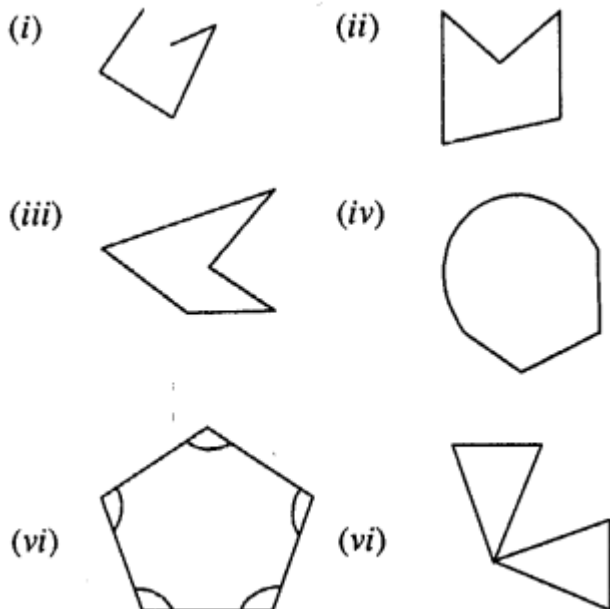


16. Understanding Shapes (Including Polygons)

EXERCISE 16(A)

Question 1.

State which of the following are polygons :



If the given figure is a polygon, name it as convex or concave.

Solution:

Only Fig. (ii), (iii) and (v) are polygons.

Fig. (ii) and (iii) are concave polygons while Fig. (v) is convex.

Question 2.

Calculate the sum of angles of a polygon with :

- (i) 10 sides
- (ii) 12 sides
- (iii) 20 sides
- (iv) 25 sides

Solution:

(i) No. of sides $n = 10$

$$\begin{aligned}\text{sum of angles of polygon} &= (n - 2) \times 180^\circ \\ &= (10 - 2) \times 180^\circ = 1440^\circ\end{aligned}$$

(ii) no. of sides $n = 12$

$$\begin{aligned}\text{sum of angles} &= (n - 2) \times 180^\circ \\ &= (12 - 2) \times 180^\circ = 10 \times 180^\circ = 1800^\circ\end{aligned}$$

(iii) $n = 20$

Sum of angles of Polygon = $(n - 2) \times 180^\circ$

= $(20 - 2) \times 180^\circ = 3240^\circ$

(iv) $n = 25$

Sum of angles of polygon = $(n - 2) \times 180^\circ$

= $(25 - 2) \times 180^\circ = 4140^\circ$

Question 3.

Find the number of sides in a polygon if the sum of its interior angles is :

(i) 900°

(ii) 1620°

(iii) 16 right-angles

(iv) 32 right-angles.

Solution:

(i) Let no. of sides = n

Sum of angles of polygon = 900°

$(n - 2) \times 180^\circ = 900^\circ$

$n - 2 = \frac{900}{180}$

$n - 2 = 5$

$n = 5 + 2$

$n = 7$

(ii) Let no. of sides = n

Sum of angles of polygon = 1620°

$(n - 2) \times 180^\circ = 1620^\circ$

$n - 2 = \frac{1620}{180}$

$n - 2 = 9$

$n = 9 + 2$

$n = 11$

(iii) Let no. of sides = n

Sum of angles of polygon = 16 right angles = $16 \times 90 = 1440^\circ$

$(n - 2) \times 180^\circ = 1440^\circ$

$n - 2 = \frac{1440}{180}$

$n - 2 = 8$

$n = 8 + 2$

$n = 10$

(iv) Let no. of sides = n

Sum of angles of polygon = 32 right angles = $32 \times 90 = 2880^\circ$

$(n - 2) \times 180^\circ = 2880^\circ$

$n - 2 = \frac{2880}{180}$

$n - 2 = 16$

$n = 16 + 2$

$n = 18$

Question 4.

Is it possible to have a polygon ; whose sum of interior angles is :

- (i) 870°
- (ii) 2340°
- (iii) 7 right-angles
- (iv) 4500°

Solution:

(i) Let no. of sides = n

Sum of angles = 870°

$$(n - 2) \times 180^\circ = 870^\circ$$

$$n - 2 = \frac{870}{180}$$

$$n - 2 = \frac{29}{6}$$

$$n = \frac{29}{6} + 2$$

$$n = \frac{41}{6}$$

Which is not a whole number.

Hence it is not possible to have a polygon, the sum of whose interior angles is 870°

(ii) Let no. of sides = n

Sum of angles = 2340°

$$(n - 2) \times 180^\circ = 2340^\circ$$

$$n - 2 = \frac{2340}{180}$$

$$n - 2 = 13$$

$$n = 13 + 2 = 15$$

Which is a whole number.

Hence it is possible to have a polygon, the sum of whose interior angles is 2340° .

(iii) Let no. of sides = n

Sum of angles = 7 right angles = $7 \times 90 = 630^\circ$

$$(n - 2) \times 180^\circ = 630^\circ$$

$$n - 2 = \frac{630}{180}$$

$$n - 2 = \frac{7}{2}$$

$$n = \frac{7}{2} + 2$$

$$n = \frac{11}{2}$$

Which is not a whole number. Hence it is not possible to have a polygon, the sum of whose interior angles is 7 right-angles.

(iv) Let no. of sides = n

$$(n - 2) \times 180^\circ = 4500^\circ$$

$$n - 2 = \frac{4500}{180}$$

$$n - 2 = 25$$

$$n = 25 + 2$$

$$n = 27$$

Which is a whole number.

Hence it is possible to have a polygon, the sum of whose interior angles is 4500° .

Question 5.

- (i) If all the angles of a hexagon are equal ; find the measure of each angle.
 (ii) If all the angles of a 14-sided figure are equal ; find the measure of each angle.

Solution:

(i) No. of sides of hexagon, $n = 6$

Let each angle be $= x^\circ$

Sum of angles $= 6x^\circ$

$(n - 2) \times 180^\circ = \text{Sum of angles}$

$(6 - 2) \times 180^\circ = 6x^\circ$

$4 \times 180 = 6x$

$$x = \frac{4 \times 180}{6}$$

$$x = 120^\circ$$

$\therefore$ Each angle of hexagon $= 120^\circ$ Ans.

(ii) No. of sides of polygon, $n = 14$

Let each angle $= x^\circ$

$\therefore$ Sum of angles $= 14x^\circ$

$\therefore (n-2) \times 180^\circ = \text{Sum of angles of polygon}$

$\therefore (14-2) \times 180^\circ = 14x$

$12 \times 180^\circ = 14x$

$$x = \frac{12 \times 180}{14}$$

$$x = \frac{1080}{7}$$

$$x = \left(154\frac{2}{7}\right)^\circ \text{ Ans.}$$

Question 6.

Find the sum of exterior angles obtained on producing, in order, the sides of a polygon with :

- (i) 7 sides
 (ii) 10 sides
 (iii) 250 sides.

Solution:

(i) No. of sides $n = 7$

Sum of interior & exterior angles at one vertex $= 180^\circ$

$$\begin{aligned}\text{Sum of all interior \& exterior angles} &= 7 \times 180^\circ \\ &= 1260^\circ\end{aligned}$$

$$\begin{aligned}\text{Sum of interior angles} &= (n-2) \times 180^\circ \\ &= (7-2) \times 180^\circ \\ &= 900^\circ\end{aligned}$$

$$\begin{aligned}\therefore \text{Sum of exterior angles} &= 1260^\circ - 900^\circ \\ &= 360^\circ \text{ Ans.}\end{aligned}$$

(ii) No. of sides $n = 10$

$$\begin{aligned}\text{Sum of interior and exterior angles} &= 10 \times 180^\circ \\ &= 1800^\circ\end{aligned}$$

$$\begin{aligned}\text{But sum of interior angles} &= (n-2) \times 180^\circ \\ &= (10-2) \times 180^\circ \\ &= 1440^\circ\end{aligned}$$

$$\begin{aligned}\therefore \text{Sum of exterior angles} &= 1800 - 1440 \\ &= 360^\circ \text{ Ans.}\end{aligned}$$

(iii) No. of side $n = 250$

$$\begin{aligned}\text{Sum of all interior and exterior angles} &= 250 \times 180^\circ \\ &= 45000^\circ\end{aligned}$$

$$\begin{aligned}\text{But sum of interior angles} &= (n-2) \times 180^\circ \\ &= (250-2) \times 180^\circ \\ &= 248 \times 180^\circ \\ &= 44640^\circ\end{aligned}$$

$$\begin{aligned}\therefore \text{Sum of exterior angles} &= 45000 - 44640 \\ &= 360^\circ\end{aligned}$$

Question 7.

The sides of a hexagon are produced in order. If the measures of exterior angles so obtained are $(6x - 1)^\circ$, $(10x + 2)^\circ$, $(8x + 2)^\circ$, $(9x - 3)^\circ$, $(5x + 4)^\circ$ and $(12x + 6)^\circ$; find each exterior angle.

Solution:

Sum of exterior angles of hexagon formed by producing sides of order = 360°

$$\therefore (6x-1)^\circ + (10x+2)^\circ + (8x+2)^\circ + (9x-3)^\circ \\ + (5x+4)^\circ + (12x+6)^\circ = 360^\circ$$

$$50x + 10^\circ = 360^\circ$$

$$50x = 360^\circ - 10^\circ$$

$$50x = 350^\circ$$

$$x = \frac{350}{50}$$

$$x = 7$$

$\therefore$ Angles are

$$(6x-1)^\circ ; (10x+2)^\circ ; (8x+2)^\circ ; (9x-3)^\circ ; \\ (5x+4)^\circ \text{ and } (12x+6)^\circ$$

$$\text{i.e. } (6 \times 7 - 1)^\circ ; (10 \times 7 + 2)^\circ ; (8 \times 7 + 2)^\circ ; \\ (9 \times 7 - 3)^\circ ; (5 \times 7 + 4)^\circ ; (12 \times 7 + 6)^\circ$$

i.e. $41^\circ ; 72^\circ, 58^\circ ; 60^\circ ; 39^\circ$ and 90°

Question 8.

The interior angles of a pentagon are in the ratio 4 : 5 : 6 : 7 : 5. Find each angle of the pentagon.

Solution:

Let the interior angles of the pentagon be $4x, 5x, 6x, 7x, 5x$.

Their sum = $4x + 5x + 6x + 7x + 5x = 21x$

Sum of interior angles of a polygon = $(n - 2) \times$

$$180^\circ = (5 - 2) \times 180^\circ = 540^\circ$$

$$\therefore 21x = 540 \Rightarrow x = \frac{540}{21} \Rightarrow x = 20^\circ$$

$$\therefore \text{Angles are } 4 \times 20^\circ = 80^\circ$$

$$5 \times 20^\circ = 100^\circ$$

$$6 \times 20^\circ = 120^\circ$$

$$7 \times 20^\circ = 140^\circ$$

$$5 \times 20 = 100^\circ$$

Question 9.

Two angles of a hexagon are 120° and 160° . If the remaining four angles are equal, find each equal angle.

Solution:

Two angles of a hexagon are $120^\circ, 160^\circ$

Let remaining four angles be x , x , x and x .

Their sum $= 4x + 280^\circ$

But sum of all the interior angles of a hexagon

$$= (6 - 2) \times 180^\circ$$

$$= 4 \times 180^\circ = 720^\circ$$

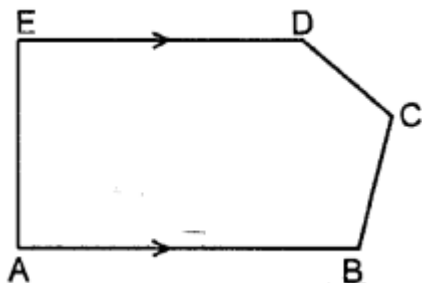
$$\therefore 4x + 280^\circ = 720^\circ$$

$$\Rightarrow 4x = 720^\circ - 280^\circ = 440^\circ \Rightarrow x = 110^\circ$$

$\therefore$ Equal angles are 110° (each)

Question 10.

The figure, given below, shows a pentagon ABCDE with sides AB and ED parallel to each other, and $\angle B : \angle C : \angle D = 5 : 6 : 7$.



(i) Using formula, find the sum of interior angles of the pentagon.

(ii) Write the value of $\angle A + \angle E$

(iii) Find angles B, C and D.

Solution:

(i) Sum of interior angles of the pentagon

$$= (5 - 2) \times 180^\circ$$

$$= 3 \times 180^\circ = 540^\circ \quad [\because \text{sum for a polygon of } x \text{ sides} = (x - 2) \times 180^\circ]$$

(ii) Since $AB \parallel ED$

$$\therefore \angle A + \angle E = 180^\circ$$

(iii) Let $\angle B = 5x$ $\angle C = 6x$ $\angle D = 7x$

$$\therefore 5x + 6x + 7x + 180^\circ = 540^\circ$$

$$(\angle A + \angle E = 180^\circ)$$

Proved in (ii)

$$18x = 540^\circ - 180^\circ$$

$$\Rightarrow 18x = 360^\circ \Rightarrow x = 20^\circ$$

$$\therefore \angle B = 5 \times 20^\circ = 100^\circ, \angle C = 6 \times 20 = 120^\circ$$

$$\angle D = 7 \times 20 = 140^\circ$$

Question 11.

Two angles of a polygon are right angles and the remaining are 120° each. Find the number of sides in it.

Solution:

Let number of sides = n

$$\begin{aligned}\text{Sum of interior angles} &= (n-2) \times 180^\circ \\ &= 180n - 360^\circ\end{aligned}$$

$$\begin{aligned}\text{Sum of 2 right angles} &= 2 \times 90^\circ \\ &= 180^\circ\end{aligned}$$

$$\begin{aligned}\therefore \text{Sum of other angles} &= 180n - 360^\circ - 180^\circ \\ &= 180n - 540\end{aligned}$$

$$\begin{aligned}\text{No. of vertices at which these angles are formed} \\ &= n - 2\end{aligned}$$

$$\therefore \text{Each interior angle} = \frac{180n - 540}{n - 2}$$

$$\therefore \frac{180n - 540}{n - 2} = 120^\circ$$

$$180n - 540 = 120n - 240$$

$$180n - 120n = -240 + 540$$

$$60n = 300$$

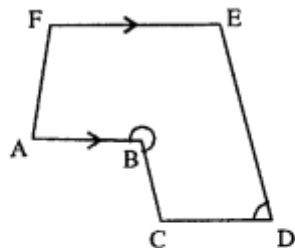
$$n = \frac{300}{60}$$

$$n = 5$$

Question 12.

In a hexagon ABCDEF, side AB is parallel to side FE and $\angle B : \angle C : \angle D : \angle E = 6 : 4 : 2 : 3$. Find $\angle B$ and $\angle D$.

Solution:



Given : Hexagon ABCDEF in which $AB \parallel EF$ and $\angle B : \angle C : \angle D : \angle E = 6 : 4 : 2 : 3$.

To find : $\angle B$ and $\angle D$

Proof : No. of sides $n = 6$

$$\therefore \text{Sum of interior angles} = (n-2) \times 180^\circ \\ = (6-2) \times 180^\circ = 720^\circ$$

$\because AB \parallel EF$ (Given)

$$\therefore \angle A + \angle F = 180^\circ$$

$$\text{But } \angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 720^\circ \\ \text{(Proved)}$$

$$\angle B + \angle C + \angle D + \angle E + 180^\circ = 720^\circ$$

$$\therefore \angle B + \angle C + \angle D + \angle E = 720^\circ - 180^\circ = 540^\circ$$

$$\text{Ratio} = 6 : 4 : 2 : 3$$

$$\text{Sum of parts} = 6 + 4 + 2 + 3 = 15$$

$$\therefore \angle B = \frac{6}{15} \times 540 = 216^\circ$$

$$\angle D = \frac{2}{15} \times 540 = 72^\circ$$

Hence $\angle B = 216^\circ$; $\angle D = 72^\circ$ **Ans.**

Question 13.

the angles of a hexagon are $x + 10^\circ$, $2x + 20^\circ$, $2x - 20^\circ$, $3x - 50^\circ$, $x + 40^\circ$ and $x + 20^\circ$. Find x .

Solution:

Sol. Angles of a hexagon are $x + 10^\circ$, $2x + 20^\circ$, $2x - 20^\circ$, $3x - 50^\circ$, $x + 40^\circ$ and $x + 20^\circ$

$\therefore$ But sum of angles of a hexagon $= (x - 2) \times 180^\circ$
 $= (6 - 2) \times 180^\circ = 4 \times 180^\circ = 720^\circ$

But sum $= x + 10 + 2x + 20^\circ + 2x - 20^\circ + 3x - 50^\circ + x + 40 + x + 20$
 $= 10x + 90 - 70 = 10x + 20$

$\therefore 10x + 20 = 720^\circ \Rightarrow 10x = 720 - 20 = 700$

$\Rightarrow x = \frac{700^\circ}{10} = 70^\circ$

$\therefore x = 70^\circ$

Question 14.

In a pentagon, two angles are 40° and 60° , and the rest are in the ratio 1 : 3 : 7. Find the biggest angle of the pentagon.

Solution:

In a pentagon, two angles are 40° and 60° Sum of remaining 3 angles $= 3 \times 180^\circ$
 $= 540^\circ - 40^\circ - 60^\circ = 540^\circ - 100^\circ = 440^\circ$

Ratio in these 3 angles $= 1 : 3 : 7$

Sum of ratios $= 1 + 3 + 7 = 11$

Biggest angle $= \frac{440 \times 7}{11} = 280^\circ$

EXERCISE 16(B)

Question 1.

Fill in the blanks :

In case of regular polygon, with :

no. of sides	each exterior angle	each interior angle
(i)8....		
(ii)12....		
(iii)	72°....	
(iv)	45°....	
(v)		150°....
(vi)		140°....

Solution:

no. of sides	each exterior angle	each interior angle
(i) 8	45°	135°
(ii) 12	30°	150°
(iii) 5	72°	108°
(iv) 8	45°	135°
(v) 12	30°	150°
(vi) 9	40°	140°

Explanation

(i) Each exterior angle = $\frac{360^\circ}{8} = 45^\circ$

Each interior angle = $180^\circ - 45^\circ = 135^\circ$

(ii) Each exterior angle = $\frac{360^\circ}{12} = 30^\circ$

Each interior angle = $180^\circ - 30^\circ = 150^\circ$

(iii) Since each exterior = 72°

$\therefore$ Number of sides = $\frac{360^\circ}{72^\circ} = 5$

Also interior angle = $180^\circ - 72^\circ = 108^\circ$

(iv) Since each exterior angle = 45°

$\therefore$ Number of sides = $\frac{360^\circ}{45^\circ} = 8$

Interior angle = $180^\circ - 45^\circ = 135^\circ$

(v) Since interior angle = 150°

$\therefore$ Exterior angle = $180^\circ - 150^\circ = 30^\circ$

$\therefore$ Number of sides = $\frac{360^\circ}{30^\circ} = 12$

(vi) Since interior angle = 140°

$\therefore$ Exterior angle = $180^\circ - 140^\circ = 40^\circ$

$\therefore$ Number of sides = $\frac{360^\circ}{40^\circ} = 9$

Question 2.

Find the number of sides in a regular polygon, if its each interior angle is :

(i) 160°

(ii) 135°

(iii) $1\frac{1}{5}$ of a right-angle

Solution:

(i) Let no. of sides of regular polygon be n .

Each interior angle = 160°

$$\therefore \frac{(n-2)}{n} \times 180^\circ = 160^\circ$$

$$180n - 360^\circ = 160n$$

$$180n - 160n = 360^\circ$$

$$20n = 360^\circ$$

$$n = 18 \text{ Ans.}$$

(ii) No. of sides = n

Each interior angle = 135°

$$\frac{(n-2)}{n} \times 180^\circ = 135^\circ$$

$$180n - 360^\circ = 135n$$

$$180n - 135n = 360^\circ$$

$$45n = 360^\circ$$

$$n = 8 \text{ Ans.}$$

(iii) No. of sides = n

Each interior angle = $1\frac{1}{5}$ right angles

$$= \frac{6}{5} \times 90$$

$$= 108^\circ$$

$$\therefore \frac{(n-2)}{n} \times 180^\circ = 108^\circ$$

$$180n - 360^\circ = 108n$$

$$180n - 108n = 360^\circ$$

$$72n = 360^\circ$$

$$n = 5 \text{ Ans.}$$

Question 3.

Find the number of sides in a regular polygon, if its each exterior angle is :

- (i) $\frac{1}{3}$ of a right angle
- (ii) two-fifth of a right-angle.

Solution:

- (i) Each exterior angle = $\frac{1}{3}$ of a right angle
 $= \frac{1}{3} \times 90$

$$\therefore \frac{360^\circ}{n} = 30^\circ$$

$$\therefore n = \frac{360^\circ}{30^\circ}$$

$$n = 12 \text{ Ans.}$$

- (ii) Each exterior angle = $\frac{2}{5}$ of a right-angle

$$= \frac{2}{5} \times 90^\circ$$

$$= 36^\circ$$

Let number of sides = n

$$\therefore \frac{360^\circ}{n} = 36^\circ$$

$$n = \frac{360^\circ}{36^\circ}$$

$$n = 10 \text{ Ans.}$$

Question 4.

Is it possible to have a regular polygon whose each interior angle is :

- (i) 170°
- (ii) 138°

Solution:

- (i) No. of sides = n
each interior angle = 170°

$$\begin{aligned}\therefore \quad \frac{(n-2)}{n} \times 180^\circ &= 170^\circ \\ 180n - 360^\circ &= 170n \\ 180n - 170n &= 360^\circ \\ 10n &= 360^\circ \\ n &= \frac{360^\circ}{10} \\ n &= 36\end{aligned}$$

which is a whole number.

Hence it is possible to have a regular polygon whose interior angle is 170° .

(ii) Let no. of sides = n
each interior angle = 138°

$$\begin{aligned}\therefore \quad \frac{(n-2)}{n} \times 180^\circ &= 138^\circ \\ 180n - 360^\circ &= 138n \\ 180n - 138n &= 360^\circ \\ 42n &= 360^\circ \\ n &= \frac{360^\circ}{42} \\ n &= \frac{60^\circ}{7}\end{aligned}$$

Which is not a whole number.

Hence it is not possible to have a regular polygon having interior angle of 138° .

Question 5.

Is it possible to have a regular polygon whose each exterior angle is :

- (i) 80°
- (ii) 40% of a right angle.

Solution:

(i) Let no. of sides = n each exterior angle = 80°

$$\frac{360^\circ}{n} = 80^\circ$$

$$n = \frac{360^\circ}{80^\circ}$$

$$n = \frac{9}{2}$$

Which is not a whole number.

Hence it is not possible to have a regular polygon whose each exterior angle is of 80° .

(ii) Let number of sides = n

Each exterior angle = 40% of a right angle

$$= \frac{40}{100} \times 90$$

$$= 36^\circ$$

$$n = \frac{360^\circ}{36^\circ}$$

$$n = 10$$

Which is a whole number.

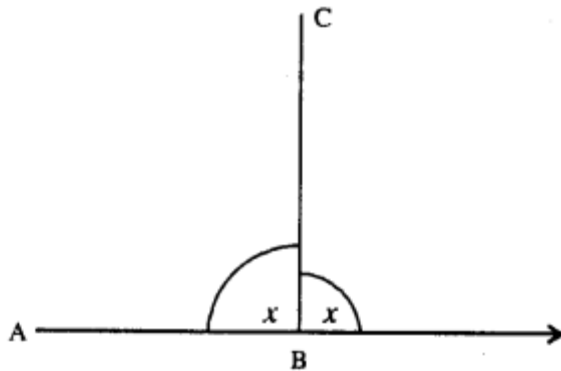
Hence it is possible to have a regular polygon whose each exterior angle is 40% of a right angle.

Question 6.

Find the number of sides in a regular polygon, if its interior angle is equal to its exterior angle.

Solution:

Let each exterior angle or interior angle be $= x^\circ$



$$\therefore x + x = 180^\circ$$

$$2x = 180^\circ$$

$$x = 90^\circ$$

Now, let no. of sides = n

$$\therefore \text{each exterior angle} = \frac{360^\circ}{n}$$

$$\therefore 90^\circ = \frac{360^\circ}{n}$$

$$n = \frac{360^\circ}{90^\circ}$$

$$n = 4 \quad \text{Ans.}$$

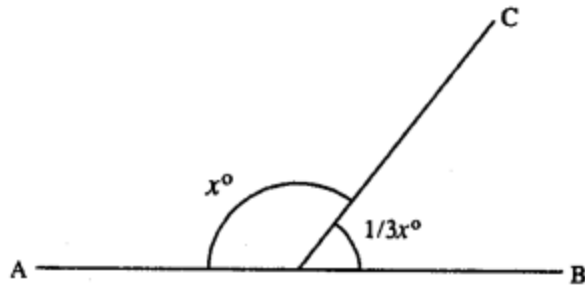
Question 7.

The exterior angle of a regular polygon is one-third of its interior angle. Find the number of sides in the polygon.

Solution:

Let interior angle = x°

Exterior angle = $\frac{1}{3}x^\circ$



$$\therefore x + \frac{1}{3}x = 180^\circ$$

$$3x + x = 540$$

$$4x = 540$$

$$x = \frac{540}{4}$$

$$x = 135^\circ$$

$$\therefore \text{Exterior angle} = \frac{1}{3} \times 135^\circ$$
$$= 45^\circ$$

Let no. of sides = n

$$\therefore \text{each exterior angle} = \frac{360^\circ}{n}$$

$$\therefore 45^\circ = \frac{360^\circ}{n}$$

$$\therefore n = \frac{360^\circ}{45^\circ}$$

$$n = 8 \text{ Ans.}$$

Question 8.

The measure of each interior angle of a regular polygon is five times the measure of its exterior angle. Find :

- (i) measure of each interior angle ;
- (ii) measure of each exterior angle and
- (iii) number of sides in the polygon.

Solution:

Let exterior angle = x°

Interior angle = $5x^\circ$

$$x + 5x = 180^\circ$$

$$6x = 180^\circ$$

$$x = 30^\circ$$

Each exterior angle = 30°

Each interior angle = $5 \times 30^\circ = 150^\circ$

Let no. of sides = n

$$\therefore \text{each exterior angle} = \frac{360^\circ}{n}$$

$$30^\circ = \frac{360^\circ}{n}$$

$$n = \frac{360^\circ}{30^\circ}$$

$$n = 12$$

Hence (i) 150° (ii) 30° (iii) 12

Question 9.

The ratio between the interior angle and the exterior angle of a regular polygon is 2 : 1.

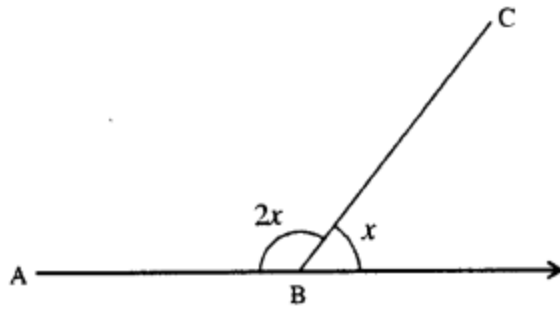
Find :

- (i) each exterior angle of the polygon ;
- (ii) number of sides in the polygon

Solution:

Interior angle : exterior angle = 2 : 1

Let interior angle = $2x^\circ$ & exterior angle = x°



$$\therefore 2x^\circ + x^\circ = 180^\circ$$

$$3x = 180^\circ$$

$$(i) \quad x = 60^\circ$$

$$\therefore \text{Each exterior angle} = 60^\circ$$

Let no. of sides = n

$$\therefore \frac{360^\circ}{n} = 60^\circ$$

$$n = \frac{360^\circ}{60^\circ}$$

$$(ii) \quad n = 6$$

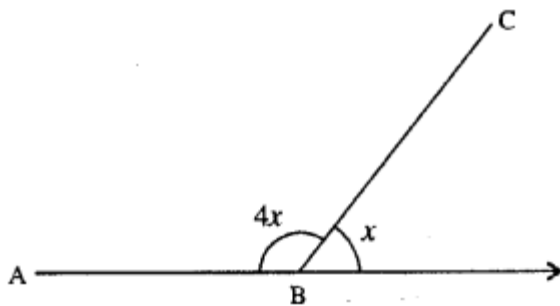
$$\therefore (i) 60^\circ \quad (ii) 6 \quad \text{Ans.}$$

Question 10.

The ratio between the exterior angle and the interior angle of a regular polygon is 1 : 4. Find the number of sides in the polygon.

Solution:

Let exterior angle = x° & interior angle = $4x^\circ$



$$\therefore 4x + x = 180^\circ$$

$$5x = 180^\circ$$

$$x = 36^\circ$$

$$\therefore \text{Each exterior angle} = 36^\circ$$

Let no. of sides = n

$$\therefore \frac{360^\circ}{n} = 36^\circ$$

$$n = \frac{360^\circ}{36}$$

$$n = 10 \quad \text{Ans.}$$

Question 11.

The sum of interior angles of a regular polygon is twice the sum of its exterior angles.
Find the number of sides of the polygon.

Solution:

Let number of sides = n

Sum of exterior angles = 360°

Sum of interior angles = $360^\circ \times 2 = 720^\circ$

Sum of interior angles = $(n - 2) \times 180^\circ$

$720^\circ = (n - 2) \times 180^\circ$

$n - 2 = \frac{720}{180}$

$n - 2 = 4$

$n = 4 + 2$

$n = 6$

Question 12.

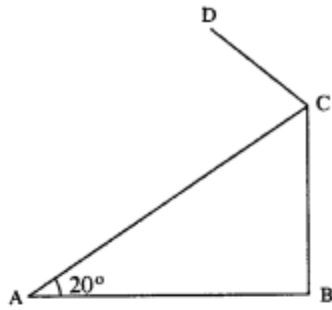
AB, BC and CD are three consecutive sides of a regular polygon. If angle BAC = 20° ;
find :

(i) its each interior angle,

(ii) its each exterior angle

(iii) the number of sides in the polygon.

Solution:



∴ Polygon is regular (Given)

$$\therefore AB = BC$$

$$\Rightarrow \angle BAC = \angle BCA$$

[$\angle$ s opp. to equal sides]

But $\angle BAC = 20^\circ$

$$\therefore \angle BCA = 20^\circ$$

i.e. In $\triangle ABC$,

$$\angle B + \angle BAC + \angle BCA = 180^\circ$$

$$\angle B + 20^\circ + 20^\circ = 180^\circ$$

$$\angle B = 180^\circ - 40^\circ$$

$$\angle B = 140^\circ$$

(i) each interior angle = 140°

(ii) each exterior angle = $180^\circ - 140^\circ$
 $= 40^\circ$

(iii) Let no. of sides = n

$$\therefore \frac{360^\circ}{n} = 40^\circ$$

$$n = \frac{360^\circ}{40^\circ} = 9,$$

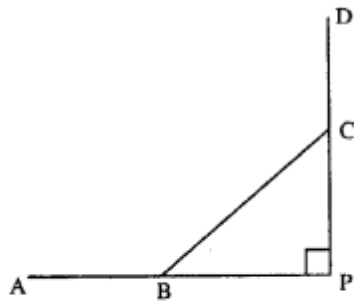
$$n = 9$$

∴ (i) 140° (ii) 9 **Ans.**

Question 13.

Two alternate sides of a regular polygon, when produced, meet at the right angle. Calculate the number of sides in the polygon.

Solution:



Let number of sides of regular polygon = n
AB & DC when produced meet at P such that
 $\angle P = 90^\circ$

$\therefore$ Interior angles are equal.

$$\therefore \angle ABC = \angle BCD$$

$$\therefore 180^\circ - \angle ABC = 180^\circ - \angle BCD$$

$$\therefore \angle PBC = \angle BCP$$

$$\text{But } \angle P = 90^\circ \text{ (Given)}$$

$$\therefore \angle PBC + \angle BCP = 180^\circ - 90^\circ = 90^\circ$$

$$\therefore \angle PBC = \angle BCP$$

$$= \frac{1}{2} \times 90^\circ = 45^\circ$$

$$\therefore \text{Each exterior angle} = 45^\circ$$

$$\therefore 45^\circ = \frac{360^\circ}{n}$$

$$n = \frac{360^\circ}{45^\circ}$$

$$n = 8 \text{ Ans.}$$

Question 14.

In a regular pentagon ABCDE, draw a diagonal BE and then find the measure of:

(i) $\angle BAE$

(ii) $\angle ABE$

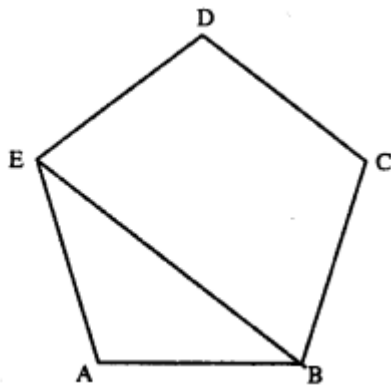
(iii) $\angle BED$

Solution:

(i) Since number of sides in the pentagon = 5

$$\text{Each exterior angle} = \frac{360}{5} = 72^\circ$$

$$\angle BAE = 180^\circ - 72^\circ = 108^\circ$$



(ii) In $\triangle ABE$, $AB = AE$

$$\therefore \angle ABE = \angle AEB$$

$$\text{But } \angle BAE + \angle ABE + \angle AEB = 180^\circ$$

$$\therefore 108^\circ + 2 \angle ABE = 180^\circ - 108^\circ = 72^\circ$$

$$\Rightarrow \angle ABE = 36^\circ$$

(iii) Since $\angle AED = 108^\circ$

$$[\because \text{each interior angle} = 108^\circ]$$

$$\Rightarrow \angle AEB = 36^\circ$$

$$\Rightarrow \angle BED = 108^\circ - 36^\circ = 72^\circ$$

Question 15.

The difference between the exterior angles of two regular polygons, having the sides equal to $(n - 1)$ and $(n + 1)$ is 9° . Find the value of n .

Solution:

We know that sum of exterior angles of a polygon is 360°

(i) If sides of a regular polygon = $n - 1$

$$\text{Then each angle} = \frac{360^\circ}{n-1}$$

and if sides are $n + 1$, then

$$\text{each angle} = \frac{360^\circ}{n+1}$$

According to the condition,

$$\frac{360^\circ}{n-1} - \frac{360^\circ}{n+1} = 9$$

$$\Rightarrow 360 \left[\frac{1}{n-1} - \frac{1}{n+1} \right] = 9$$

$$\Rightarrow 360 \left[\frac{n+1-n+1}{(n-1)(n+1)} \right] = 9$$

$$\Rightarrow \frac{2 \times 360}{n^2 - 1} = 9 \Rightarrow n^2 - 1 = \frac{2 \times 360}{9} = 80$$

$$\Rightarrow n^2 - 1 = 80 \Rightarrow n^2 = 1 + 80 = 81$$

$$\Rightarrow n^2 - 81 = 0$$

$$\Rightarrow (n)^2 - (9)^2 = 0$$

$$\Rightarrow (n + 9)(n - 9) = 0$$

Either $n + 9 = 0$, then $n = -9$ which is not possible being negative,

or $n - 9 = 0$, then $n = 9$

$$\therefore n = 9$$

$$\therefore \text{No. of sides of a regular polygon} = 9$$

Question 16.

If the difference between the exterior angle of a n sided regular polygon and an $(n + 1)$ sided regular polygon is 12° , find the value of n .

Solution:

We know that sum of exterior angles of a polygon = 360°

Each exterior angle of a regular polygon of 360°

$$n \text{ sides} = \frac{360^\circ}{n}$$

and exterior angle of the regular polygon of

$$(n + 1) \text{ sides} = \frac{360^\circ}{n + 1}$$

$$\therefore \frac{360^\circ}{n} - \frac{360^\circ}{n + 1} = 12$$

$$\Rightarrow 360 \left[\frac{1}{n} - \frac{1}{n + 1} \right] = 12 \Rightarrow 360 \left[\frac{n + 1 - n}{n(n + 1)} \right] = 12$$

$$\Rightarrow \frac{360 \times 1}{n^2 + n} = 12 \Rightarrow 12(n^2 + n) = 360^\circ$$

$$\Rightarrow n^2 + n = 30 \quad (\text{Dividing by } 12)$$

$$\Rightarrow n^2 + n - 30 = 0$$

$$\Rightarrow n^2 + 6n - 5n - 30 = 0 \left\{ \begin{array}{l} \because -30 = 6 \times (-5) \\ 1 = 6 - 5 \end{array} \right\}$$

$$\Rightarrow n(n+6) - 5(n+6) = 0$$

$$\Rightarrow (n+6)(n-5) = 0$$

Either $n+6=0$, then $n=-6$ which is not possible being negative

or $n-5=0$, then $n=5$

Hence $n=5$.

Question 17.

The ratio between the number of sides of two regular polygons is 3 : 4 and the ratio between the sum of their interior angles is 2 : 3. Find the number of sides in each polygon.

Solution:

Ratio of sides of two regular polygons = 3 : 4

Let sides of first polygon = $3n$

and sides of second polygon = $4n$

Sum of interior angles of first polygon

$$= (2 \times 3n - 4) \times 90^\circ = (6n - 4) \times 90^\circ$$

and sum of interior angle of second polygon

$$= (2 \times 4n - 4) \times 90^\circ = (8n - 4) \times 90^\circ$$

$$\therefore \frac{(6n-4) \times 90^\circ}{(8n-4) \times 90^\circ} = \frac{2}{3}$$

$$\Rightarrow \frac{6n-4}{8n-4} = \frac{2}{3}$$

$$\Rightarrow 18n - 12 = 16n - 8$$

$$\Rightarrow 18n - 16n = -8 + 12$$

$$\Rightarrow 2n = 4$$

$$\Rightarrow n = 2$$

$\therefore$ No. of sides of first polygon

$$= 3n = 3 \times 2 = 6$$

and no. of sides of second polygon

$$= 4n = 4 \times 2 = 8$$

Question 18.

Three of the exterior angles of a hexagon are 40° , 51° and 86° . If each of the remaining exterior angles is x° , find the value of x .

Solution:

Sum of exterior angles of a hexagon = $4 \times 90^\circ = 360^\circ$

Three angles are 40° , 51° and 86°

Sum of three angles = $40^\circ + 51^\circ + 86^\circ = 177^\circ$

Sum of other three angles = $360^\circ - 177^\circ = 183^\circ$

Each angle is x°

$$3x = 183^\circ$$

$$x = \frac{183}{3}$$

Hence $x = 61$

Question 19.

Calculate the number of sides of a regular polygon, if:

(i) its interior angle is five times its exterior angle.

(ii) the ratio between its exterior angle and interior angle is 2 : 7.

(iii) its exterior angle exceeds its interior angle by 60° .

Solution:

Let number of sides of a regular polygon = n

(i) Let exterior angle = x

Then interior angle = $5x$

$$x + 5x = 180^\circ$$

$$\Rightarrow 6x = 180^\circ$$

$$\Rightarrow x = \frac{180^\circ}{6} = 30^\circ$$

$$\therefore \text{Number of sides } (n) = \frac{360^\circ}{30} = 12$$

(ii) Ratio between exterior angle and interior angle
= 2 : 7

Let exterior angle = $2x$

Then interior angle = $7x$

$$\therefore 2x + 7x = 180^\circ$$

$$\Rightarrow 9x = 180^\circ$$

$$\Rightarrow x = \frac{180^\circ}{9} = 20^\circ$$

$$\therefore \text{Ext. angle} = 2x = 2 \times 20^\circ = 40^\circ$$

$$\therefore \text{No. of sides} = \frac{360^\circ}{40} = 9$$

(iii) Let interior angle = x

Then exterior angle = $x + 60$

$$\therefore x + x + 60^\circ = 180^\circ$$

$$\Rightarrow 2x = 180^\circ - 60^\circ = 120^\circ \Rightarrow x = \frac{120^\circ}{2} = 60^\circ$$

$$\therefore \text{Exterior angle} = 60^\circ + 60^\circ = 120^\circ$$

$$\therefore \text{Number of sides} = \frac{360^\circ}{120^\circ} = 3$$

Question 20.

The sum of interior angles of a regular polygon is thrice the sum of its exterior angles. Find the number of sides in the polygon.

Solution:

Sum of interior angles = 3 x Sum of exterior angles

Let exterior angle = x

The interior angle = $3x$

$$x + 3x = 180^\circ$$

$$\Rightarrow 4x = 180^\circ$$

$$\Rightarrow x = \frac{180}{4}$$

$$\Rightarrow x = 45^\circ$$

$$\text{Number of sides} = \frac{360}{45} = 8$$

EXERCISE 16(C)

Question 1.

Two angles of a quadrilateral are 89° and 113° . If the other two angles are equal; find the equal angles.

Solution:

Let the other angle = x°

According to given,

$$89^\circ + 113^\circ + x^\circ + x^\circ = 360^\circ$$

$$2x^\circ = 360^\circ - 202^\circ$$

$$2x^\circ = 158^\circ$$

$$x^\circ = \frac{158}{2}$$

other two angles = 79° each

Question 2.

Two angles of a quadrilateral are 68° and 76° . If the other two angles are in the ratio 5 : 7; find the measure of each of them.

Solution:

Two angles are 68° and 76°

Let other two angles be $5x$ and $7x$

$$68^\circ + 76^\circ + 5x + 7x = 360^\circ$$

$$12x + 144^\circ = 360^\circ$$

$$12x = 360^\circ - 144^\circ$$

$$12x = 216^\circ$$

$$x = 18^\circ$$

angles are $5x$ and $7x$

i.e. $5 \times 18^\circ$ and $7 \times 18^\circ$ i.e. 90° and 126°

Question 3.

Angles of a quadrilateral are $(4x)^\circ$, $5(x+2)^\circ$, $(7x-20)^\circ$ and $6(x+3)^\circ$. Find :

(i) the value of x .

(ii) each angle of the quadrilateral.

Solution:

Angles of quadrilateral are,

$$(4x)^\circ, 5(x+2)^\circ, (7x-20)^\circ \text{ and } 6(x+3)^\circ.$$

$$\therefore 4x + 5(x+2) + (7x-20) + 6(x+3) = 360^\circ$$

$$4x + 5x + 10 + 7x - 20 + 6x + 18 = 360^\circ$$

$$22x + 8 = 360^\circ$$

$$22x = 360^\circ - 8^\circ$$

$$22x = 352^\circ$$

$$x = 16^\circ \text{ Ans.}$$

Hence angles are,

$$(4x)^\circ = (4 \times 16)^\circ = 64^\circ,$$

$$5(x+2)^\circ = 5(16+2)^\circ = 90^\circ,$$

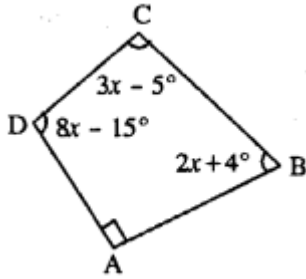
$$(7x-20)^\circ = (7 \times 16 - 20)^\circ = 92^\circ$$

$$6(x+3)^\circ = 6(16+3) = 114^\circ \text{ Ans.}$$

Question 4.

Use the information given in the following figure to find :

- (i) x
- (ii) $\angle B$ and $\angle C$

**Solution:**

$$\therefore \angle A = 90^\circ \quad (\text{Given})$$

$$\angle B = (2x + 4^\circ)$$

$$\angle C = (3x - 5^\circ)$$

$$\angle D = (8x - 15^\circ)$$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$90^\circ + (2x + 4^\circ) + (3x - 5^\circ) + (8x - 15^\circ) = 360^\circ$$

$$90^\circ + 2x + 4^\circ + 3x - 5^\circ + 8x - 15^\circ = 360^\circ$$

$$\Rightarrow 74^\circ + 13x = 360^\circ$$

$$\Rightarrow 13x = 360^\circ - 74^\circ$$

$$\Rightarrow 13x = 286^\circ$$

$$\Rightarrow x = 22^\circ$$

$$\therefore \angle B = 2x + 4 = 2 \times 22^\circ + 4 = 48^\circ$$

$$\angle C = 3x - 5 = 3 \times 22^\circ - 5 = 61^\circ$$

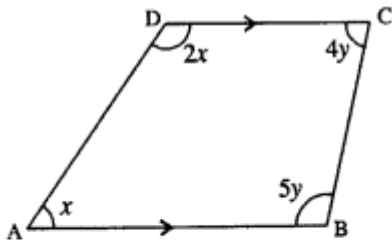
Hence (i) 22° (ii) $\angle B = 48^\circ$, $\angle C = 61^\circ$ Ans.

Question 5.

In quadrilateral ABCD, side AB is parallel to side DC. If $\angle A : \angle D = 1 : 2$ and $\angle C : \angle B = 4 : 5$

- (i) Calculate each angle of the quadrilateral.
- (ii) Assign a special name to quadrilateral ABCD

Solution:



$$\therefore \angle A : \angle D = 1 : 2$$

$$\text{Let } \angle A = x \text{ and } \angle B = 2x$$

$$\therefore \angle C : \angle B = 4 : 5$$

$$\text{Let } \angle C = 4y \text{ and } \angle B = 5y$$

$$\therefore AB \parallel DC$$

$$\therefore \angle A + \angle D = 180^\circ$$

$$x + 2x = 180^\circ$$

$$3x = 180^\circ$$

$$x = 60^\circ$$

$$\therefore A = 60^\circ$$

$$\angle D = 2x = 2 \times 60 = 120^\circ$$

$$\text{Again } \angle B + \angle C = 180^\circ$$

$$5y + 4y = 180^\circ$$

$$9y = 180^\circ$$

$$y = 20^\circ$$

$$\therefore \angle B = 5y = 5 \times 20 = 100^\circ$$

$$\angle C = 4y = 4 \times 20 = 80^\circ$$

Hence $\angle A = 60^\circ$; $\angle B = 100^\circ$; $\angle C = 80^\circ$
and $\angle D = 120^\circ$ **Ans.**

(ii) Quadrilateral ABCD is a trapezium because
one pair of opposite side is parallel

Question 6.

From the following figure find ;

(i) x

(ii) $\angle ABC$

(iii) $\angle ACD$

Solution:

$$x = \frac{312}{12} = 26^\circ$$

$$(ii) \quad \angle ABC = 4x$$

$$4 \times 26 = 104^\circ$$

$$(iii) \quad \angle ACD = 180^\circ - 4x - 48^\circ$$

$$= 180^\circ - 4 \times 26^\circ - 48^\circ$$

$$= 180^\circ - 104^\circ - 48^\circ$$

$$= 180^\circ - 152^\circ = 28^\circ$$

(i) In Quadrilateral ABCD,

$$x + 4x + 3x + 4x + 48^\circ = 360^\circ$$

$$12x = 360^\circ - 48^\circ$$

$$12x = 312$$

Question 7.

Given : In quadrilateral ABCD ; $\angle C = 64^\circ$, $\angle D = \angle C - 8^\circ$; $\angle A = 5(a+2)^\circ$ and $\angle B = 2(2a+7)^\circ$.

Calculate $\angle A$.

Solution:

$$\angle C = 64^\circ \text{ (Given)}$$

$$\angle D = \angle C - 8^\circ = 64^\circ - 8^\circ = 56^\circ$$

$$\angle A = 5(a+2)^\circ$$

$$\angle B = 2(2a+7)^\circ$$

$$\text{Now } \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$5(a+2)^\circ + 2(2a+7)^\circ + 64^\circ + 56^\circ = 360^\circ$$

$$5a + 10 + 4a + 14^\circ + 64^\circ + 56^\circ = 360^\circ$$

$$9a + 144^\circ = 360^\circ$$

$$9a = 360^\circ - 144^\circ$$

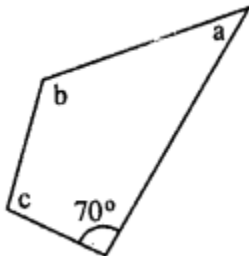
$$9a = 216^\circ$$

$$a = 24^\circ$$

$$\angle A = 5(a + 2) = 5(24+2) = 130^\circ$$

Question 8.

In the given figure : $\angle b = 2a + 15$ and $\angle c = 3a + 5$; find the values of b and c.



Solution:

Sum of angles of quadrilateral = 360°

$$70^\circ + a + 2a + 15 + 3a + 5 = 360^\circ$$

$$6a + 90^\circ = 360^\circ$$

$$6a = 270^\circ$$

$$a = 45^\circ$$

$$b = 2a + 15 = 2 \times 45 + 15 = 105^\circ$$

$$c = 3a + 5 = 3 \times 45 + 5 = 140^\circ$$

Hence $\angle b$ and $\angle c$ are 105° and 140°

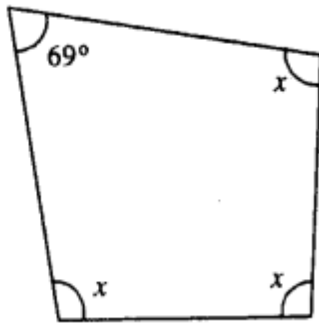
Question 9.

Three angles of a quadrilateral are equal. If the fourth angle is 69° ; find the measure of equal angles.

Solution:

Let each equal angle be x°

$$x + x + x + 69^\circ = 360^\circ$$



$$3x = 360^\circ - 69$$

$$3x = 291$$

$$x = 97^\circ$$

Each, equal angle = 97°

Question 10.

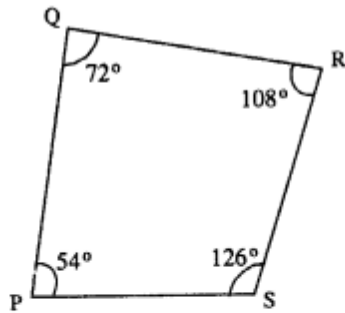
In quadrilateral PQRS, $\angle P : \angle Q : \angle R : \angle S = 3 : 4 : 6 : 7$.

Calculate each angle of the quadrilateral and then prove that PQ and SR are parallel to each other

(i) Is PS also parallel to QR ?

(ii) Assign a special name to quadrilateral PQRS.

Solution:



$$\therefore \angle P : \angle Q : \angle R : \angle S = 3 : 4 : 6 : 7$$

Let $\angle P = 3x$

$$\angle Q = 4x$$

$$\angle R = 6x$$

& $\angle S = 7x$

$$\therefore \angle P + \angle Q + \angle R + \angle S = 360^\circ$$

$$3x + 4x + 6x + 7x = 360^\circ$$

$$20x = 360^\circ$$

$$x = 18^\circ$$

$$\therefore \angle P = 3x = 3 \times 18 = 54^\circ$$

$$\angle Q = 4x = 4 \times 18 = 72^\circ$$

$$\angle R = 6x = 6 \times 18 = 108^\circ$$

$$\angle S = 7x = 7 \times 18 = 126^\circ$$

$$\angle Q + \angle R = 72^\circ + 108^\circ = 180^\circ$$

or $\angle P + \angle S = 54^\circ + 126^\circ = 180^\circ$

Hence $PQ \parallel SR$

$$\text{As } \angle P + \angle Q = 72^\circ + 54^\circ = 126^\circ$$

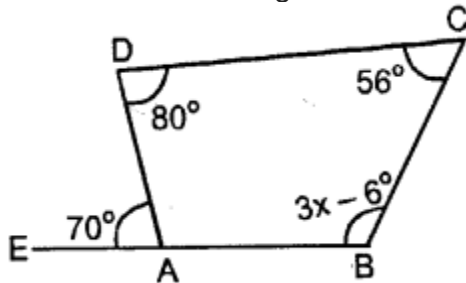
Which is $\neq 180^\circ$.

$\therefore$ PS and QR are not parallel.

(ii) PQRS is a Trapezium as its one pair of opposite side is parallel.

Question 11.

Use the informations given in the following figure to find the value of x.



Solution:

Take A, B, C, D as the vertices of Quadrilateral and BA is produced to E (say).

Since $\angle EAD = 70^\circ$

$\angle DAB = 180^\circ - 70^\circ = 110^\circ$

[EAB is a straight line and AD stands on it $\angle EAD + \angle DAB = 180^\circ$]

$110^\circ + 80^\circ + 56^\circ + 3x - 6^\circ = 360^\circ$

[sum of interior angles of a quadrilateral = 360°]

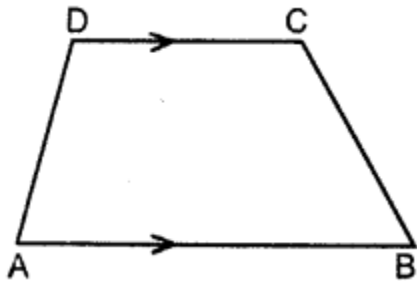
$3x = 360^\circ - 110^\circ - 80^\circ - 56^\circ + 6^\circ$

$3x = 360^\circ - 240^\circ = 120^\circ$

$x = 40^\circ$

Question 12.

The following figure shows a quadrilateral in which sides AB and DC are parallel. If $\angle A : \angle D = 4 : 5$, $\angle B = (3x - 15)^\circ$ and $\angle C = (4x + 20)^\circ$, find each angle of the quadrilateral ABCD.



Solution:

Let $\angle A = 4x$

$\angle D = 5x$

Since $\angle A + \angle D = 180^\circ$ [AB||DC]

$4x + 5x = 180^\circ$

$\Rightarrow 9x = 180^\circ$

$\Rightarrow x = 20^\circ$

$\angle A = 4(20) = 80^\circ$,

$\angle D = 5(20) = 100^\circ$

Again $\angle B + \angle C = 180^\circ$ [AB||DC]

$3x - 15^\circ + 4x + 20^\circ = 180^\circ$

$7x = 180^\circ - 5^\circ$

$\Rightarrow 7x = 175^\circ$

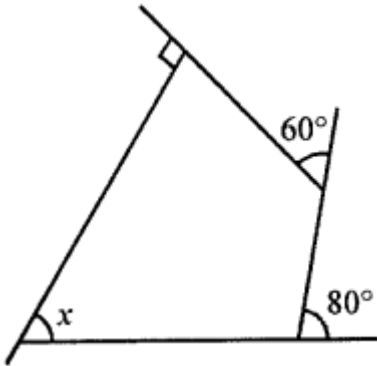
$\Rightarrow x = 25^\circ$

$\angle B = 75^\circ - 15^\circ = 60^\circ$

and $\angle C = 4(25) + 20 = 100^\circ + 20^\circ = 120^\circ$

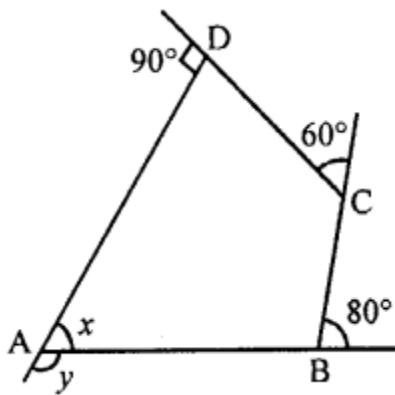
Question 13.

Use the following figure to find the value of x



Solution:

The sum of exterior angles of a quadrilateral



$$\Rightarrow y + 80^\circ + 60^\circ + 90^\circ = 360^\circ$$

$$\Rightarrow y + 230^\circ = 360^\circ$$

$$\Rightarrow y = 360^\circ - 230^\circ = 130^\circ$$

At vertex A,

$$\angle y + \angle x = 180^\circ \text{ (Linear pair)}$$

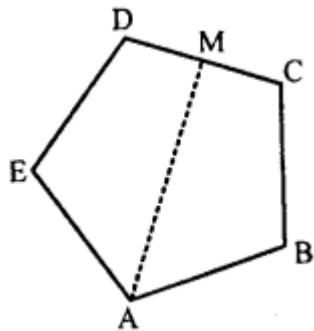
$$x = 180^\circ - 130^\circ$$

$$\Rightarrow x = 50^\circ$$

Question 14.

ABCDE is a regular pentagon. The bisector of angle A of the pentagon meets the side CD in point M. Show that $\angle AMC = 90^\circ$.

Solution:



Given : ABCDE is a regular pentagon.

The bisector $\angle A$ of the pentagon meets the side CD at point M.

To prove : $\angle AMC = 90^\circ$

Proof: We know that, the measure of each interior angle of a regular pentagon is 108° .

$$\angle BAM = \frac{1}{2} \times 108^\circ = 54^\circ$$

Since, we know that the sum of a quadrilateral is 360°

In quadrilateral ABCM, we have

$$\angle BAM + \angle ABC + \angle BCM + \angle AMC = 360^\circ$$

$$54^\circ + 108^\circ + 108^\circ + \angle AMC = 360^\circ$$

$$\angle AMC = 360^\circ - 270^\circ$$

$$\angle AMC = 90^\circ$$

Question 15.

In a quadrilateral ABCD, AO and BO are bisectors of angle A and angle B respectively.

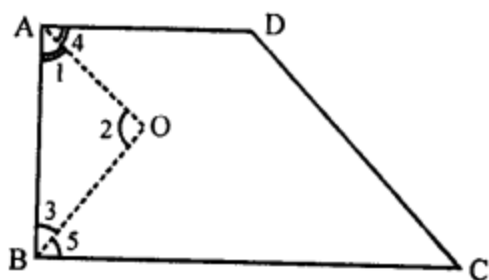
Show that:

$$\angle AOB = \frac{1}{2} (\angle C + \angle D)$$

Solution:

Given : AO and BO are the bisectors of $\angle A$ and $\angle B$ respectively.

$$\angle 1 = \angle 4 \text{ and } \angle 3 = \angle 5 \dots\dots\dots(i)$$



$$\text{To prove : } \angle AOB = \frac{1}{2} (\angle C + \angle D)$$

Proof: In quadrilateral ABCD

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\frac{1}{2} (\angle A + \angle B + \angle C + \angle D) = 180^\circ \dots\dots\dots(ii)$$

Now in $\triangle AOB$

$$\angle 1 + \angle 2 + \angle 3 = 180^\circ \dots\dots\dots(iii)$$

Equating equation (ii) and equation (iii), we get

$$\angle 1 + \angle 2 + \angle 3 = \angle A + \angle B + \frac{1}{2} (\angle C + \angle D)$$

$$\angle 1 + \angle 2 + \angle 3 = \angle 1 + \angle 3 + \frac{1}{2} (\angle C + \angle D)$$

$$\angle 2 = \frac{1}{2} (\angle C + \angle D)$$

$$\angle AOB = \frac{1}{2} (\angle C + \angle D)$$

Hence proved.