 + 15x + 30 = 2400$$

$$\Rightarrow 12x + 20x + 15x = 2400 - 20 - 30$$

$$\Rightarrow 47x = 2350$$

$$\Rightarrow x = \frac{2350}{47}$$

$$x = 50$$

$$x + 1 = 50 + 1 = 51$$

$$x + 2 = 50 + 2 = 52$$

Three consecutive whole numbers are 50, 51 and 52

Question 19.

If the same number be added to the numbers 5, 11, 15 and 31, the resulting numbers are in proportion. Find the number.

Solution:

Let x be added to each number, then the numbers will be $5 + x$, $11 + x$, $15 + x$ and $31 + x$

According to the condition

$$\frac{5+x}{11+x} = \frac{15+x}{31+x}$$

By cross multiplication,

$$(5+x)(31+x) = (15+x)(11+x)$$

$$\Rightarrow 155 + 5x + 31x + x^2 = 165 + 11x + 15x + x^2$$

$$\Rightarrow 155 + 36x + x^2 = 165 + 26x + x^2$$

$$\Rightarrow 36x + x^2 - 26x - x^2 = 165 - 155$$

$$\Rightarrow 10x = 10 \Rightarrow x = \frac{10}{10} = 1$$

Question 20.

The present age of a man is twice that of his son. Eight years hence, their ages will be in the ratio 7 : 4. Find their present ages.

Solution:

Let present age of son = x year

Then age of his father = $2x$

8 years hence,

Age of son = $(x + 8)$ years and age of father = $(2x + 8)$ years

According to the condition,

$$\frac{2x+8}{x+8} = \frac{7}{4}$$

$$\Rightarrow 8x + 32 = 7x + 56$$

$$\Rightarrow 8x - 7x = 56 - 32$$

$$\Rightarrow x = 24$$

Present age of son = 24 years

and age of father = $2x = 2 \times 24 = 48$ years

Hence age of man = 48 years and age of his son = 24 years

EXERCISE 14(C)**Question 1.**

Solve:

$$(i) \frac{1}{3}x - 6 = \frac{5}{2}$$

$$(ii) \frac{2x}{3} - \frac{3x}{8} = \frac{7}{12}$$

$$(iii) (x + 2)(x + 3) + (x - 3)(x - 2) - 2x(x + 1) = 0$$

$$(iv) \frac{1}{10} - \frac{7}{x} = 35$$

$$(v) 13(x - 4) - 3(x - 9) - 5(x + 4) = 0$$

$$(vi) x + 7 - \frac{8x}{3} = \frac{17x}{6} - \frac{5x}{8}$$

$$(vii) \frac{3x-2}{4} - \frac{2x+3}{3} = \frac{2}{3} - x$$

$$(viii) \frac{x+2}{6} - \left(\frac{11-x}{3} - \frac{1}{4} \right) = \frac{3x-4}{12}$$

$$(ix) \frac{2}{5x} - \frac{5}{3x} = \frac{1}{15}$$

$$(x) \frac{x+2}{3} - \frac{x+1}{5} = \frac{x-3}{4} - 1$$

$$(xi) \frac{3x-2}{3} + \frac{2x+3}{2} = x + \frac{7}{6}$$

$$(xii) x - \frac{x-1}{2} = 1 - \frac{x-2}{3}$$

$$(xiii) \frac{9x+7}{2} - \left(x - \frac{x-2}{7} \right) = 36$$

$$(xiv) \frac{6x+1}{2} + 1 = \frac{7x-3}{3}$$

Solution:

$$(i) \frac{1}{3}x - 6 = \frac{5}{2}$$

$$\Rightarrow \frac{1}{3}x = \frac{5}{2} + \frac{6}{1}$$

$$\Rightarrow \frac{1}{3}x = \frac{5 \times 1}{2 \times 1} + \frac{6 \times 2}{1 \times 2}$$

$$\Rightarrow \frac{1}{3}x = \frac{5}{2} + \frac{12}{2}$$

$$\Rightarrow \frac{1}{3}x = \frac{5+12}{2}$$

$$= \frac{1}{3}x = \frac{17}{2}$$

$$= x = \frac{17 \times 3}{2 \times 1} = \frac{51}{2} = 25\frac{1}{2}$$

$$(ii) \frac{2x}{3} - \frac{3x}{8} = \frac{7}{12}$$

2	3, 8
2	3, 4
2	3, 2
3	3, 1
	1

$$\text{L.C.M. of 3 and 8} = 2 \times 2 \times 2 \times 3 = 24$$

$$\therefore \frac{2x \times 8}{3 \times 8} - \frac{3x \times 3}{8 \times 3} = \frac{7}{12}$$

$$= \frac{16x}{24} - \frac{9x}{24} = \frac{7}{12}$$

$$= \frac{16x}{24} - \frac{9x}{24} = \frac{7}{12}$$

$$= \frac{16x - 9x}{24} = \frac{7}{12}$$

$$= \frac{7x}{24} = \frac{7}{12}$$

$$= x = \frac{7 \times 24}{12 \times 7} = 2$$

$$\therefore x = 2$$

$$(iii) (x+2)(x+3) + (x-3)(x-2) - 2x(x+1) = 0$$

$$\text{Sol. } (x+2)(x+3) + (x-3)(x-2) - 2x(x+1) = 0$$

$$\Rightarrow [x^2 + (2+3)x + 2 \times 3] + [x^2 + (-3-2)x + (-3)(-2)] - 2x^2 - 2x = 0$$

$$\Rightarrow x^2 + 5x + 6 + x^2 - 5x + 6 - 2x^2 - 2x = 0$$

$$\Rightarrow x^2 + x^2 - 2x^2 + 5x - 5x - 2x + 6 + 6 = 0$$

$$= -2x + 12 = 0$$

Subtracting 12 from both sides,

$$-2x + 12 - 12 = 0 - 12 \Rightarrow -2x = -12$$

Dividing by -2

$$\frac{-2x}{-2} = \frac{-12}{-2} \Rightarrow x = 6$$

$$\therefore x = 6$$

Verification

$$\text{L.H.S.} = (x+2)(x+3) + (x-3)(x-2) - 2x(x+1)$$

$$= (6+2)(6+3) + (6-3)(6-2) - 2 \times 6(6+1)$$

$$= 8 \times 9 + 3 \times 4 - 12 \times 7$$

$$= 72 + 12 - 84 = 84 - 84 = 0 = \text{R.H.S.}$$

$$(iv) \frac{1}{10} - \frac{7}{x} = 35$$

$$\Rightarrow \frac{-7}{x} = 35 - \frac{1}{10}$$

$$\Rightarrow \frac{-7}{x} = \frac{35 \times 10}{1 \times 10} - \frac{1 \times 1}{10 \times 1}$$

$$\Rightarrow \frac{-7}{x} = \frac{350 - 1}{10}$$

$$\Rightarrow \frac{1}{x} = \frac{350 - 1}{10 \times (-7)}$$

$$\Rightarrow x = \frac{349}{(-70)} = \frac{-70}{349}$$

$$(v) 13(x - 4) - 3(x - 9) - 5(x + 4) = 0$$

$$\Rightarrow 13(x - 4) - 3(x - 9) - 5(x + 4) = 0$$

$$\Rightarrow 13x - 52 - 3x + 27 - 5x - 20 = 0$$

$$\Rightarrow 13x - 3x - 5x - 52 + 27 - 20 = 0$$

$$\Rightarrow 13x - 8x - 72 + 27 = 0$$

$$\Rightarrow 5x - 45 = 0$$

Dividing by 5,

$$\frac{5x}{5} - \frac{45}{5} = 0 \Rightarrow x - 9 = 0 \Rightarrow x = 9$$

Verification,

$$\begin{aligned}\text{L.H.S.} &= 13(x-4) - 3(x-9) - 5(x+4) \\ &= 13(9-4) - 3(9-9) - 5(9+4) \\ &= 13 \times 5 - 3 \times 0 - 5 \times 13 \\ &= 65 - 0 - 65 = 0 = \text{R.H.S.}\end{aligned}$$

$$(vi) \quad x + 7 - \frac{8x}{3} = \frac{17x}{6} - \frac{5x}{8}$$

$$\Rightarrow \frac{3(x+7) - 8x}{3} = \frac{17x \times 4 - 5x \times 3}{24}$$

$$\Rightarrow \frac{3x + 21 - 8x}{3} = \frac{68x - 15x}{24}$$

$$\Rightarrow \frac{-5x + 21}{3} = \frac{53x}{24}$$

$$\Rightarrow 3 \times 53x = 24(-5x + 21)$$

$$\Rightarrow 159x = -120x + 504$$

$$\Rightarrow 159x + 120x = 504$$

$$\Rightarrow 279x = 504$$

$$\Rightarrow x = \frac{504}{279} = \frac{168}{93} = \frac{56}{31}$$

$$\therefore x = 1 \frac{25}{31}$$

$$(vii) \quad \frac{3x-2}{4} - \frac{2x+3}{3} = \frac{2}{3} - x$$

$$= \frac{3x-2}{4} - \frac{2x+3}{3} = \frac{2}{3} - \frac{x}{1}$$

$$= \frac{3(3x-2) - 4(2x+3)}{12} = \frac{2 \times 1}{3 \times 1} - \frac{x \times 3}{1 \times 3}$$

$$= \frac{9x-6-8x-12}{12} = \frac{2-3x}{3}$$

$$= \frac{(x-18)}{12} = \frac{2-3x}{3}$$

$$= 3(x-18) = 12(2-3x)$$

$$= 3x - 54 = 24 - 36x$$

$$= 3x + 36x = 24 + 54$$

$$= 39x = 78$$

$$x = \frac{78}{39} = 2$$

$$\therefore x = 2$$

$$(viii) \frac{x+2}{6} - \left(\frac{11-x}{3} - \frac{1}{4} \right) = \frac{3x-4}{12}$$

$$\Rightarrow \frac{x+2}{6} - \left(\frac{4(11-x) - 1 \times 3}{12} \right) = \frac{3x-4}{12}$$

$$\Rightarrow \frac{x+2}{6} - \frac{44+4x+3}{12} = \frac{3x-4}{12}$$

$$\Rightarrow \frac{2(x+2) - 41 + 4x}{12} = \frac{3x-4}{12}$$

$$\Rightarrow \frac{2x+4-41+4x}{12} = \frac{3x-4}{12}$$

$$\Rightarrow \frac{6x-37}{12} = \frac{3x-4}{12}$$

$$\Rightarrow 12(6x-37) = 12(3x-4)$$

$$\Rightarrow 72x - 444 = 36x - 48$$

$$\Rightarrow 72x - 36x = -48 + 444$$

$$\Rightarrow 36x = 396$$

$$\Rightarrow x = \frac{396}{36} = 11$$

$$\therefore x = 11$$

$$(ix) \frac{2}{5x} - \frac{5}{3x} = \frac{1}{15}$$

$$\Rightarrow \frac{2 \times 3}{5x \times 3} - \frac{5 \times 5}{3x \times 5} = \frac{1}{15}$$

$$\Rightarrow \frac{6-25}{15x} = \frac{1}{15}$$

$$\Rightarrow \frac{-19}{15x} = \frac{1}{15}$$

$$\Rightarrow \frac{-19}{x} = \frac{15}{15}$$

$$\Rightarrow -19 = x$$

$$\therefore x = -19$$

$$(x) \frac{x+2}{3} - \frac{x+1}{5} = \frac{x-3}{4} - 1$$

(L.C.M. of 3 and 5 = 15)

$$\Rightarrow \frac{5(x+2) - 3(x+1)}{15} = \frac{x-3-4}{4}$$

$$\Rightarrow \frac{5x+10-3x-3}{15} = \frac{x-7}{4}$$

$$\Rightarrow \frac{2x+7}{15} = \frac{x-7}{4}$$

$$\Rightarrow 4(2x+7) = 15(x-7)$$

$$\Rightarrow 8x+28 = 15x-105$$

$$\Rightarrow 8x-15x = -105-28$$

$$\Rightarrow -7x = -133$$

$$x = \frac{-133}{-7}$$

$$\therefore x = 19$$

$$(xi) \frac{3x-2}{3} + \frac{2x+3}{2} = x + \frac{7}{6}$$

$$\Rightarrow \frac{2(3x-2) + 3(2x+3)}{6} = x + \frac{7}{6}$$

$$\Rightarrow \frac{6x-4+6x+9}{6} = \frac{6x+7}{6}$$

$$\Rightarrow \frac{12x+5}{6} = \frac{6x+7}{6}$$

$$\Rightarrow 6(12x+5) = 6(6x+7)$$

$$\Rightarrow 72x+30 = 36x-42$$

$$\Rightarrow 72x-36x = 42-30$$

$$\Rightarrow 36x = 12$$

$$x = \frac{12}{36}$$

$$\therefore x = \frac{1}{3}$$

$$\begin{aligned}
 \text{(xii)} \quad x - \frac{x-1}{2} &= 1 - \frac{x-2}{3} \\
 \Rightarrow \frac{2(x) - 1(x-1)}{2} &= \frac{3(1) - 1(x-2)}{3} \\
 \Rightarrow \frac{2x - x + 1}{2} &= \frac{3 - x + 2}{3} \\
 \Rightarrow \frac{1x + 1}{2} &= \frac{5 - x}{3} \\
 \Rightarrow 3(x + 1) &= 2(5 - x) \\
 \Rightarrow 3x + 3 &= 10 - 2x \\
 \Rightarrow 3x + 2x &= 10 - 3 \\
 \Rightarrow 5x &= 7 \\
 \therefore x &= \frac{7}{5}
 \end{aligned}$$

$$\begin{aligned}
 \text{(xiii)} \quad \frac{9x+7}{2} - \left(x - \frac{x-2}{7} \right) &= 36 \\
 \Rightarrow \frac{9x+7}{2} - \left(\frac{7 \times x - 1(x-2)}{7} \right) &= 36 \\
 \Rightarrow \frac{9x+7}{2} - \left(\frac{7x - x - 2}{7} \right) &= 36 \\
 \Rightarrow \frac{9x+7}{2} - \left(\frac{6x-2}{7} \right) &= 36 \\
 \Rightarrow \frac{7(9x+7) + 2(-6x+2)}{14} &= 36 \\
 \Rightarrow \frac{63x + 49 - 12x + 4}{14} &= 36 \\
 \Rightarrow \frac{51x + 53}{14} &= 36 \\
 \Rightarrow 51x + 53 &= 14 \times 36 \\
 \Rightarrow 51x &= 504 - 53 \\
 \Rightarrow 51x &= 459 \\
 \Rightarrow x &= \frac{459}{51}
 \end{aligned}$$

$$\therefore x = 9$$

$$(xiv) \frac{6x+1}{2} + 1 = \frac{7x-3}{3}$$

$$\Rightarrow \frac{(6x+1)+1 \times 2}{2} = \frac{7x-3}{3}$$

$$\Rightarrow \frac{6x+1+2}{2} = \frac{7x-3}{3}$$

$$\Rightarrow \frac{6x+3}{2} = \frac{7x-3}{3}$$

$$\Rightarrow 3(6x+3) = 2(7x-3)$$

$$\Rightarrow 18x+9 = 14x-6$$

$$\Rightarrow 18x-14x = -6-9$$

$$\Rightarrow 4x = -15$$

$$\therefore x = \frac{-15}{4}$$

Question 2.

After 12 years, I shall be 3 times as old as I was 4 years ago. Find my present age.

Solution:

Let present age = x years

According to question,

$$(x+12) = 3(x-4)$$

$$x+12 = 3x-12$$

$$2x = 24$$

$$\Rightarrow x = 12 \text{ years}$$

Present age = 12 years

Question 3.

A man sold an article for ₹ 7396 and gained 10% on it. Find the cost price of the article

Solution:

S.P. of article = ₹ 396

Gain = 10%

Let cost price = ₹ x

$$\therefore \text{S.P.} = \frac{x \times (100 + 10)}{100} = \frac{110}{100}x$$

$$\therefore \frac{110}{100}x = 396$$

$$\Rightarrow x = \frac{396 \times 100}{110} = 360$$

Cost price of an article = ₹ 360

Question 4.

The sum of two numbers is 4500. If 10% of one number is 12.5% of the other, find the numbers.

Solution:

Let the first number = x

and the second number = y

According to question,

$$x + y = 4500 \dots\dots(i)$$

$$\text{and } 10\% x = 12.5\% y$$

$$\text{i.e. } 10x = 12.5y$$

$$x = \frac{12.5}{10} y \dots(ii)$$

Substitute the value of x in equation (i),

$$\frac{12.5}{10} y + y = 45,000$$

$$12.5y + 10y = 45,000$$

$$22.5y = 45,000$$

$$y = \frac{45,000}{22.5} = 2000$$

Now, put the value of y in equation (ii)

$$x = \frac{12.5}{10} \times 2000$$

$$x = 2500$$

Hence, the numbers are 2500 and 2000

Question 5.

The sum of two numbers is 405 and their ratio is 8 : 7. Find the numbers.

Solution:

Let the first number = x

and the second number = 7

According to the question, $x + y = 405$ (i)

and the numbers are in the ratio $8 : 7$

$$\text{i.e. } \frac{8x}{7y} = 1$$

$$\Rightarrow 8x = 7y$$

$$\Rightarrow x = \frac{7}{8}y$$

Now, substitute the value of x in equation (i)

$$\frac{7}{8}y + y = 405$$

$$7y + 8y = 405 \times 8$$

$$15y = 3240$$

$$y = \frac{3240}{15}$$

$$y = 216$$

Now, put the value of y in equation (ii)

$$x = \frac{7}{8} \times 216$$

$$x = 189$$

Hence, the numbers are 189 and 216

Question 6.

The ages of A and B are in the ratio $7 : 5$. Ten years hence, the ratio of their ages will be $9 : 7$. Find their present ages.

Solution:

Ratio in the present ages of A and B = $7 : 5$

Let age of A = $7x$ years

Let age of B = $5x$ years

10 years hence,

Then age of A = $7x + 10$ years

and age of B = $5x + 10$ years

According to the condition,

$$\frac{7x+10}{5x+10} = \frac{9}{7}$$

By crossing multiplication

$$7(7x + 10) = 9(5x + 10)$$

$$\Rightarrow 49x + 70 = 45x + 90$$

$$\Rightarrow 49x - 45x = 90 - 70$$

$$\Rightarrow 4x = 20$$

$$\Rightarrow x = 5$$

Present age of A = $7x = 7 \times 5 = 35$ years

and present age of B = $5x = 5 \times 5 = 25$ years

Question 7.

Find the number whose double is 45 greater than its half.

Solution:

Let the required number = x

Double of it = $2x$

$$\text{and half of it} = \frac{x}{2}$$

According to the condition,

$$2x - \frac{x}{2} = 45$$

$$\Rightarrow \frac{4x - x}{2} = 45 \Rightarrow \frac{3}{2}x = 45$$

$$\Rightarrow x = \frac{45 \times 2}{3} = 30$$

Required number = 30

Question 8.

The difference between the squares of two consecutive numbers is 31. Find the numbers.

Solution:

Let first number = x

and The second number = $x + 1$

According to the condition,

$$\therefore (x + 1)^2 - (x)^2 = 31$$

$$\Rightarrow x^2 + 2x + 1 - x^2 = 31$$

$$\Rightarrow 2x = 31 - 1 = 30$$

$$\Rightarrow x = \frac{30}{2} = 15$$

First number = 15

and second number = $15 + 1 = 16$

Hence, the numbers are 15, 16

Question 9.

Find a number such that when 5 is subtracted from 5 times the number, the result is 4 more than twice the number.

Solution:

Let the required number = x

5 times of it = $5x$

Twice of it = $2x$

According to the condition,

$$5x - 5 = 2x + 4$$

$$\Rightarrow 5x - 2x = 4 + 5$$

$$\Rightarrow 3x = 9$$

$$\Rightarrow x = 3$$

Required number = 3

Question 10.

The numerator of a fraction is 5 less than its denominator. If 3 is added to the numerator, and denominator both, the fraction becomes $\frac{2}{3}$. Find the original fraction.

Solution:

Let denominator of the original fraction = x

Then numerator = $x - 5$

$$\text{and fraction} = \frac{x-5}{x}$$

According to the condition,

$$\frac{x-5+3}{x+3} = \frac{2}{3}$$

$$\Rightarrow \frac{x-2}{x+3} = \frac{2}{3}$$

$$\Rightarrow 5(x-2) = 4x + 12$$

(By cross multiplication)

$$\Rightarrow 5x - 10 = 4x + 12$$

$$\Rightarrow x = 22$$

$$\therefore \text{Original fraction} = \frac{x-5}{x}$$

$$= \frac{22-5}{22} = \frac{17}{22}$$