

ISC SEMESTER 2 EXAMINATION
SAMPLE PAPER - 1
MATHEMATICS

Maximum Marks: 40

Time allowed: One and a half hour

*Candidates are allowed an additional 10 minutes for **only** reading the paper.*

*They must **Not** start writing during this time.*

The Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B** **OR** **Section C***

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

Mathematical tables and graph papers are provided.

Section-A

Question 1.

Choose the correct option for the following questions.

(i) $\int_0^a f(x) dx = \int_0^\infty f(k-x) dx$, then the value of k is :

- (a) 0 (b) a (c) $2a$ (d) None of these

(ii) If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then $P(\bar{A} \cap \bar{B})$ equals :

- (a) $\frac{4}{15}$ (b) $\frac{8}{45}$ (c) $\frac{1}{3}$ (d) $\frac{2}{9}$

(iii) $\int \frac{x}{4+x^4} dx$ is equal to :

- (a) $\frac{1}{4} \tan^{-1} x^2 + c$ (b) $\frac{1}{4} \tan^{-1} \left(\frac{x^2}{2} \right) + c$ (c) $\frac{1}{2} \tan^{-1} \left(\frac{x^2}{2} \right)$ (d) None of these

(iv) Write the order and degree of the differential equation $a^2 \frac{d^2 y}{dx^2} = \left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^{1/4}$.

- (a) 2, 1 (b) 2, 2 (c) 2, 4 (d) None of these

(v) $\int \frac{(\sin x + \cos x)}{\sqrt{1 + \sin 2x}} dx =$

- (a) $\frac{-x}{2} + c$ (b) $\frac{x}{2} + c$ (c) $x + c$ (d) None of these

(vi) A four-digit number is formed using the digits 1, 2, 3, 5 with no repetition. Find the probability that the number is divisible by 5.

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{24}$ (d) $\frac{1}{6}$

Question 2.

Evaluate : $\int \frac{\cos x}{\sin x + \sqrt{\sin x}} dx.$

OR

Evaluate : $\int_0^{2\pi} \sqrt{1 + \sin \frac{x}{2}} dx.$

Question 3.

Solve : $dy = (5x - 4y) dx$ where $y = 0$ and $x = 0$.

OR

Solve : $\frac{dy}{dx} + y = 1$

Question 4.

Evaluate : $\int_0^{\pi/2} \log \sin x dx.$

Question 5.

A problem in mathematics is given to three students A, B and C. The chances of solving it by A, B and C are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Calculate the probability that the problem will be solved.

OR

Bag A contains 5 white and 4 black balls and Bag B contains 7 white and 6 black balls. One ball is drawn from the bag A and, without noticing its colour, is put in the bag B. If a ball is then drawn from bag B, find the probability that it is black in colour.

Question 6.

Evaluate : $\int_0^{\pi/4} \log (1 + \tan x) dx.$

Question 7.

A class consists of 50 students out of which there are 10 girls. In the class, 2 girls and 5 boys are rank holders in an examination. If a student is selected at random from the class and is found to be a rank holder, what is the probability that the student selected is a girl ?

Question 8.

Evaluate : $\int \tan 2x \tan 3x \tan 5x dx.$

OR

Evaluate : $\int \frac{\sin (x - \alpha)}{\sin (x + \alpha)} dx.$

Section-B**Question 9.**

Choose the correct option for the following questions :

- (i) The intercepts made on the coordinate axes by the plane $2x + 3y + 6z = 18$ are :
 (a) 9, 9, 3 (b) 9, 6, 3 (c) 3, 6, 9 (d) None of these
- (ii) The distance between the planes $2x + 2y - z + 2 = 0$ and $4x + 4y - 2z + 5 = 0$ is :
 (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) 1

Question 10.

Find the equation of the plane through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$.

Question 11.

Find the area bounded by the curves $y = 6x - x^2$ and $y = x^2 - 2x$.

Section-C**Question 12.**

Choose the correct option for the following questions.

- (i) Find the coefficient of correlation between x and y if the regression line $2x + 5y - 9 = 0$ is y on x and the regression coefficient x on y is -0.7 .

(a) -0.529 (b) 0.52 (c) 5.2 (d) -0.40

- (ii) Equations of two regression lines are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Find mean of x and mean of y .

(a) $-\frac{4}{7}, \frac{11}{7}$ (b) $\frac{4}{7}, -\frac{11}{7}$ (c) $-\frac{4}{7}, -\frac{11}{7}$ (d) $\frac{4}{7}, \frac{11}{7}$

Question 13.

Find b_{xy} and b_{yx} from the following pairs of observations on X and Y :

$(1, 2), (2, 3), (3, 5), (4, 6), (5, 4)$

Question 14.

A company manufacture two types of products A and B. Each unit of A requires 3 gm of nickel and 1 gm of chromium, while each unit of B requires 1 gm of nickel and 2 gm of chromium. The firm can produce 9 gm of nickel and 8 gm of chromium. The profit is ₹ 40 on each unit of product A and ₹ 50 on each unit of product B. How many units of each type should the company manufacture so as to earn maximize profit ?

**Section-A****Answer 1.**

- (i) (b) a

Explanation :

From fundamental property of definite integral, we know that,

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

- (ii) (d) $\frac{2}{9}$

Explanation :

Given : $P(A) = \frac{3}{5}, P(B) = \frac{4}{9}$

$$\begin{aligned} P(\bar{A} \cap \bar{B}) &= P(A \cup B)' = 1 - P(A \cup B) \\ &= 1 - \{P(A) + P(B) - P(A \cap B)\} \\ &= 1 - \{P(A) + P(B) - P(A).P(B)\} \\ &= 1 - \left(\frac{3}{5} + \frac{4}{9} - \frac{3}{5} \times \frac{4}{9} \right) \\ &= 1 - \frac{35}{45} \\ &= \frac{10}{45} = \frac{2}{9} \end{aligned}$$

(iii) (b) $\frac{1}{4} \tan^{-1} \left(\frac{x^2}{2} \right) + c$

Explanation :

Let
$$I = \int \frac{x}{4+x^4} dx$$
$$= \int \frac{x}{(2)^2 + (x^2)^2} dx$$

Putting $x^2 = t \Rightarrow 2x dx = dt \Rightarrow x dx = \frac{dt}{2}$

$$\therefore I = \frac{1}{2} \int \frac{dt}{(2)^2 + t^2}$$
$$= \frac{1}{2} \left[\frac{1}{2} \tan^{-1} \left(\frac{t}{2} \right) \right] + c$$
$$= \frac{1}{4} \tan^{-1} \left(\frac{x^2}{2} \right) + c$$

$\left(\because \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} \right)$
 $(\because t = x^2)$

(iv) (c) 2, 4

Explanation :

$$a^2 \frac{d^2 y}{dx^2} = \left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^{1/4}$$

$$\left(a^2 \frac{d^2 y}{dx^2} \right)^4 = 1 + \left(\frac{dy}{dx} \right)^2$$

$\therefore$ Order of the differential equation is 2.
and degree of the differential equation is 4.

(v) (c) $x + c$

Explanation :

$$\int \frac{(\sin x + \cos x)}{\sqrt{1 + \sin 2x}} dx = 1$$

Since
$$1 + \sin 2x = \sin^2 x + \cos^2 x + 2 \sin x \cdot \cos x$$
$$= (\sin x + \cos x)^2$$
$$I = \int \frac{(\sin x + \cos x)}{\sqrt{(\sin x + \cos x)^2}} dx$$
$$= \int \left(\frac{\sin x + \cos x}{\sin x + \cos x} \right) dx$$
$$= \int dx = x + c$$

(vi) (b) $\frac{1}{4}$

Explanation :

Total four-digit numbers formed by 1, 2, 3 and 5 = $4!$

$\therefore n(S) = 4! = 24$

Total number of four digit numbers divisible by 5 = $3!$

$\therefore n(E) = 3! = 6$

$\therefore$ Probability that the number is divisible by 5

$$= \frac{n(E)}{n(S)} = \frac{6}{24} = \frac{1}{4}$$

Answer 2.

$$\int \frac{\cos x}{\sin x + \sqrt{\sin x}} dx = \int \frac{\cos x}{\sqrt{\sin x} (\sqrt{\sin x} + 1)} dx$$

Let

$$\sqrt{\sin x} = t$$

$\Rightarrow$

$$\frac{1}{2\sqrt{\sin x}} \cdot \cos x = \frac{dt}{dx}$$

$\Rightarrow$

$$\frac{\cos x}{\sqrt{\sin x}} dx = 2 dt$$

$\therefore$

$$\int \frac{\cos x}{\sin x + \sqrt{\sin x}} dx = \int \frac{2}{(t+1)} dt$$

$$= 2 \log |t+1| + c$$

$$= 2 \log |\sqrt{\sin x} + 1| + c.$$

Ans.

OR

Let

$$I = \int_0^{2\pi} \sqrt{1 + \sin \frac{x}{2}} dx$$

Let $\frac{x}{2} = t \Rightarrow \frac{dx}{2} = dt$ or $dx = 2dt$

When $x = 0$, $t = 0$

When $x = 2\pi$, $t = \pi$

$\therefore$

$$\begin{aligned} I &= \int_0^{\pi} 2\sqrt{1 + \sin t} dt \\ &= 2 \int_0^{\pi} \sqrt{\cos^2 \frac{t}{2} + \sin^2 \frac{t}{2} + 2 \sin \frac{t}{2} \cos \frac{t}{2}} dt \\ &= 2 \int_0^{\pi} \sqrt{\left(\cos \frac{t}{2} + \sin \frac{t}{2}\right)^2} dt \end{aligned}$$

or

$$I = 2 \int_0^{\pi} \left(\cos \frac{t}{2} + \sin \frac{t}{2}\right) dt$$

Let $\frac{t}{2} = u \Rightarrow \frac{dt}{2} = du$ or $dt = 2du$

When $t = 0$, $u = 0$

When $t = \pi$, $u = \frac{\pi}{2}$

$\therefore$

$$\begin{aligned} I &= 2 \int_0^{\pi/2} (\cos u + \sin u) (2du) \\ &= 4 \int_0^{\pi/2} (\cos u + \sin u) du \\ &= 4 [\sin u - \cos u]_0^{\pi/2} \\ &= 4 \left[\sin \frac{\pi}{2} - \cos \frac{\pi}{2} - \sin 0 + \cos 0 \right] \\ &= 4[1 - 0 - 0 + 1] = 4 \times 2 = 8. \end{aligned}$$

Ans.

$$dy = (5x - 4y) dx$$

Answer 3.

Given :

$$dy = (5x - 4y) dx$$

 $\Rightarrow$

$$\frac{dy}{dx} = 5x - 4y$$

 $\Rightarrow$

$$\frac{dy}{dx} + 4y = 5x$$

Compare with

$$\frac{dy}{dx} + Py = Q(x)$$

 $\therefore$

$$P = 4 \text{ and } Q(x) = 5x$$

$$\text{I.F.} = e^{\int P dx} = e^{\int 4 dx} = e^{4x}$$

Solution of differential equation is given by,

$$\text{I.F.} \times y = \int \text{I.F.} \times Q(x) dx$$

 $\Rightarrow$

$$ye^{4x} = \int e^{4x} \cdot 5x dx$$

 $\Rightarrow$

$$ye^{4x} = 5 \left[x \int e^{4x} dx - \int \left[\frac{d}{dx} x \int e^{4x} dx \right] dx \right]$$

 $\Rightarrow$

$$ye^{4x} = 5 \left[\frac{xe^{4x}}{4} - \int \frac{e^{4x}}{4} dx \right]$$

 $\Rightarrow$

$$ye^{4x} = \frac{5xe^{4x}}{4} - \frac{5e^{4x}}{16} + c$$

When $x = 0, y = 0$

$$0 = 0 - \frac{5}{16} \times 1 + c$$

 $\Rightarrow$

$$c = \frac{5}{16}$$

 $\therefore$

$$ye^{4x} = \frac{5xe^{4x}}{4} - \frac{5e^{4x}}{16} + \frac{5}{16}$$

 $\Rightarrow$

$$16y = 20x + 5e^{-4x} - 5.$$

Ans.**OR**

Given,

$$\frac{dy}{dx} + y = 1$$

 $\Rightarrow$

$$\frac{dy}{dx} = 1 - y$$

 $\Rightarrow$

$$\frac{dx}{dy} = \frac{1}{1-y}$$

 $\Rightarrow$

$$dx = \frac{1}{1-y} dy$$

Intergrating both sides, we get

$$\int dx = \int \frac{1}{1-y} dy$$

$$x = -\log |1-y| + c$$

Ans.**Answer 4.**

Let

$$I = \int_0^{\pi/2} \log \sin x dx \quad \dots(i)$$

Using property,

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$I = \int_0^{\pi/2} \log \sin \left(\frac{\pi}{2} - x \right) dx$$

$$= \int_0^{\pi/2} \log \cos x dx \quad \dots(ii)$$

Adding (i) and (ii), we get

$$2I =$$

$$\begin{aligned}
& \int_0^{\pi/2} [\log (\sin x) + \log (\cos x)] dx \\
&= \int_0^{\pi/2} \log \sin x \cos x dx \\
&= \int_0^{\pi/2} \log \frac{\sin 2x}{2} dx \\
&= \int_0^{\pi/2} (\log \sin 2x - \log 2) dx \\
&= \int_0^{\pi/2} \log \sin 2x dx - \int_0^{\pi/2} \log 2 dx \\
&\quad \text{[Putting } t = 2x, dt = 2 dx] \\
&= \frac{1}{2} \int_0^{\pi} \log \sin t dt - \log 2 [x]_0^{\pi/2} \\
&= \frac{1}{2} \cdot 2 \int_0^{\pi/2} \log \sin t dt - \frac{\pi}{2} \log 2 [\because \sin (\pi - t) = \sin t]
\end{aligned}$$

$$\begin{aligned}
2I &= I - \frac{\pi}{2} \log 2 & \left[\because \int_0^a f(x) dx = \int_0^a f(t) dt \right] \\
I &= -\frac{\pi}{2} \log 2. & \text{Ans.}
\end{aligned}$$

Answer 5.

Chances of solving the problem by A, $P(A) = \frac{1}{2}$

$\therefore$ Probability that problem will not be solved by A i.e.,

$$P(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2}$$

Similarly

$$P(\bar{B}) = 1 - \frac{1}{3} = \frac{2}{3}$$

and

$$P(\bar{C}) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\begin{aligned}
\therefore P(\text{Problem will be solved}) &= 1 - P(\text{None of them can solve the problem}) \\
&= 1 - P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C}) \\
&= 1 - \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \\
&= 1 - \frac{1}{4} = \frac{3}{4}
\end{aligned}$$

$\therefore$ Probability of the problem will be solved = $\frac{3}{4}$.

Ans.

OR

Given :

Bag A contains : 5 White and 4 Black balls

and Bag B contains : 7 White and 6 Black balls

Let X be the event that black ball is drawn from bag B.

Let E_1 be the event that white ball is drawn from bag A.

$$\therefore P(E_1) = \frac{5}{9}$$

Now, probability that black ball is drawn from bag B i.e.,

$$P(X/E_1) = \frac{6}{14}$$

Let E_2 be the event that black ball drawn from bag A.

$$\therefore P(E_2) = \frac{4}{9}$$

Now, probability that black ball is drawn from bag B

$$P(X/E_2) = \frac{7}{14}$$

Now, probability that black ball is drawn from bag B

$$\begin{aligned} &= P(E_1) \cdot P(X/E_1) + P(E_2) \cdot P(X/E_2) \\ &= \frac{5}{9} \times \frac{6}{14} + \frac{4}{9} \times \frac{7}{14} \\ &= \frac{30}{126} + \frac{28}{126} = \frac{58}{126} \end{aligned}$$

$$\therefore \text{Required probability} = \frac{29}{63}.$$

Ans.

Answer 6.

Let

$$I = \int_0^{\pi/4} \log(1 + \tan x) dx$$

$$I = \int_0^{\pi/4} \log \left[1 + \tan \left(\frac{\pi}{4} - x \right) \right] dx \quad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$\Rightarrow$

$$I = \int_0^{\pi/4} \log \left(1 + \frac{1 - \tan x}{1 + \tan x} \right) dx$$

$\Rightarrow$

$$I = \int_0^{\pi/4} \log \left(\frac{1 + \tan x + 1 - \tan x}{1 + \tan x} \right) dx$$

$\Rightarrow$

$$I = \int_0^{\pi/4} \log \frac{2}{1 + \tan x} dx$$

$\Rightarrow$

$$I = \int_0^{\pi/4} \log 2 dx - \int_0^{\pi/4} \log(1 + \tan x) dx$$

$\Rightarrow$

$$I = \int_0^{\pi/4} \log 2 dx - I$$

$\Rightarrow$

$$2I = \log 2 \int_0^{\pi/4} dx = \log 2 [x]_0^{\pi/4}$$

$\Rightarrow$

$$2I = \frac{\pi}{4} \log 2$$

$\therefore$

$$I = \frac{\pi}{8} \log 2.$$

Ans.

Answer 7.

Given :

Total number of students = 50

Total number of girls = 10

Total number of boys = 40

Total number of rank holders = 2 girls and 5 boys

Let E_1 be the event to select a boy, E_2 be the event to select a girl and A be the event that selected student is a rank holder.

So

$$P(E_1) = \frac{40}{50} = \frac{4}{5}$$

$$P(E_2) = \frac{10}{50} = \frac{1}{5}$$

$$P(A/E_1) = \frac{5}{40} = \frac{1}{8}$$

$$P(A/E_2) = \frac{2}{10} = \frac{1}{5}$$

∴ Required probability that selected rank holder is a girl

$$\begin{aligned} &= \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)} \\ &= \frac{\frac{1}{5} \times \frac{1}{5}}{\frac{4}{5} \times \frac{1}{8} + \frac{1}{5} \times \frac{1}{5}} = \frac{\frac{1}{25}}{\frac{1}{10} + \frac{1}{25}} \\ &= \frac{\frac{1}{25}}{\frac{25+10}{250}} = \frac{1}{25} \times \frac{250}{35} \end{aligned}$$

∴ Required probability = $\frac{2}{7}$.

Ans.

Answer 8.

$$\int \tan 2x \tan 3x \tan 5x \, dx$$

∴

$$5x = 2x + 3x$$

⇒

$$\tan 5x = \tan (2x + 3x)$$

⇒

$$\tan 5x = \frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x}$$

⇒

$$\tan 5x - \tan 5x \tan 2x \tan 3x = \tan 2x + \tan 3x$$

⇒

$$\tan 5x - \tan 2x - \tan 3x = \tan 5x \tan 2x \tan 3x$$

∴

$$\int \tan 2x \tan 3x \tan 5x \, dx = \int \tan 5x \, dx - \int \tan 2x \, dx - \int \tan 3x \, dx$$

$$= -\frac{1}{5} \log \cos 5x + \frac{1}{2} \log \cos 2x + \frac{1}{3} \log \cos 3x + c$$

$$= \frac{1}{2} \log \cos 2x + \frac{1}{3} \log \cos 3x - \frac{1}{5} \log \cos 5x + c$$

Ans.

OR

$$\int \frac{\sin(x - \alpha)}{\sin(x + \alpha)} \, dx$$

$$\text{Let } x + \alpha = t \therefore dx = dt$$

∴

$$\int \frac{\sin(x - \alpha)}{\sin(x + \alpha)} \, dx = \int \frac{\sin(t - 2\alpha)}{\sin t} \, dt$$

$$= \int \frac{\sin t \cos 2\alpha - \cos t \sin 2\alpha}{\sin t} \, dt$$

$$= \int \frac{\sin t \cos 2\alpha}{\sin t} \, dt - \int \frac{\cos t \sin 2\alpha}{\sin t} \, dt$$

$$= \cos 2\alpha \int dt - \sin 2\alpha \int \cot t \, dt$$

$$= t \cos 2\alpha - \sin 2\alpha \log \sin t + c$$

$$= (x + \alpha) \cos 2\alpha - \sin 2\alpha \log \sin(x + \alpha) + c$$

$$= x \cos 2\alpha - \sin 2\alpha \log \sin(x + \alpha) + c + \alpha \cos 2\alpha$$

$$= x \cos 2\alpha - \sin 2\alpha \log \sin(x + \alpha) + c_1$$

Ans.

where

$$c_1 = c + \alpha \cos 2\alpha.$$

Section-B

Answer 9.

(i) (b) 9, 6, 3

Explanation :

Given, $2x + 3y + 6z = 18$

Divide by 18 on both sides, we get

$$\frac{x}{9} + \frac{y}{6} + \frac{z}{3} = 1 \quad \dots(i)$$

Compare equation (i) with intercept form $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, we get

$$a = 9, b = 6 \text{ and } c = 3$$

(ii) (a) $\frac{1}{2}$

Explanation :

Given : $2x + 2y - z + 2 = 0$

and $4x + 4y - 2z + 5 = 0$

or $2x + 2y - z + 5/2 = 0$

Then we see that these are parallel planes because d.r.'s are same

$$\begin{aligned} \therefore \text{Distance between them} &= d_2 - d_1 \\ &= \frac{5}{2} - 2 = \frac{1}{2} \end{aligned}$$

Answer 10.

Let the direction ratios of the plane be $\langle A, B, C \rangle$.

It passes through $(2, 2, 1)$.

$$\therefore A(x - 2) + B(y - 2) + C(z - 1) = 0 \quad \dots(i)$$

Plane also passes through $(9, 3, 6)$.

$$\begin{aligned} \therefore A(9 - 2) + B(3 - 2) + C(6 - 1) &= 0 \\ \Rightarrow 7A + B + 5C &= 0 \quad \dots(ii) \end{aligned}$$

Plane (i) is perpendicular to the plane $2x + 6y + 6z = 9$

$\therefore$ Direction ratios of both the planes are equal.

$$\therefore 2A + 6B + 6C = 0 \quad \dots(iii)$$

Solving equations (ii) and (iii) by cross-multiplication method, we get

$$\begin{aligned} \frac{A}{6 - 30} &= \frac{B}{10 - 42} = \frac{C}{42 - 2} \\ \Rightarrow \frac{A}{-24} &= \frac{B}{-32} = \frac{C}{40} \end{aligned}$$

$$\Rightarrow \frac{A}{3} = \frac{B}{4} = \frac{C}{-5}$$

$$\therefore A = 3k, B = 4k \text{ and } C = -5k$$

From equation (i),

$$3k(x - 2) + 4k(y - 2) - 5k(z - 1) = 0$$

$$\Rightarrow 3(x - 2) + 4(y - 2) - 5(z - 1) = 0$$

$$\Rightarrow 3x - 6 + 4y - 8 - 5z + 5 = 0$$

$$\Rightarrow 3x + 4y - 5z - 9 = 0$$

$$\Rightarrow 3x + 4y - 5z = 9.$$

Ans.

Answer 11.

Given :

$$\begin{aligned}y &= 6x - x^2 \\x^2 - 6x &= -y \\x^2 - 6x + 9 &= -y + 9 \\(x - 3)^2 &= -(y - 9)\end{aligned}$$

This represent a parabola with vertex (3, 9).

$y = 6x - x^2$ intersect X-axis at (0, 0) and (6, 0).

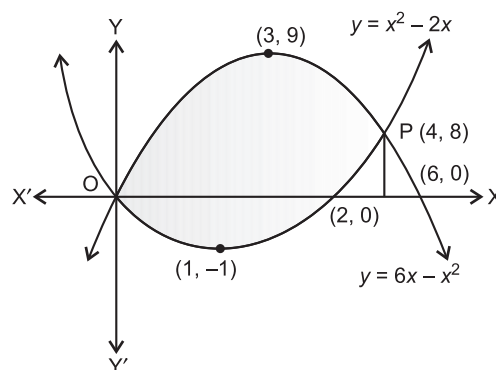
Also,

$$\begin{aligned}y &= x^2 - 2x \\x^2 - 2x + 1 &= y + 1 \\(x - 1)^2 &= y + 1\end{aligned}$$

This represent a parabola with vertex (1, -1).

$y = x^2 - 2x$ intersect the X-axis at (0, 0) and (2, 0).

The point of intersection of two parabolas is (4, 8).



$$\begin{aligned}\text{Required shaded area} &= \int_0^4 (6x - x^2) dx + \int_0^2 (x^2 - 2x) dx - \int_2^4 (x^2 - 2x) dx \\&= \left[3x^2 - \frac{x^3}{3} \right]_0^4 + \left[\frac{x^3}{3} - x^2 \right]_0^2 - \left[\frac{x^3}{3} - x^2 \right]_2^4 \\&= \left[3 \times 16 - \frac{64}{3} \right] + \left[\frac{8}{3} - 4 \right] - \left[\left(\frac{64}{3} - 16 \right) - \left(\frac{8}{3} - 4 \right) \right] \\&= \left[48 - \frac{64}{3} \right] + \left[\frac{8 - 12}{3} \right] - \left[\left(\frac{64 - 48}{3} \right) - \left(\frac{8 - 12}{3} \right) \right] \\&= \left[\frac{144 - 64}{3} - \frac{4}{3} \right] - \left[\frac{16}{3} + \frac{4}{3} \right] = \left[\frac{80}{3} - \frac{4}{3} \right] - \frac{20}{3} = \frac{76}{3} - \frac{20}{3} \\&= \frac{56}{3} \text{ sq. units}\end{aligned}$$

Ans.

Section-C

Answer 12.

(i) (a) - 0.529

Explanation :

Given,

$$b_{xy} = -0.7$$

and regression line y on x is

$$2x + 5y = 9$$

$\Rightarrow$

$$5y = -2x + 9$$

$\Rightarrow$

$$y = \frac{-2}{5}x + \frac{9}{5}$$

$\therefore$

$$b_{yx} = \frac{-2}{5} = -0.4$$

and,

$\rho(x, y)$ = coefficient of correlation between x and y

$$= -\sqrt{b_{xy} \cdot b_{yx}}$$

$$= -\sqrt{(-0.7)(-0.4)}$$

$$\rho(x, y) = -0.529$$

[$\because \rho(x, y), b_{xy}$ and b_{yx} have same sign]

(ii) (c) $\frac{-4}{7}, \frac{-11}{7}$

Explanation :

Given : $4x + 3y + 7 = 0$

and $3x + 4y + 8 = 0$

Solving the two equations, we get

$$\frac{x}{24 - 28} = \frac{y}{21 - 32} = \frac{1}{16 - 9}$$

$$\frac{x}{-4} = \frac{y}{-11} = \frac{1}{7}$$

$$\therefore x = -\frac{4}{7} \text{ and } y = -\frac{11}{7}$$

Hence, mean of x series is $-\frac{4}{7}$ and mean of y series is $-\frac{11}{7}$.

Answer 13.

X	Y	X ²	Y ²	XY
1	2	1	4	2
2	3	4	9	6
3	5	9	25	15
4	6	16	36	24
5	4	25	16	20
$\Sigma X = 15$	$\Sigma Y = 20$	$\Sigma X^2 = 55$	$\Sigma Y^2 = 90$	$\Sigma XY = 67$

Here,

$$N = 5$$

$$\therefore b_{xy} = \frac{\Sigma XY - \frac{1}{N} \Sigma X \Sigma Y}{\Sigma Y^2 - \frac{1}{N} (\Sigma Y)^2}$$

$$\Rightarrow b_{xy} = \frac{67 - \frac{1}{5} \times 15 \times 20}{90 - \frac{1}{5} (20)^2}$$

$$\Rightarrow b_{xy} = \frac{67 - 60}{90 - 80} = \frac{7}{10} = 0.7$$

Ans.

$$\text{and } b_{yx} = \frac{\Sigma XY - \frac{1}{N} \Sigma X \Sigma Y}{\Sigma X^2 - \frac{1}{N} (\Sigma X)^2} = \frac{67 - \frac{1}{5} \times 15 \times 20}{55 - \frac{1}{5} (15)^2} = \frac{67 - 60}{55 - 45}$$

$$\therefore b_{yx} = \frac{7}{10} = 0.7$$

Ans.

Answer 14.

Let the company manufactures x units of product A and y units of product B.

Object : To maximise profit

$$Z = 40x + 50y$$

Constraint :

$$3x + y \leq 9$$

$$x + 2y \leq 8$$

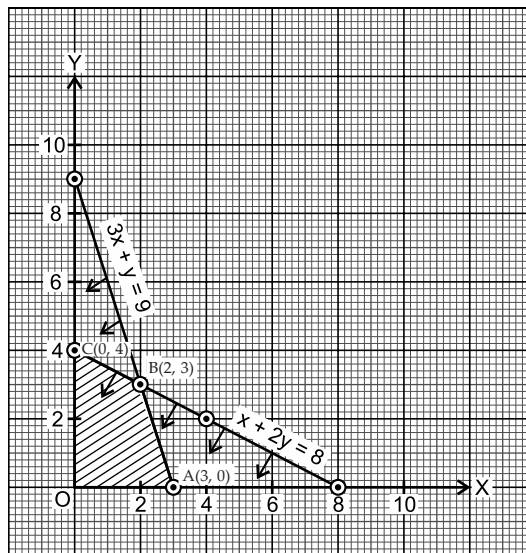
$$x \geq 0, y \geq 0$$

$$3x + y = 9$$

$$x + 2y = 8$$

x	0	3	2
y	9	0	3

x	0	8	4
y	4	0	2



The shaded region is the required feasible region.

Maximum profit $Z = 40x + 50y$

At $A(3, 0)$, $Z = 40 \times 3 + 50 \times 0 = ₹ 120$

At $B(2, 3)$, $Z = 40 \times 2 + 50 \times 3 = ₹ 230$

At $C(0, 4)$, $Z = 40 \times 0 + 50 \times 4 = ₹ 200$

So, the company should manufacture 2 units of product A and 3 units product B to earn maximum profit.

Ans.

□□