

15. Linear Inequations (Including Number Lines)

Exercise 15 (A)

Question 1.

If the replacement set is the set of natural numbers, solve.

(i) $x - 5 < 0$

(ii) $x + 1 < 7$

(iii) $3x - 4 > 6$

(iv) $4x + 1 > 17$

Solution:

(i) $x - 5 < 0$

$$x - 5 + 5 < 0 + 5 \text{(Adding 5)}$$

$$\Rightarrow x < 5$$

$$\text{Required answer} = \{1, 2, 3, 4\}$$

(ii) $x + 1 \leq 7 \Rightarrow x + 1 - 1 \leq 7 - 1$ (Subtracting 1)

$$\Rightarrow x \leq 6$$

$$\text{Required answer} = \{1, 2, 3, 4, 5, 6\}$$

(iii) $3x - 4 > 6$

$$3x - 4 + 4 > 6 + 4 \text{ (Adding 4)}$$

$$\Rightarrow 3x > 10$$

$$\frac{3x}{3} > \frac{10}{3} \text{ ... (Dividing by 3)}$$

$$\Rightarrow x > \frac{10}{3}$$

$$\Rightarrow x > 3\frac{1}{3}$$

$$\text{Required answer} = \{4, 5, 6, \dots\}$$

(iv) $4x + 1 \geq 17$

$$\Rightarrow 4x + 1 - 1 \geq 17 - 1 \text{ (Subtracting)}$$

$$\Rightarrow 4x \geq 16$$

$$\Rightarrow \frac{4x}{4} \geq \frac{16}{4} \text{ (Dividing by 4)}$$

$$\Rightarrow x \geq 4$$

$$\text{Required answer} = \{4, 5, 6, \dots\}$$

Question 2.

If the replacement set = $\{-6, -3, 0, 3, 6, 9\}$; find the truth set of the following:

(i) $2x - 1 > 9$

(ii) $3x + 7 < 1$

Solution:

(i) $2x - 1 > 9$

$$\Rightarrow 2x - 1 + 1 > 9 + 1 \text{ (Adding 1)}$$

$$\Rightarrow 2x > 10$$

$$\Rightarrow x > 5 \text{ (Dividing by 2)}$$

$$\Rightarrow x > 5$$

Required answer = {6, 9}

(ii) $3x + 7 \leq 1$

$\Rightarrow 3x + 7 - 7 \leq 1 - 7$ (Subtracting 7)

$\Rightarrow 3x \leq -6$

$\Rightarrow x \leq -2$

Required Answer = {-6, -3}

Question 3.

Solve $7 > 3x - 8$; $x \in \mathbb{N}$

Solution:

$7 > 3x - 8$

$\Rightarrow 7 - 3x > 3x - 3x - 8$ (Subtracting $3x$)

$\Rightarrow 7 - 7 - 3x > 3x - 3x - 8 - 7$ (Subtracting 7)

$\Rightarrow -3x > -15$

$\Rightarrow x < 5$ (Dividing by -3)

Required Answer = {1, 2, 3, 4}

Note : Division by negative number reverses the inequality.

Question 4.

$-17 < 9y - 8$; $y \in \mathbb{Z}$

Solution:

$-17 < 9y - 8$

$\Rightarrow -17 + 8 < 9y - 8 + 8$ (Adding 8)

$\Rightarrow -9 < 9y$

$\Rightarrow -1 < y$ (Dividing by 9)

Required number = {0, 1, 2, 3, 4, ...}

Question 5.

Solve $9x - 7 \leq 28 + 4x$; $x \in \mathbb{W}$

Solution:

$9x - 7 \leq 28 + 4x$

$\Rightarrow 9x - 4x - 7 \leq 28 + 4x - 4x$ (Subtracting $4x$)

$\Rightarrow 5x - 7 \leq 28$

$\Rightarrow 5x - 7 + 7 \leq 28 + 7$ (Adding 7)

$\Rightarrow 5x \leq 35$

$\Rightarrow x \leq 7$ (Dividing by 5)

Required answer = {0, 1, 2, 3, 4, 5, 6, 7}

Question 6.

Solve : $\frac{2}{3}x + 8 < 12$; $x \in \mathbb{W}$

Solution:

$$\text{Sol } \frac{2}{3}x + 8 < 12$$

$$\Rightarrow \frac{2}{3}x + 8 - 8 < 12 - 8$$

$$\Rightarrow \frac{2}{3}x < 4$$

$$\Rightarrow \frac{2}{3}x \times \frac{3}{2} < 4 \times \frac{3}{2} \quad (\text{Multiplying by } \frac{3}{2})$$

$$\Rightarrow x < 6$$

$$\therefore \text{Required answer} = \{0, 1, 2, 3, 4, 5\}$$

Question 7.

Solve $-5(x + 4) > 30$; $x \in \mathbb{Z}$

Solution:

$$\text{Sol. } -5(x+4) > 30$$

$$\Rightarrow \frac{-5(x+4)}{-5} < \frac{30}{-5} \quad \dots (\text{Dividing by } -5)$$

Note : Division by a negative number reverses the equality

$$\Rightarrow x + 4 < -6$$

$$\Rightarrow x + 4 - 4 < -6 - 4 \quad (\text{Subtracting } 4)$$

$$\Rightarrow x < -10$$

$$\therefore \text{Required Answer} = \{-11, -12, -13, \dots\}$$

Question 8.

Solve the ineqnation $8 - 2x > x - 5$; $x \in \mathbb{N}$.

Solution:

$$\text{Sol. } 8 - 2x \geq x - 5; x \in \mathbb{N}$$

$$\Rightarrow 8 + 5 \geq 2x + x$$

$$\Rightarrow 13 \geq 3x \Rightarrow 3x \leq 13$$

$$\Rightarrow x \leq \frac{13}{3} = 4\frac{1}{3}$$

$$x = 1, 2, 3, 4 \quad (x \in \mathbb{N})$$

$$\text{Solution set} = \{1, 2, 3, 4\}$$

Question 9.

Solve the inequality $18 - 3(2x - 5) > 12$; $x \in W$.

Solution:

$$\text{Sol. } 18 - 3(2x - 5) > 12; x \in W$$

$$\Rightarrow 18 - 6x + 15 > 12 \Rightarrow 33 - 12 > 6x$$

$$\Rightarrow 21 > 6x$$

$$\Rightarrow 6x < 21 \Rightarrow x < \frac{21}{6} = \frac{7}{2} = 3\frac{1}{2}$$

$$\text{But } x \in W, x = 0, 1, 2, 3$$

$$\therefore \text{Solution set} = \{0, 1, 2, 3\}$$

Question 10.

Solve : $\frac{2x+1}{3} + 15 < 17$; $x \in W$.

Solution:

$$\text{Sol. } \frac{2x+1}{3} + 15 \leq 17; x \in W$$

$$\Rightarrow \frac{2x+1}{3} \leq 17 - 15 = 2$$

$$\Rightarrow 2x + 1 \leq 6 \Rightarrow 2x \leq 5$$

$$\Rightarrow x \leq \frac{5}{2} = 2\frac{1}{2}$$

$$\text{But } x \in W$$

$$\therefore x = 0, 1, 2$$

$$\therefore \text{Solution set is} = \{0, 1, 2\}$$

Question 11.

Solve : $-3 + x < 2$, $x \in N$

Solution:

$$\text{Sol. } -3 + x < 2, x \in N$$

$$\Rightarrow x < 2 - (-3)$$

$$\Rightarrow x < 2 + 3$$

$$\Rightarrow x < 5$$

$$\therefore x = 1, 2, 3, 4 \quad (\because x \in N)$$

$$\therefore \text{Solution set} = \{1, 2, 3, 4\}$$

Question 12.

Solve : $4x - 5 > 10 - x$, $x \in \{0, 1, 2, 3, 4, 5, 6, 7\}$

Solution:

Sol. $4x - 5 > 10 - x$, $x \in \mathbb{N}$

$$\Rightarrow 4x + x > 10 + 5$$

$$\Rightarrow 5x > 15$$

$$\Rightarrow x > \frac{15}{5} = 3$$

$$\therefore x = 4, 5, 6, 7$$

Solution set = $\{4, 5, 6, 7\}$

Question 13.

Solve : $15 - 2(2x - 1) < 15$, $x \in \mathbb{Z}$.

Solution:

Sol. $15 - 2(2x - 1) < 15$, $x \in \mathbb{Z}$

$$\Rightarrow 15 - 4x + 2 < 15$$

$$\Rightarrow 17 - 4x < 15$$

$$\Rightarrow -4x < 15 - 17$$

$$\Rightarrow -4x < -2$$

$$\Rightarrow \frac{-4}{-4}x > \frac{-2}{-4} = \frac{1}{2} \quad (\text{Dividing by } -4)$$

$$\therefore x = 1, 2, 3, 4, 5, \dots$$

$$\therefore \text{Solution set} = \{1, 2, 3, 4, 5, \dots\}$$

Question 14.

Solve : $\frac{2x+3}{5} > \frac{4x-1}{2}$, $x \in \mathbb{W}$.

Solution:

Sol. $\frac{2x+3}{5} > \frac{4x-1}{2}$, $x \in \mathbb{W}$

$$\Rightarrow 2(2x + 3) > 5(4x - 1)$$

$$\Rightarrow 4x + 6 > 20x - 5$$

$$\Rightarrow 4x - 20x > -5 - 6$$

$$\Rightarrow -16x > -11$$

$$\Rightarrow x < \frac{-11}{-16} \quad (\text{Dividing by } -16)$$

$$\Rightarrow x < \frac{11}{16}$$

$$\therefore x = 0$$

$$\therefore \text{Solution set} = \{0\}$$

Exercise 15 (B)

Solve and graph the solution set on a number line :

Question 1.

$$x - 5 < -2 ; x \in \mathbb{N}$$

Solution:

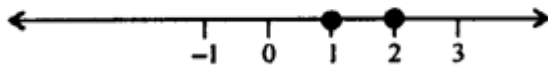
Sol. $x - 5 < -2$

$$\Rightarrow x - 5 + 5 < -2 + 5$$

(Adding 5 to both sides)

$$\Rightarrow x < 3$$

$\therefore$ The required graph is :



Question 2.

$$3x - 1 > 5 ; x \in \mathbb{W}$$

Solution:

Sol. $3x - 1 > 5$

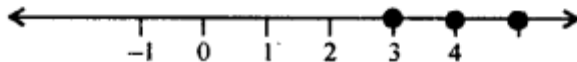
$$\Rightarrow 3x - 1 + 1 > 5 + 1 \quad (\text{Adding 1 to both sides})$$

$$\Rightarrow 3x > 6$$

$$\Rightarrow \frac{3x}{3} > \frac{6}{3} \quad (\text{Dividing both sides by 3})$$

$$\Rightarrow x > 2$$

$\therefore$ The required graph is :



Question 3.

$$-3x + 12 < -15 ; x \in \mathbb{R}$$

Sol. $-3x + 12 < -15$

$$\Rightarrow -3x + 12 - 12 < -15 - 12$$

(Subtracting 12 from both sides)

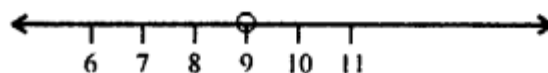
$$\Rightarrow -3x < -27$$

$$\Rightarrow \frac{-3x}{-3} > \frac{-27}{-3} \quad (\text{Dividing both sides by } -3)$$

Note : Division by a negative number reverses the inequality.

$$\Rightarrow x > 9$$

$\therefore$ The required graph is :



Question 4.

$$7 > 3x - 8 ; x \in W$$

Solution:

$$\text{Sol. } 7 \geq 3x - 8$$

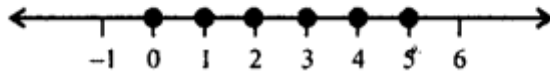
$$\Rightarrow 7 + 8 \geq 3x - 8 + 8 \quad (\text{Adding } 8 \text{ to both sides})$$

$$\Rightarrow 15 \geq 3x$$

$$\Rightarrow \frac{15}{3} \geq \frac{3x}{3} \quad (\text{Dividing both sides by } 3)$$

$$\Rightarrow 5 \geq x$$

$\therefore$ The required graph is :

**Question 5.**

$$8x - 8 < -24 ; x \in Z$$

Solution:

$$\text{Sol. } 8x - 8 \leq -24$$

$$\Rightarrow 8x - 8 + 8 \leq -24 + 8$$

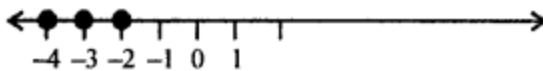
(Adding 8 to both sides)

$$\Rightarrow 8x \leq -16$$

$$\Rightarrow \frac{8x}{8} \leq \frac{-16}{8} \quad (\text{Dividing both sides by } 8)$$

$$\Rightarrow x \leq -2$$

$\therefore$ The required graph is :

**Question 6.**

$$8x - 9 > 35 - 3x ; x \in N$$

Solution:

$$\text{Sol. } 8x - 9 \geq 35 - 3x$$

$$\Rightarrow 8x + 3x - 9 \geq 35 - 3x + 3x$$

(Adding $3x$ to both sides)

$$\Rightarrow 11x - 9 \geq 35$$

$$\Rightarrow 11x - 9 + 9 \geq 35 + 9$$

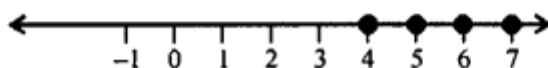
(Adding 9 to both sides)

$$\Rightarrow 11x \geq 44$$

$$\Rightarrow \frac{11x}{11} \geq \frac{44}{11} \quad (\text{Dividing both sides by } 11)$$

$$\Rightarrow x \geq 4$$

$\therefore$ The required graph is :



Question 7.

$$5x + 4 > 8x - 11 ; x \in \mathbb{Z}$$

Solution:

$$\text{Sol. } 5x + 4 > 8x - 11$$

$$\Rightarrow 5x - 5x + 4 > 8x - 5x - 11$$

(Subtracting $5x$ from both sides)

$$\Rightarrow 4 > 3x - 11$$

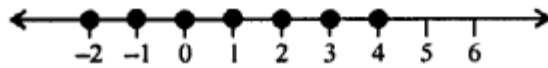
$$\Rightarrow 4 + 11 > 3x - 11 + 11$$

(Adding 11 to both sides)

$$\Rightarrow 15 > 3x$$

$$\Rightarrow \frac{15}{3} > \frac{3x}{3} \quad (\text{Dividing both sides by 3})$$

$$\Rightarrow 5 > x$$

 $\therefore$ The required graph is**Question 8.**

$$\frac{2x}{5} + 1 < -3 ; x \in \mathbb{R}$$

Solution:

$$\text{Sol. } \frac{2x}{5} + 1 < -3$$

$$\Rightarrow \frac{2x}{5} + 1 - 1 < -3 - 1$$

(Subtracting 1 from both sides)

$$\Rightarrow \frac{2x}{5} < -4$$

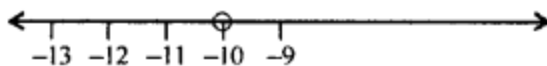
$$\Rightarrow \frac{2x}{5} \times 5 < -4 \times 5$$

(Multiplying both sides by 5)

$$\Rightarrow 2x < -20$$

$$\Rightarrow \frac{2x}{2} < \frac{-20}{2} \quad (\text{Dividing both sides by 2})$$

$$\Rightarrow x < -10$$

 $\therefore$ The required graph is :

Question 9.

$$\frac{x}{2} > -1 + \frac{3x}{4}; x \in \mathbb{N}$$

Solution:

$$\text{Sol. } \frac{x}{2} > -1 + \frac{3x}{4}$$

$$\Rightarrow \frac{x}{2} \times 4 > -1 \times 4 + \frac{3x}{4} \times 4$$

(Multiplying both sides by 4)

$$\Rightarrow 2x > -4 + 3x$$

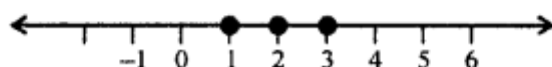
$$\Rightarrow 2x - 2x > -4 + 3x - 2x$$

(Subtracting $2x$ from both sides)

$$\Rightarrow 0 > -4 + x$$

$$\Rightarrow 0 + 4 > -4 + 4 + x \quad (\text{Adding 4 to both sides})$$

$$\Rightarrow 4 > x$$

 $\therefore$ The required graph is :**Question 10.**

$$\frac{2}{3}x + 5 \leq \frac{1}{2}x + 6; x \in \mathbb{W}$$

$$\text{Sol. } \frac{2}{3}x + 5 \leq \frac{1}{2}x + 6$$

$$\Rightarrow \frac{2}{3}x \times 6 + 5 \times 6 \leq \frac{1}{2}x \times 6 + 6 \times 6$$

(Multiplying both sides by 6)

$$\Rightarrow 4x + 30 \leq 3x + 36$$

$$\Rightarrow 4x - 3x + 30 \leq 3x - 3x + 36$$

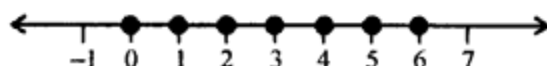
(Subtracting $3x$ from both sides)

$$\Rightarrow x + 30 \leq 36$$

$$\Rightarrow x + 30 - 30 \leq 36 - 30$$

(Subtracting 30 from both sides)

$$x \leq 6$$

 $\therefore$ The required graph is :

Question 11.

Solve the inequation $5(x - 2) > 4(x + 3) - 24$ and represent its solution on a number line.

Given the replacement set is $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$.

Solution:

Sol. $5(x - 2) > 4(x + 3) - 24$

$$\Rightarrow 5x - 10 > 4x + 12 - 24$$

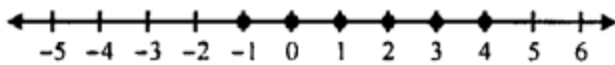
$$\Rightarrow 5x - 4x > 10 - 12$$

$$\Rightarrow x > -2$$

Since replacement set = $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$\therefore$ Solution set = $\{-1, 0, 1, 2, 3, 4\}$

$\therefore$ Solution set on a number line is shown below.

**Question 12.**

Solve $\frac{2}{3}(x - 1) + 4 < 10$ and represent its solution on a number line.

Given replacement set is $\{-8, -6, -4, 3, 6, 8, 12\}$.

Solution:

Sol. $\frac{2}{3}(x - 1) + 4 < 10$

$$\Rightarrow \frac{2}{3}(x - 1) < 10 - 4$$

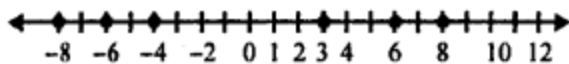
$$\Rightarrow \frac{2}{3}(x - 1) < 6$$

$$\Rightarrow 2(x - 1) < 18$$

$$\Rightarrow x - 1 < 9$$

$$\Rightarrow x - 1 + 1 < 9 + 1 \text{ (Adding 1 to both sides)}$$

$$\Rightarrow x < 10$$



Thus $x < 10$

Since, replacement set = $\{-8, -6, -4, 3, 6, 8, 12\}$

$\Rightarrow$ Solution set = $\{-8, -6, -4, 3, 6, 8\}$

Question 13.

For each inequation, given below, represent the solution on a number line :

(i) $\frac{5}{2} - 2x \geq \frac{1}{2}$; $x \in W$

(ii) $3(2x - 1) \geq 2(2x + 3)$, $x \in Z$

(iii) $2(4 - 3x) \leq 4(x - 5)$, $x \in W$

(iv) $4(3x + 1) > 2(4x - 1)$, x is a negative integer

(v) $\frac{4-x}{2} < 3$, $x \in R$

(vi) $-2(x + 8) \leq 8$, $x \in R$

Solution:

(i) $\frac{5}{2} - 2x \geq \frac{1}{2}$, $x \in W$

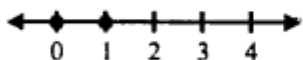
$$\Rightarrow -2x \geq \frac{1}{2} - \frac{5}{2}$$

$$\Rightarrow -2x \geq \frac{-4}{2}$$

$$\Rightarrow -2x > -2$$

$$\Rightarrow x \leq 1$$

$$\therefore x = \{0, 1\}$$



(ii) $3(2x - 1) \geq 2(2x + 3)$, $x \in Z$

$$\Rightarrow 6x - 3 \geq 4x + 6, x \in Z$$

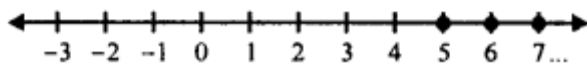
$$\Rightarrow 6x - 4x \geq 6 + 3$$

$$\Rightarrow 2x \geq 9$$

$$\Rightarrow x \geq \frac{9}{2}$$

$$\Rightarrow x \geq 4\frac{1}{2}$$

$$\therefore x = \{5, 6, 7, \dots\}$$



(iii) $2(4 - 3x) \leq 4(x - 5)$, $x \in W$

$$8 - 6x \leq 4x - 20$$

$$\Rightarrow -6x - 4x \leq -20 - 8$$

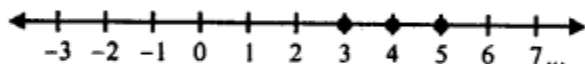
$$\Rightarrow -10x \leq -28$$

$$\Rightarrow 10x \geq 28$$

$$\Rightarrow x \geq \frac{28}{10}$$

$$\Rightarrow x \geq 2.8$$

$$\therefore x = \{3, 4, 5, \dots\}$$



(iv) $4(3x + 1) > 2(4x - 1)$, x is a negative integer

$$\Rightarrow 12x + 4 > 8x - 2$$

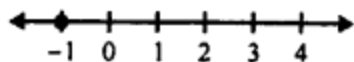
$$\Rightarrow 12x - 8x > -2 - 4$$

$$\Rightarrow 4x > -6$$

$$\Rightarrow x > \frac{-6}{4}$$

$$\Rightarrow x > -1.5$$

$$\therefore x = \{-1\}$$



(v) $\frac{4-x}{2} < 3$, $x \in \mathbb{R}$

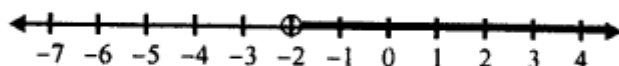
$$\Rightarrow 4 - x < 6$$

$$\Rightarrow -x < 6 - 4$$

$$\Rightarrow -x < 2$$

$$\Rightarrow x > -2$$

$$\therefore x > -2$$



(vi) $-2(x + 8) \leq 8$, $x \in \mathbb{R}$

$$-2x - 16 \leq 8$$

$$\Rightarrow -2x \leq 8 + 16$$

$$\Rightarrow -2x \leq 24$$

$$\Rightarrow x \geq \frac{-24}{2}$$

$$x \geq -12$$

