

20. Area of Trapezium and a Polygon

EXERCISE 20 (A)

Question 1.

Find the area of a triangle, whose sides are :

(i) 10 cm, 24 cm and 26 cm

(ii) 18 mm, 24 mm and 30 mm

(iii) 21 m, 28 m and 35 m

Solution:

(i) Sides of Δ are

$a = 10$ cm

$b = 24$ cm

$c = 26$ cm

$$S = \frac{a+b+c}{2} = \frac{10+24+26}{2}$$

$$= \frac{60}{2} = 30$$

$$\text{area of } \Delta = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{30(30-10)(30-24)(30-26)}$$

$$= \sqrt{30 \times 20 \times 6 \times 4}$$

$$= \sqrt{10 \times 3 \times 10 \times 2 \times 2 \times 3 \times 2 \times 2}$$

$$= \sqrt{10 \times 10 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2}$$

$$= 10 \times 3 \times 2 \times 2 = 120 \text{ cm}^2 \text{ Ans.}$$

(ii) Sides of Δ are

$a = 18$ mm

$b = 24$ mm

$c = 30$ mm

$$S = \frac{a+b+c}{2} = \frac{18+24+30}{2}$$

$$= \frac{72}{2} = 36$$

$$\begin{aligned} \text{area of } \Delta &= \sqrt{S(S-a)(S-b)(S-c)} \\ &= \sqrt{36(36-18)(36-24)(36-30)} \\ &= \sqrt{36 \times 18 \times 12 \times 6} \\ &= \sqrt{18 \times 2 \times 18 \times 2 \times 6 \times 6} \\ &= \sqrt{18 \times 18 \times 2 \times 2 \times 6 \times 6} \\ &= 18 \times 2 \times 6 = 216 \text{ mm}^2 \text{ Ans.} \end{aligned}$$

(iii) Sides of Δ are

$$a = 21 \text{ m}$$

$$b = 28 \text{ m}$$

$$c = 35 \text{ m}$$

$$S = \frac{a+b+c}{2} = \frac{21+28+35}{2}$$

$$= \frac{84}{2} = 42$$

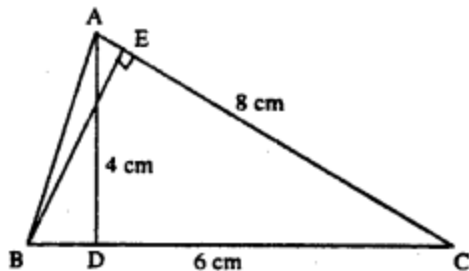
$$\begin{aligned} \text{area of } \Delta &= \sqrt{S(S-a)(S-b)(S-c)} \\ &= \sqrt{42(42-21)(42-28)(42-35)} \\ &= \sqrt{42 \times 21 \times 14 \times 7} \\ &= \sqrt{7 \times 3 \times 2 \times 3 \times 7 \times 2 \times 7 \times 7} \\ &= \sqrt{7 \times 7 \times 7 \times 7 \times 3 \times 3 \times 2 \times 2} \\ &= 7 \times 7 \times 3 \times 2 \\ &= 294 \text{ m}^2 \text{ Ans.} \end{aligned}$$

Question 2.

Two sides of a triangle are 6 cm and 8 cm. If height of the triangle corresponding to 6 cm side is 4 cm ; find :

- (i) area of the triangle
- (ii) height of the triangle corresponding to 8 cm side.

Solution:



$$BC = 6 \text{ cm}$$

$$\text{height } AD = 4 \text{ cm}$$

$$\text{area of } \Delta = \frac{1}{2} \text{ base} \times \text{height}$$

$$= \frac{1}{2} \times BC \times AD$$

$$= \frac{1}{2} \times 6 \times 4 = 12 \text{ cm}^2$$

$$\text{Again area of } \Delta = \frac{1}{2} AC \times BE$$

$$12 = \frac{1}{2} \times 8 \times BE$$

$$\therefore BE = \frac{12 \times 2}{8}$$

$$BE = 3 \text{ cm}$$

$$\therefore (i) 12 \text{ cm}^2 (ii) 3 \text{ cm Ans.}$$

Question 3.

The sides of a triangle are 16 cm, 12 cm and 20 cm. Find :

- (i) area of the triangle ;
- (ii) height of the triangle, corresponding to the largest side ;
- (iii) height of the triangle, corresponding to the smallest side.

Solution:

Sides of Δ are

$$a = 20 \text{ cm}$$

$$b = 12 \text{ cm}$$

$$c = 16 \text{ cm}$$

$$\begin{aligned} S &= \frac{a+b+c}{2} \\ &= \frac{20+12+16}{2} \\ &= \frac{48}{2} = 24 \end{aligned}$$

$$\begin{aligned} \text{area of } \Delta &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{24(24-20)(24-12)(24-16)} \\ &= \sqrt{24 \times 4 \times 12 \times 8} \\ &= \sqrt{12 \times 2 \times 4 \times 12 \times 2 \times 4} \\ &= \sqrt{12 \times 12 \times 4 \times 4 \times 2 \times 2} \\ &= 12 \times 4 \times 2 = 96 \text{ cm}^2 \end{aligned}$$

AD is height of Δ corresponding to largest side.

$$\therefore \frac{1}{2} \times BC \times AD = 96$$

$$\frac{1}{2} \times 20 \times AD = 96$$

$$AD = \frac{96 \times 2}{20}$$

$$AD = 9.6 \text{ cm}$$

BE is height of Δ corresponding to smallest side.

$$\therefore \frac{1}{2} \times AC \times BE = 96$$

$$\frac{1}{2} \times 12 \times BE = 96$$

$$BE = \frac{96 \times 2}{12}$$

$$BE = 16 \text{ cm}$$

(i) 96 cm^2 (ii) 9.6 cm (iii) 16 cm Ans.

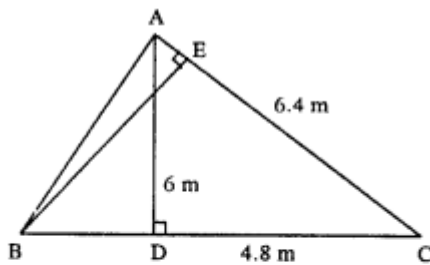
Question 4.

Two sides of a triangle are 6.4 m and 4.8 m. If height of the triangle corresponding to 4.8 m side is 6 m; find :

(i) area of the triangle ;

(ii) height of the triangle corresponding to 6.4 m side.

Solution:



ABC is the Δ in which $BC = 4.8$ m
 $AC = 6.4$ m and $AD = 6$ m

$$\therefore \text{area of } \Delta ABC = \frac{1}{2} BC \times AD$$

$$= \frac{1}{2} \times 4.8 \times 6$$

$$= 14.4 \text{ m}^2$$

BE is height of Δ corresponding to 6.4 m

$$\therefore \frac{1}{2} AC \times BE = 14.4$$

$$\frac{1}{2} \times 6.4 \times BE = 14.4$$

$$BE = \frac{14.4 \times 2}{6.4}$$

$$BE = \frac{14.4}{3.2}$$

$$= 9/2 = 4.5 \text{ m}$$

Hence (i) 14.4 m^2 (ii) 4.5 m

Question 5.

The base and the height of a triangle are in the ratio 4 : 5. If the area of the triangle is 40 m^2 ; find its base and height.

Solution:

Let base of $\Delta = 4x$ m

and height of $\Delta = 5x$ m

area of $\Delta = 40 \text{ m}^2$

$$\therefore \frac{1}{2} \text{base} \times \text{height} = \text{area of } \Delta$$

$$\frac{1}{2} \times 4x \times 5x = 40$$

$$10x^2 = 40$$

$$x^2 = 4$$

$$x = \sqrt{4}$$

$$x = 2$$

$$\therefore \text{base} = 4x = 4 \times 2 = 8 \text{ m}$$

$$\text{height} = 5x = 5 \times 2 = 10 \text{ m}$$

$\therefore$ 8 m; 10 m **Ans.**

Question 6.

The base and the height of a triangle are in the ratio 5 : 3. If the area of the triangle is 67.5 m^2 ; find its base and height.

Solution:

Let base = $5x \text{ m}$

height = $3x \text{ m}$

$$\text{Area of } \Delta = \frac{1}{2} \text{base} \times \text{height}$$

$$\therefore \frac{1}{2} \times 5x \times 3x = 67.5$$

$$x^2 = \frac{67.5 \times 2}{15}$$

$$x^2 = 4.5 \times 2$$

$$x^2 = 9.0$$

$$x = \sqrt{9}$$

$$x = 3$$

$$\text{base} = 5x = 5 \times 3 = 15 \text{ m}$$

$$\text{height} = 3x = 3 \times 3 = 9 \text{ m}$$

Question 7.

The area of an equilateral triangle is $144\sqrt{3} \text{ cm}^2$; find its perimeter.

Solution:

Let each side of an equilateral triangle = x cm

$$\therefore \text{Its area} = \frac{\sqrt{3}}{4} (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} x^2 = 144\sqrt{3} \text{ (given)}$$

$$\Rightarrow x^2 = 144\sqrt{3} \times \frac{4}{\sqrt{3}}$$

$$\Rightarrow x^2 = 144 \times 4 \Rightarrow x^2 = 576$$

$$\Rightarrow x = \sqrt{576} = 24 \text{ cm.}$$

$$\Rightarrow \text{Each side} = 24 \text{ cm}$$

$$\text{Hence perimeter} = 3(24) = 72 \text{ cm.}$$

Question 8.

The area of an equilateral triangle is numerically equal to its perimeter. Find its perimeter correct to 2 decimal places.

Solution:

Let each side of the equilateral triangle = x

$$\therefore \text{Its area} = \frac{\sqrt{3}}{4} x^2$$

$$\text{Area perimeter} = 3x$$

$$\text{By the given condition} = \frac{\sqrt{3}}{4} x^2 = 3x$$

$$x^2 = 3x \times \frac{4}{\sqrt{3}}$$

$$x^2 = \frac{3x \times 4 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{3x \times 4 \times \sqrt{3}}{3} = 4x\sqrt{3}$$

$$\Rightarrow x^2 = \sqrt{3} (4x) \Rightarrow x = 4\sqrt{3} \quad [\because x \neq 0]$$

$$\therefore \text{Perimeter} = 12\sqrt{3} \text{ units}$$

$$= 12 (1.732) = 20.784 = 20.78 \text{ units}$$

Question 9.

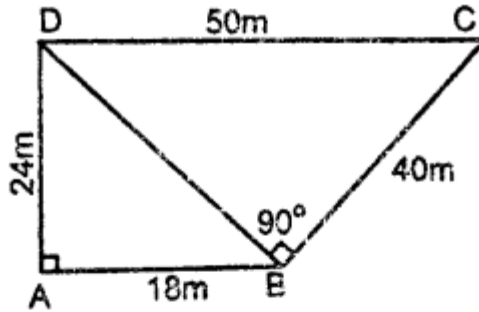
A field is in the shape of a quadrilateral ABCD in which side AB = 18 m, side AD = 24 m, side BC = 40m, DC = 50 m and angle A = 90°. Find the area of the field.

Solution:

Since $\angle A = 90^\circ$

By Pythagorus Theorem,

In $\triangle ABD$,



$$BD = \sqrt{AB^2 + AD^2} = \sqrt{18^2 + 24^2}$$

$$= \sqrt{324 + 576} = \sqrt{900} = 30 \text{ m.}$$

$$\text{Now, area of } \triangle ABD = \frac{1}{2} (18) (24)$$

$$= (18) (12) = 216 \text{ m}^2$$

Again in $\triangle BCD$; sides are 30, 40, 50

$\Rightarrow$ By Pythagoras Theorem $\angle CBD = 90^\circ$

[$\therefore DC^2 = BD^2 + BC^2$, Since $(50)^2 = (30)^2 + (40)^2$]

$$\therefore \text{Area of } \triangle BCD = \frac{1}{2} (40) (30) = 600 \text{ m}^2$$

$$\text{Hence, area of quadrilateral ABCD} = \text{Area of } \triangle ABD + \text{area of } \triangle BCD$$

$$= 216 + 600 = 816 \text{ m}^2.$$

Question 10.

The lengths of the sides of a triangle are in the ratio 4 : 5 : 3 and its perimeter is 96 cm. Find its area.

Solution:

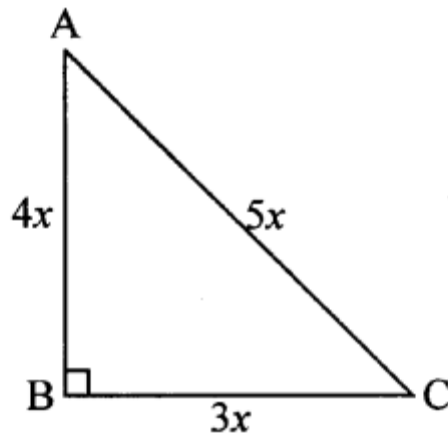
Let the sides of the triangle ABC be 4x, 5x and 3x

Let AB = 4x, AC = 5x and BC = 3x

$$\text{Perimeter} = 4x + 5x + 3x = 96$$

$$\Rightarrow 12x = 96$$

$$\Rightarrow x = \frac{96}{12}$$



$$\therefore x = 8$$

$\therefore$ Sides are

$$BC = 3(8) = 24 \text{ cm}, AB = 4(8) = 32 \text{ cm},$$

$$AC = 5(8) = 40 \text{ cm}$$

$$\text{Since } (AC)^2 = (AB)^2 + (BC)^2$$

$$[\because (5x)^2 = (3x)^2 + (4x)^2]$$

$\therefore$ By Pythagorus Theorem, $\angle B = 90^\circ$

$$\therefore \text{Area of } \triangle ABC = \frac{1}{2} (BC) (AB) = \frac{1}{2} (24) (32)$$

$$= 12 \times 32 = 384 \text{ cm}^2$$

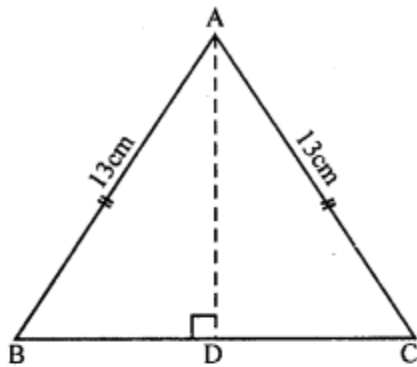
Question 11.

One of the equal sides of an isosceles triangle is 13 cm and its perimeter is 50 cm. Find the area of the triangle.

Solution:

In isosceles $\triangle ABC$

AB = AC = 13 cm But perimeter = 50 cm



$$\begin{aligned}\therefore BC &= 50 - (13 + 13) \text{ cm} \\ &= 50 - 26 = 24 \text{ cm}\end{aligned}$$

$AD \perp BC$

$$\therefore AD = DC = \frac{24}{2} = 12 \text{ cm.}$$

In right $\triangle ABD$,

$$AB^2 = AD^2 + BD^2 \quad (\text{Pythagoras Theorem})$$

$$(13)^2 = AD^2 + (12)^2$$

$$\Rightarrow 169 = AD^2 + 144$$

$$\Rightarrow AD^2 = 169 - 144$$

$$= 25 = (5)^2$$

$$\therefore AD = 5 \text{ cm.}$$

$$\text{Now area of } \triangle ABC = \frac{1}{2} \text{ Base} \times \text{Altitude}$$

$$= \frac{1}{2} \times BC \times AD$$

$$= \frac{1}{2} \times 24 \times 5 = 60 \text{ cm}^2$$

Question 12.

The altitude and the base of a triangular field are in the ratio 6 : 5. If its cost is ₹ 49,57,200 at the rate of ₹ 36,720 per hectare and 1 hectare = 10,000 sq. m, find (in metre) dimensions of the field,

Solution:

Total cost = ₹ 49,57,200

Rate = ₹ 36,720 per hectare

Total area of the triangular field

$$= \frac{4957200}{36720} \times 10000 \text{ m}^2 = 1350000 \text{ m}^2$$

Ratio in altitude and base of the field = 6 : 5

Let altitude = $6x$

and base = $5x$

$$\therefore \text{Area} = \frac{1}{2} \text{Base} \times \text{Altitude}$$

$$\Rightarrow 1350000 = \frac{1}{2} \times 5x \times 6x$$

$$\Rightarrow 15x^2 = 1350000 \Rightarrow x^2 = \frac{1350000}{15}$$

$$\Rightarrow x^2 = 90000 = (300)^2$$

$$\therefore x = 300$$

$$\therefore \text{Base} = 5x = 5 \times 300 = 1500 \text{ m}$$

$$\text{and altitude} = 6x = 6 \times 300 = 1800 \text{ m}$$

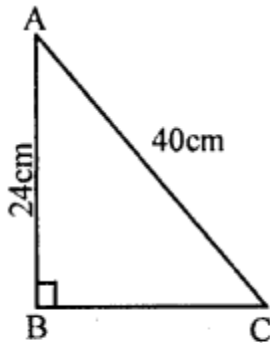
Question 13.

Find the area of the right-angled triangle with hypotenuse 40 cm and one of the other two sides 24 cm.

Solution:

In right angled triangle ABC Hypotenuse AC = 40 cm
One side AB = 24 cm

$$\begin{aligned} \therefore BC &= \sqrt{AC^2 - AB^2} \\ &= \sqrt{40^2 - 24^2} = \sqrt{1600 - 576} \\ &= \sqrt{1024} = 32 \text{ cm} \\ \therefore \text{Area} \end{aligned}$$

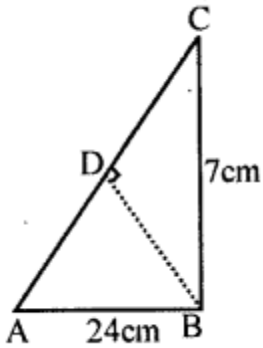


$$= \frac{1}{2} AB \times BC = \frac{1}{2} \times 24 \times 32 \text{ cm}^2 = 384 \text{ cm}^2$$

Question 14.

Use the information given in the adjoining figure to find :

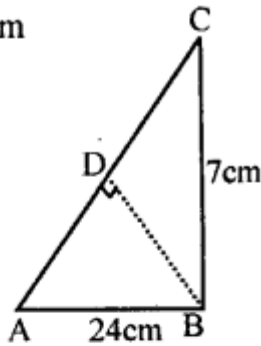
- (i) the length of AC.
- (ii) the area of a $\triangle ABC$
- (iii) the length of BD, correct to one decimal place.



Solution:

Sol. $AB = 24 \text{ cm}$, $BC = 7 \text{ cm}$

$$\begin{aligned} \text{(i) } AC &= \sqrt{AB^2 + BC^2} \\ &= \sqrt{24^2 + 7^2} \\ &= \sqrt{576 + 49} \\ &= \sqrt{625} = 25 \text{ cm} \end{aligned}$$



$$\text{(ii) Area of } \triangle ABC = \frac{1}{2} AB \times BC$$

$$= \frac{1}{2} \times 24 \times 7 = 84 \text{ cm}^2$$

(iii) $BD \perp AC$

$$\text{Area } \triangle ABC = \frac{1}{2} AC \times BD$$

$$84 = \frac{1}{2} \times 25 \times BD$$

$$\begin{aligned} \Rightarrow BD &= \frac{84 \times 2}{25} = \frac{168}{25} = 6.72 \text{ cm} \\ &= 6.7 \text{ cm} \end{aligned}$$

EXERCISE 20(B)

Question 1.

Find the length and perimeter of a rectangle, whose area = 120 cm^2 and breadth = 8 cm

Solution:

area of rectangle = 120 cm^2

breadth, $b = 8 \text{ cm}$

Area = $l \times b$

$l \times 8 = 120$

$l = 120/8 = 15 \text{ cm}$

Perimeter = $2(l+b) = 2(15+8) = 2 \times 23 = 46 \text{ cm}$

Length = 15 cm; Perimeter = 46 cm

Question 2.

The perimeter of a rectangle is 46 m and its length is 15 m. Find its :

(i) breadth

(ii) area

(iii) diagonal.

Solution:

(i) Perimeter of rectangle = 46 m

length, $l = 15 \text{ m}$

$2(l+b) = 46$

$2(15 + b) = 46$

$15+b = 46/2 = 23$

$b = 23 - 15$

$b = 8 \text{ m}$

(ii) area = $l \times b = 15 \times 8 = 120 \text{ m}^2$

(iii) diagonal = $\sqrt{l^2 + b^2} = \sqrt{15^2 + 8^2}$

$= \sqrt{225 + 64} = \sqrt{289} = 17 \text{ m}$

Hence (i) 8 m (ii) 120 m^2 (iii) 17 m Ans.

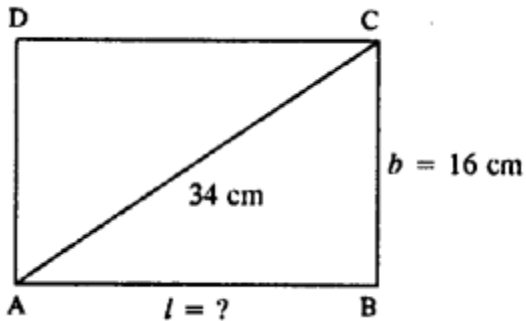
Question 3.

The diagonal of a rectangle is 34 cm. If its breadth is 16 cm; find its :

(i) length

(ii) area

Solution:



$AC^2 = AB^2 + BC^2$ (By Pythagoras theorem)

$$(34)^2 = l^2 + (16)^2$$

$$1156 = l^2 + 256$$

$$l^2 = 1156 - 256$$

$$l^2 = 900$$

$$l = \sqrt{900} = 30 \text{ cm}$$

$$\text{area} = l \times b = 30 \times 16 = 480 \text{ cm}^2$$

(i) 30 cm (ii) 480 cm²

Question 4.

The area of a small rectangular plot is 84 m². If the difference between its length and the breadth is 5 m; find its perimeter.

Solution:

Area of a rectangular plot = 84 m²

Let breadth = x m

Then length = (x + 5) m

Area = l × b

$$x(x + 5) = 84$$

$$x^2 + 5x - 84 = 0$$

$$\Rightarrow x^2 + 12x - 7x - 84 = 0$$

$$\Rightarrow x(x + 12) - 7(x + 12) = 0$$

$$\Rightarrow (x + 12)(x - 7) = 0$$

Either $x + 12 = 0$, then $x = -12$ which is not possible being negative

or $x - 7 = 0$, then $x = 7$

$$\text{Length} = x + 5 = 7 + 5 = 12 \text{ m}$$

and breadth = x = 7 m

$$\text{Perimeter} = 2(l + b) = 2(12 + 7) = 2 \times 19 \text{ m} = 38 \text{ m}$$

Question 5.

The perimeter of a square is 36 cm; find its area

Solution:

Perimeter of Square = 36 cm

$$\text{Side} = \frac{\text{Perimeter}}{4} = \frac{36}{4} = 9 \text{ cm}$$

$$\begin{aligned}\therefore \text{Area of Square} &= \text{side} \times \text{side} \\ &= 9 \times 9 \\ &= 81 \text{ cm}^2\end{aligned}$$

Question 6.

Find the perimeter of a square; whose area is : 1.69 m²

Solution:

Area of square = 1.69 m²

Side = $\sqrt{\text{area}} = \sqrt{1.69} = 1.3 \text{ m}$

Perimeter = 4 x side = 4 x 1.3 = 5.2 m

Question 7.

The diagonal of a square is 12 cm long; find its area and length of one side.

Solution:

Let side of square = a cm

diagonal = 12 cm

By Pythagoras Theorem, $a^2 + a^2 = (12)^2$

$$2a^2 = 144$$

$$a^2 = 72$$

Area of square = $a^2 = 72 \text{ cm}^2$

$$a^2 = 72$$

$$a = \sqrt{72} = 8.49 \text{ cm}$$

Question 8.

The diagonal of a square is 15 m; find the length of its one side and perimeter.

Solution:

Diagonal of square = 15 m

Let side of square = a

$$a^2 + a^2 = (15)^2 = 225$$

$$a^2 = 225/2 = 112.50$$

$$a = \sqrt{112.50} = 10.6 \text{ m}$$

$$\text{Perimeter} = 4 \times a = 10.6 \times 4 = 42.4 \text{ m}$$

Question 9.

The area of a square is 169 cm². Find its:

(i) one side

(ii) perimeter

Solution:

Let each side of the square be x cm.

$$\text{Its area} = x^2 = 169 \text{ (given)}$$

$$x = \sqrt{169}$$

$$x = 13 \text{ cm}$$

(i) Thus, side of the square = 13 cm

(ii) Again perimeter = 4 (side) = $4 \times 13 = 52 \text{ cm}$

Question 10.

The length of a rectangle is 16 cm and its perimeter is equal to the perimeter of a square with side 12.5 cm. Find the area of the rectangle.

Solution:

Length of the rectangle = 16 cm

Let its breadth be x cm

$$\text{Perimeter} = 2(16 + x) = 32 + 2x$$

$$\text{Also perimeter} = 4(12.5) = 50 \text{ cm.}$$

According to statement,

$$32 + 2x = 50$$

$$\Rightarrow 2x = 50 - 32 = 18$$

$$\Rightarrow x = 9$$

Breadth of the rectangle = 9 cm.

$$\text{Area of the rectangle (l} \times \text{b)} = 16 \times 9 = 144 \text{ cm}^2$$

Question 11.

The perimeter of a square is numerically equal to its area. Find its area.

Solution:

Let each side of the square be x cm.

$$\text{Its perimeter} = 4x,$$

$$\text{Area} = x^2$$

By the given condition $4x = x^2$

$$\Rightarrow x^2 - 4x = 0$$

$$\Rightarrow x(x - 4) = 0$$

$$\Rightarrow x = 4 \text{ [} x \neq 0 \text{]}$$

$$\text{Area} = x^2 = (4)^2 = 4 \times 4 = 16 \text{ sq.units.}$$

Question 12.

Each side of a rectangle is doubled. Find the ratio between :

(i) perimeters of the original rectangle and the resulting rectangle.

(ii) areas of the original rectangle and the resulting rectangle.

Solution:

Let length of the rectangle = x

and breadth of the rectangle = y

$$(i) \text{ Perimeter } P = 2(x + y)$$

Again, new length = $2x$

New breadth = $2y$

$$\begin{aligned}\therefore \text{New perimeter } P' &= 2(2x + 2y) \\ &= 4(x + y) = 2 \cdot 2(x + y) = 2P\end{aligned}$$

$$\therefore \frac{P}{P'} = \frac{1}{2} \text{ i.e. } P : P' = 1 : 2$$

(ii) Area $A = xy$

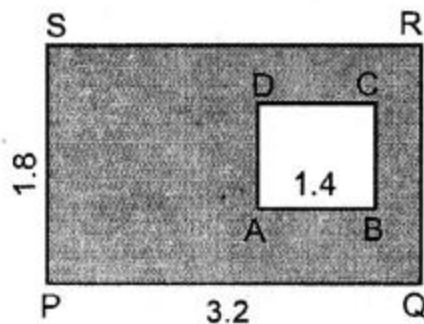
$$\text{New Area } A' = (2x)(2y) = 4xy = 4A$$

$$\therefore \frac{A}{A'} = \frac{1}{4} \text{ i.e. } A : A' = 1 : 4$$

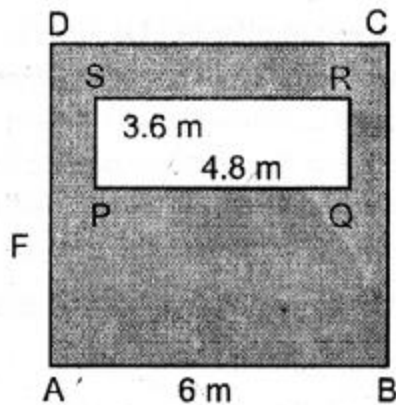
Question 13.

In each of the following cases ABCD is a square and PQRS is a rectangle. Find, in each case, the area of the shaded portion.
(All measurements are in metre).

(i)



(ii)



Solution:

(i) Area of the shaded portion

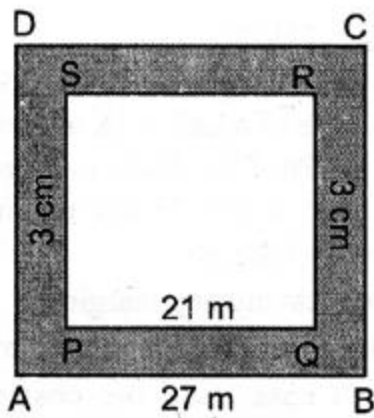
$$\begin{aligned}
 &= \text{Area of the rectangle PQRS} - \text{Area of square ABCD} \\
 &= 3.2 \times 1.8 - (1.4)^2 \quad (\because PQ = 3.2 \text{ and } PS = 1.8) \text{ Side of square AB} = 1.4 \\
 &= 5.76 - 1.96 = 3.80 = 3.8 \text{ m}^2 \\
 \text{(ii) Area of the shaded portion} &= \text{Area of square ABCD} - \text{Area of rectangle PQRS} \\
 &= 6 \times 6 - (3.6) (4.8) = 36 - 17.28 = 18.72 \text{ m}^2
 \end{aligned}$$

Question 14.

A path of uniform width, 3 m, runs around the outside of a square field of side 21 m. Find the area of the path.

Solution:

According to the given information the figure will be as shown alongside. Clearly, length of the square field excluding path = 21 m.



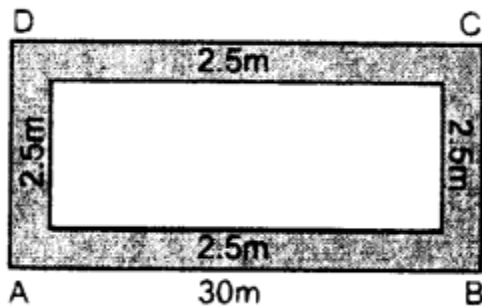
$$\begin{aligned}
 \text{Area of the square side excluding the path} &= 21 \times 21 = 441 \text{ m}^2 \\
 \text{Again, length of the square field including the path} &= 21 + 3 + 3 = 27 \text{ m} \\
 \text{Area of the square field including the path} &= 27 \times 27 = 729 \text{ m}^2 \\
 \text{Area of the path} &= 729 - 441 = 288 \text{ m}^2
 \end{aligned}$$

Question 15.

A path of uniform width, 2.5 m, runs around the inside of a rectangular field 30 m by 27 m. Find the area of the path.

Solution:

According to the given statement the figure will be as shown alongside.



Clearly, the length of the rectangular field including the path = 30 m.
Breadth = 27 m.

Its Area = $30 \times 27 = 810 \text{ m}^2$

Width of the path = 2.5 m

Length of the rectangular field including the path = $30 - 2.5 - 2.5 = 25 \text{ m}$.

Breadth = $27 - 2.5 - 2.5 = 22 \text{ m}$

Area of the rectangular field including the path = $25 \times 22 = 550 \text{ m}^2$

Hence, area of the path = $810 - 550 = 260 \text{ m}^2$.

Question 16.

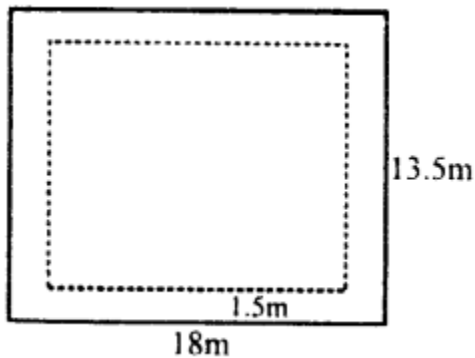
The length of a hall is 18 m and its width is 13.5 m . Find the least number of square tiles, each of side 25 cm , required to cover the floor of the hall,

(i) without leaving any margin.

(ii) leaving a margin of width 1.5 m all around. In each case, find the cost of the tiles required at the rate of Rs. 6 per tile

Solution:

(i) Length of hall (l) = 18 m and breadth (b) = 13.5 m



$$\begin{aligned}\therefore \text{Area of the floor} &= l \times b \\ &= 18 \times 13.5 \text{ m}^2 = 243.0 \text{ m}^2 \\ \text{Side of each square tiles (a)} &= 25 \text{ cm} \\ &= \frac{25}{100} = \frac{1}{4} \text{ m}\end{aligned}$$

$$\begin{aligned}\therefore \text{Area of one tile} &= a^2 = \frac{1}{4} \times \frac{1}{4} \\ &= \frac{1}{16} \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{No. of tiles required} &= 243 \div \frac{1}{16} \\ &= \frac{243 \times 16}{1} = 3888\end{aligned}$$

Rate of tiles = Rs. 6 per tile

$$\begin{aligned}\therefore \text{Total cost} &= \text{Rs. } 3888 \times 6 \\ &= \text{Rs. } 23328\end{aligned}$$

ii) Width of margin left in side = 1.5 m

$$\begin{aligned}\therefore \text{Inner length} &= 18 - 2 \times 1.5 = 18 - 3 = 15 \text{ m} \\ \text{and breadth} &= 13.5 - 2 \times 1.5 = 13.5 - 3 \\ &= 10.5 \text{ m}\end{aligned}$$

$$\begin{aligned}\therefore \text{Inner area} &= 15 \times 10.5 \text{ m}^2 \\ &= 157.5 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\therefore \text{No. of tiles} &= 157.5 \div \frac{1}{16} \\ &= 157.5 \times 16 = 2520\end{aligned}$$

$$\therefore \text{Cost of tiles} = 2520 \times 6 = \text{Rs. } 15120$$

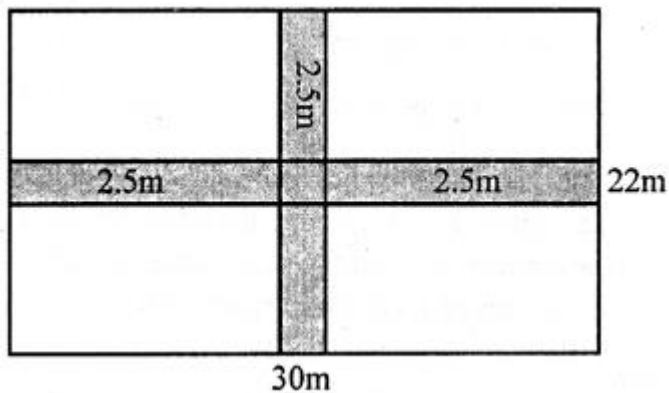
Question 17.

A rectangular field is 30 m in length and 22m in width. Two mutually perpendicular roads, each 2.5 m wide, are drawn inside the field so that one road is parallel to the length of the field and the other road is parallel to its width. Calculate the area of the crossroads.

Solution:

Length of rectangular field (l) = 30 m and breadth (b) = 22m

width of parallel roads perpendicular to each other inside the field = 2.5m



$$\begin{aligned}
 \text{Area of cross roads} &= \text{width of roads (Length + breadth)} - \text{area of middle square} \\
 &= 2.5 (30 + 22) - (2.5)^2 \\
 &= 2.5 \times 52 - 6.25 \text{ m}^2 \\
 &= (130 - 6.25) \text{ m} = 123.75 \text{ m}^2
 \end{aligned}$$

Question 18.

The length and the breadth of a rectangular field are in the ratio 5 : 4 and its area is 3380 m². Find the cost of fencing it at the rate of ₹75 per m.

Solution:

Ratio in length and breadth = 5 : 4

Area of rectangular field = 3380 m²

Let length = 5x and breadth = 4x

$$5x \times 4x = 3380$$

$$\Rightarrow 20x^2 = 3380$$

$$x^2 = 3380/20 = 169 = (13)^2$$

$$x = 13$$

$$\text{Length} = 13 \times 5 = 65 \text{ m}$$

$$\text{Breadth} = 13 \times 4 = 52 \text{ m}$$

$$\text{Perimeter} = (l + b) \times 2 = 2 \times (65 + 52) \text{ m} = 2 \times 117 = 234 \text{ m}$$

$$\text{Rate of fencing} = ₹ 75 \text{ per m}$$

$$\text{Total cost} = 234 \times 75 = ₹ 17550$$

Question 19.

The length and the breadth of a conference hall are in the ratio 7 : 4 and its perimeter is 110 m. Find:

(i) area of the floor of the hall.

(ii) number of tiles, each a rectangle of size 25 cm x 20 cm, required for flooring of the hall.

(iii) the cost of the tiles at the rate of ₹ 1,400 per hundred tiles.

Solution:

Ratio in length and breadth = 7 : 4

Perimeter = 110 m

$$\therefore \text{Length} + \text{Breadth} = \frac{110}{2} = 55 \text{ m}$$

$$\text{Sum of ratios} = 7 + 4 = 11$$

$$\therefore \text{Length} = \frac{55 \times 7}{11} = 35 \text{ m}$$

$$\text{and breadth} = \frac{55 \times 4}{11} = 20 \text{ m}$$

$$(i) \text{ Area of floor} = l \times b$$

$$= 35 \times 20 = 700 \text{ m}^2$$

$$(ii) \text{ Size of tile} = 25 \text{ cm} \times 20 \text{ cm}$$

$$= \frac{25 \times 20}{100 \times 100}$$

$$= \frac{1}{20} \text{ m}^2$$

$$\therefore \text{Number of tiles}$$

$$= \frac{\text{Area of floor}}{\text{Area of one tile}}$$

$$= \frac{700 \times 20}{1} = 14000$$

$$(iii) \text{ Cost of tiles} = ₹ 1400 \text{ per } 100 \text{ tiles}$$

$$\therefore \text{Total cost} = \frac{14000 \times 1400}{100}$$

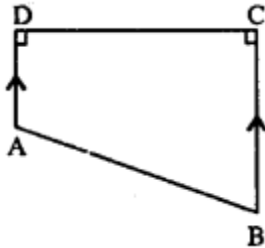
$$= ₹ 196000$$

EXERCISE 20(C)

Question 1.

The following figure shows the cross-section ABCD of a swimming pool which is trapezium in shape.

If the width DC, of the swimming pool is 6.4cm, depth (AD) at the shallow end is 80 cm and depth (BC) at deepest end is 2.4m, find Its area of the cross-section.



Solution:

Area of the cross-section = Area of trapezium ABCD

$$= \frac{1}{2} (\text{Sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2} (80 + 240) \times 6.4$$

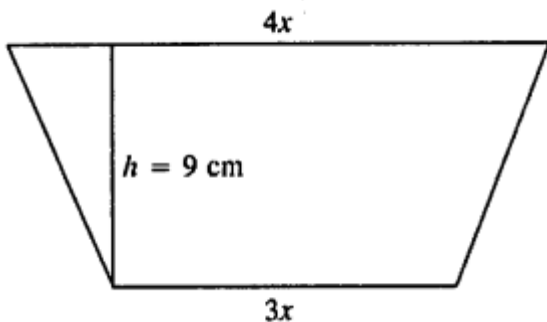
$$= (320)(3.2) = (32) (32)$$

$$= 1024 \text{ cm}^2 \text{ or } = 10.24 \text{ sq.m.}$$

Question 2.

The parallel sides of a trapezium are in the ratio 3 : 4. If the distance between the parallel sides is 9 dm and its area is 126 dm² ; find the lengths of its parallel sides.

Solution:



Let parallel sides of trapezium be

$$a = 3x$$

$$b = 4x$$

Distance between parallel sides, $h = 9 \text{ dm}$

area of trapezium = 126 dm^2

$$\frac{1}{2}(a + b) \times h = 126$$

$$\frac{1}{2}(3x + 4x) \times 9 = 126$$

$$7x \times 9 = 126 \times 2$$

$$x = \frac{126 \times 2}{7 \times 9}$$

$$x = 4$$

$$a = 3x = 3 \times 4 = 12 \text{ dm}$$

$$b = 4x = 4 \times 4 = 16 \text{ dm}$$

12 dm , 16 dm Ans.

Question 3.

The two parallel sides and the distance between them are in the ratio 3 : 4 : 2. If the area of the trapezium is 175 cm^2 , find its height.

Solution:

Let the two parallel sides and the distance between them be $3x$, $4x$, $2x$ cm respectively

Area = $\frac{1}{2}$ (sum of parallel sides) $\times$ (distance between parallel sides)

$$= \frac{1}{2} (3x + 4x) \times 2x = 175 \text{ (given)}$$

$$\Rightarrow 7x \times x = 175$$

$$\Rightarrow 7x^2 = 175$$

$$\Rightarrow x^2 = 25$$

$$\Rightarrow x = 5$$

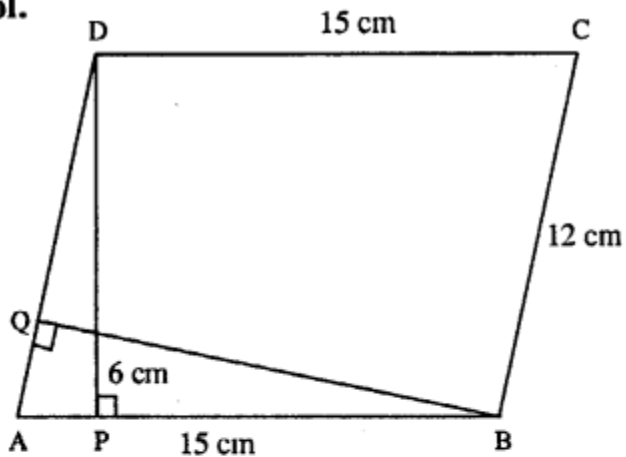
Height i.e. distance between parallel sides = $2x = 2 \times 5 = 10 \text{ cm}$

Question 4.

A parallelogram has sides of 15 cm and 12 cm; if the distance between the 15 cm sides is 6 cm; find the distance between 12 cm sides.

Solution:

Sol.



Base, $AB = 15 \text{ cm}$

Distance between 15 cm sides

i.e. height $DP = 6 \text{ cm}$

$$\begin{aligned}\therefore \text{Area of } \parallel\text{gm} &= \text{Base} \times \text{height} \\ &= AB \times DP \\ &= 15 \times 6 = 90 \text{ cm}^2\end{aligned}$$

Let BQ be distance between 12 cm sides

$$\therefore AD \times BQ = \text{area of } \parallel\text{gm } ABCD$$

$$\therefore 12 \times BQ = 90$$

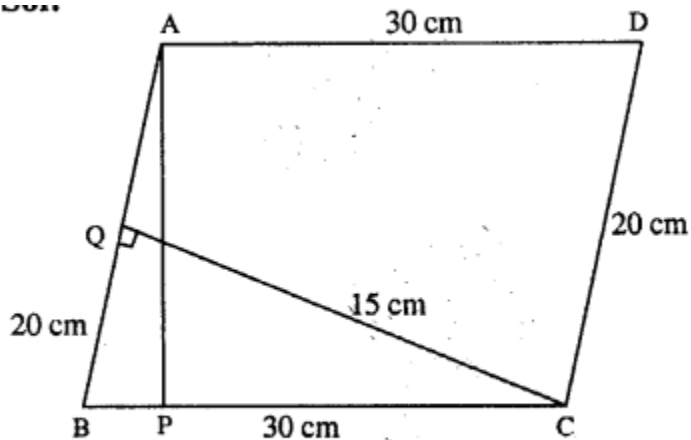
$$BQ = \frac{90}{12}$$

$$BQ = \frac{15}{2} = 7.5 \text{ cm}$$

Question 5.

A parallelogram has sides of 20 cm and 30 cm. If the distance between its shorter sides is 15 cm; find the distance between the longer sides.

Solution:



Let ABCD be the ||gm in which $BC = 30$ cm
and $CD = 20$ cm

Distance between shorter sides,

i.e. $CQ = 15$ cm

$$\begin{aligned}\therefore \text{area of ||gm} &= AB \times CQ \\ &= 20 \times 15 \\ &= 300 \text{ cm}^2\end{aligned}$$

Again $BC \times AP = \text{Area of || gm}$

$$30 \times AP = 300$$

$$AP = \frac{300}{30}$$

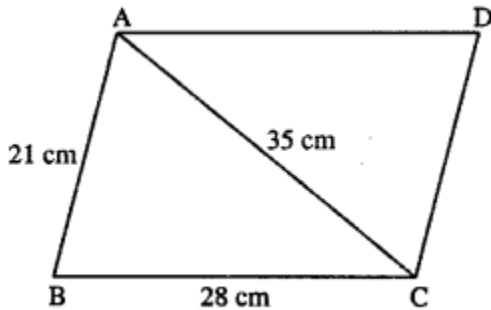
$$AP = 10 \text{ cm}$$

$\therefore$ Distance between larger sides is = 10 cm Ans.

Question 6.

The adjacent sides of a parallelogram are 21 cm and 28 cm. If its one diagonal is 35 cm; find the area of the parallelogram.

Solution:



First we find area of $\triangle ABC$.

Sides are, $a = 28$ cm

$b = 35$ cm

and $c = 21$ cm

$$\begin{aligned} S &= \frac{a+b+c}{2} \\ &= \frac{28+35+21}{2} \\ &= \frac{84}{2} = 42 \text{ cm} \end{aligned}$$

$$\text{area of } \triangle ABD = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{42(42-28)(42-35)(42-21)}$$

$$= \sqrt{42 \times 14 \times 7 \times 21}$$

$$= \sqrt{2 \times 21 \times 2 \times 7 \times 7 \times 21}$$

$$= \sqrt{2 \times 2 \times 21 \times 21 \times 7 \times 7}$$

$$= 2 \times 21 \times 7$$

$$= 294 \text{ cm}^2$$

$\therefore$ Diagonal of ||gm divides it into two equal parts.

$$\therefore \text{ area of ||gm} = 2 \times \text{area of } \triangle ABC$$

$$= 2 \times 294$$

$$= 588 \text{ cm}^2 \text{ Ans.}$$

Question 7.

The diagonals of a rhombus are 18 cm and 24 cm. Find:

(i) its area ;

(ii) length of its sides.

(iii) its perimeter;

Solution:

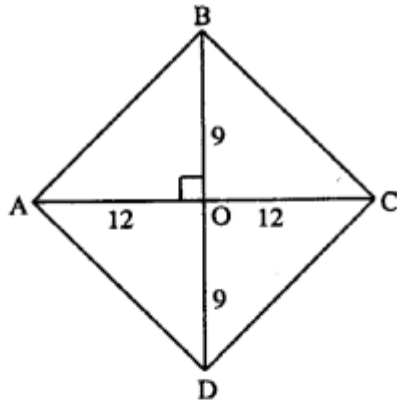
Diagonal of rhombus are 18 cm and 24 cm.

area of rhombus = $\frac{1}{2} \times$ Product of diagonals

$$= \frac{1}{2} \times 18 \times 24$$

$$= 216 \text{ cm}^2$$

(ii) Diagonals of rhombus bisect each other at right angles.



$$\therefore \quad OA = \frac{1}{2} \times 24 = 12 \text{ cm}$$

$$OB = \frac{1}{2} \times 18 = 9 \text{ cm}$$

In right $\angle$ d $\triangle AOB$

$$\begin{aligned} AB &= \sqrt{OA^2 + OB^2} \\ &= \sqrt{(12)^2 + (9)^2} \\ &= \sqrt{144 + 81} \\ &= \sqrt{225} = 15 \text{ cm} \end{aligned}$$

$\therefore$ Side of rhombus = 15 cm

$$\begin{aligned} \text{(iii) Perimeter of rhombus} &= 4 \times \text{side} \\ &= 4 \times 15 = 60 \text{ cm} \end{aligned}$$

(i) 216 cm^2 (ii) 15 cm (iii) 60 cm **Ans.**

Question 8.

The perimeter of a rhombus is 40 cm. If one diagonal is 16 cm; find :

(i) its another diagonal

(ii) area

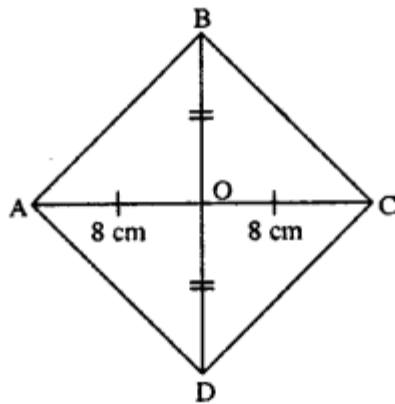
Solution:

(i) Perimeter of rhombus = 40 cm

side = $\frac{1}{4} \times 40 = 10$ cm

One diagonal = 16 cm

Diagonals of rhombus bisect each other at right angles.



$$OB = \sqrt{AB^2 - OA^2}$$

$$= \sqrt{(10)^2 - (8)^2}$$

$$= \sqrt{100 - 64}$$

$$= \sqrt{36}$$

$$= 6 \text{ cm}$$

$$\therefore \text{diagonal BD} = 6 \times 2 = 12 \text{ cm}$$

(ii) Area of rhombus

$$= \frac{1}{2} \times \text{product of diagonals}$$

$$= \frac{1}{2} \times 12 \times 16$$

$$= 96 \text{ cm}^2$$

$$\therefore \text{(i) 12 cm (ii) 96 cm}^2 \text{ Ans.}$$

Question 9.

Each side of a rhombus is 18 cm. If the distance between two parallel sides is 12 cm, find its area.

Solution:

Each side of the rhombus = 18 cm

base of the rhombus = 18 cm

Distance between two parallel sides = 12 cm

Height = 12 cm

Area of the rhombus = base x height = $18 \times 12 = 216 \text{ cm}^2$

Question 10.

The length of the diagonals of a rhombus is in the ratio 4 : 3. If its area is 384 cm^2 , find its side.

Solution:

Let the lengths of the diagonals of rhombus are $4x, 3x$.

$$\therefore \text{Area of the rhombus} = \frac{1}{2} (\text{Product of its diagonals})$$

$$= \frac{1}{2} (4x \times 3x) = 384 \text{ (given)}$$

$$\Rightarrow 6x^2 = 384 \Rightarrow x^2 = 64$$

$$\Rightarrow x = 8 \text{ cm}$$

$$\therefore \text{Diagonals are } 4 \times 8 = 32 \text{ cm}$$

$$\text{and } 3(8) = 24 \text{ cm.}$$

$$\therefore OC = 16 \text{ cm and } OD = 12 \text{ cm}$$

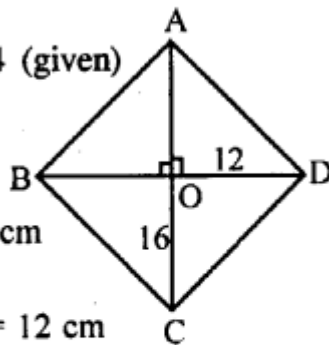
$$\therefore \text{Side DC} = \sqrt{OC^2 + OD^2}$$

$$\therefore \text{Side DC} = \sqrt{16^2 + 12^2}$$

[By Pythagoras Theorem in $\triangle DOC$]

$$= \sqrt{256 + 144} = \sqrt{400} = 20 \text{ cm}$$

Hence, side of the rhombus = 20 cm.



Question 11.

A thin metal iron-sheet is rhombus in shape, with each side 10 m. If one of its diagonals is 16 m, find the cost of painting its both sides at the rate of ₹ 6 per m^2 .

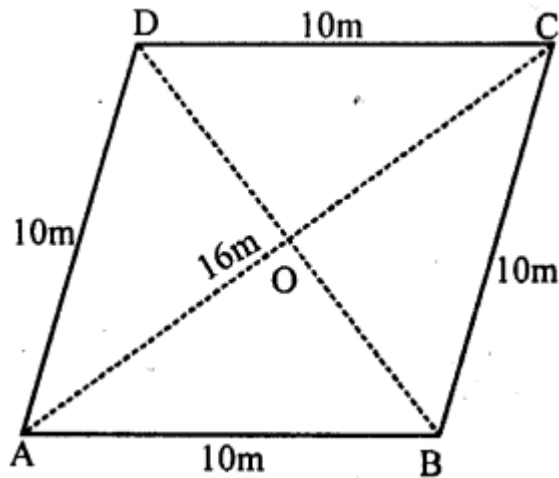
Also, find the distance between the opposite sides of this rhombus.

Solution:

Side of rhombus shaped iron sheet = 10 m and one diagonals (AC) = 16 m

Join BD diagonal which bisects AC at O

The diagonals of a rhombus bisect each other at right angle



$$\therefore AO = OC = \frac{16}{2} = 8 \text{ m.}$$

Now in right $\triangle AOB$

$$AB^2 = AO^2 + BO^2 \Rightarrow (10)^2 = (8)^2 + BO^2$$

$$\Rightarrow 100 = 64 + BO^2 \Rightarrow BO^2 = 100 - 64 = 36 \\ = (6)^2$$

$$\therefore BO = 6 \text{ m}$$

$$\therefore BD = 2 \times BO = 2 \times 6 = 12 \text{ m}$$

$$\text{Now, area of rhombus} = \frac{d_1 \times d_2}{2}$$

$$= \frac{16 \times 12}{2} = 96 \text{ m}^2$$

Rate of painting = ₹ 6 per m^2

$\therefore$ Total cost of painting both sides,

$$= 2 \times 96 \times 6 = ₹ 1152$$

Distance between two opposite sides,

$$= \frac{\text{Area}}{\text{Base}} = \frac{96}{10} = 9.6 \text{ m}$$

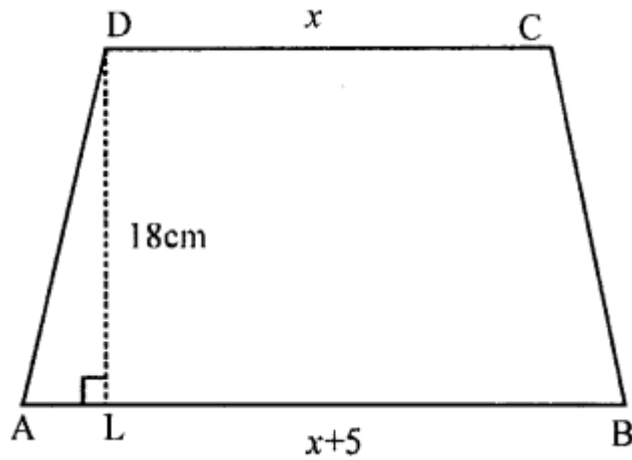
Question 12.

The area of a trapezium is 279 sq.cm and the distance between its two parallel sides is 18 cm. If one of its parallel sides is longer than the other side by 5 cm, find the lengths of its parallel sides.

Solution:

Area of trapezium = 279 sq.cm

Distance between two parallel lines (h) = 18 cm



$$\therefore \text{Sum of parallel sides} = \frac{\text{Area} \times 2}{\text{Height}}$$

$$= \frac{279 \times 2}{18} = 31 \text{ m}$$

Let shorter side, $CD = x$

Then longer side $= x + 5$

$$\therefore x + x + 5 = 31$$

$$\Rightarrow 2x = 31 - 5 = 26$$

$$\Rightarrow x = \frac{26}{2} = 13$$

$\therefore$ Shorter side = 13 cm

and longer side $= 13 + 5 = 18 \text{ cm}$

Question 13.

The area of a rhombus is equal to the area of a triangle. If base of Δ is 24 cm, its corresponding altitude is 16 cm and one of the diagonals of the rhombus is 19.2 cm. Find its other diagonal.

Solution:

Area of a rhombus = Area of a triangle
Base of triangle = 24 cm

and altitude = 16 cm

$$\therefore \text{Area} = \frac{1}{2} \text{base} \times \text{altitude}$$

$$= \frac{1}{2} \times 24 \times 16 = 192 \text{ cm}^2$$

$$\therefore \text{Area of rhombus} = 192 \text{ cm}^2$$

$$\text{One diagonal} = 19.2 \text{ cm}$$

$$\therefore \text{Second diagonal} = \frac{\text{Area} \times 2}{\text{One diagonal}}$$

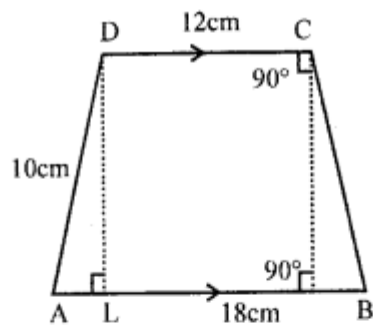
$$= \frac{192 \times 2}{19.2} = \frac{192 \times 10 \times 2}{192} = 20 \text{ cm}$$

Question 14.

Find the area of the trapezium ABCD in which $AB \parallel DC$, $AB = 18 \text{ cm}$, $\angle B = \angle C = 90^\circ$, $CD = 12 \text{ cm}$ and $AD = 10 \text{ cm}$.

Solution:

In trapezium ABCD,



$AB \parallel DC$, $AB = 18 \text{ cm}$

$\angle B = \angle C = 90^\circ$, $CD = 12 \text{ cm}$ and $AD = 10 \text{ cm}$

Area of trapezium ABCD

Draw $DL \perp AB$

$$\therefore AL = 18 - 12 = 6 \text{ cm}$$

$$AL = BC$$

$$AL = \sqrt{AD^2 - DL^2}$$

$$= \sqrt{10^2 - 6^2} = \sqrt{100 - 36}$$

$$= \sqrt{64} = 8 \text{ cm}$$

$$\text{Now area of trapezium} = \frac{1}{2} (AB + CD) \times AL$$

$$= \frac{1}{2} (18 + 12) \times 8 \text{ cm}^2$$

$$= \frac{1}{2} \times 30 \times 8 = 120 \text{ cm}^2$$

EXERCISE 20 (D)

Question 1.

Find the radius and area of a circle, whose circumference is :

(i) 132 cm

(ii) 22 m

Solution:

(i) Circumference of circle = 132 cm

$$2\pi r = 132$$

$$2 \times \frac{22}{7} \times r = 132$$

$$r = \frac{132 \times 7}{2 \times 22}$$

$$r = 21 \text{ cm}$$

$$\begin{aligned}\therefore \text{Area of circle} &= \pi r^2 \\ &= \frac{22}{7} \times 21 \times 21 \\ &= 1386 \text{ cm}^2 \text{ Ans.}\end{aligned}$$

(ii) Circumference of circle = 22 m

$$\therefore 2\pi r = 22$$

$$2 \times \frac{22}{7} \times r = 22$$

$$r = \frac{22 \times 7}{2 \times 22}$$

$$r = \frac{7}{2}$$

$$r = 3.5 \text{ m}$$

$$\begin{aligned}\text{Area of circle} &= \pi r^2 \\ &= \frac{22}{7} \times 3.5 \times 3.5 \\ &= 38.5 \text{ m}^2 \text{ Ans.}\end{aligned}$$

Question 2.

Find the radius and circumference of a circle, whose area is :

(i) 154 cm²

(ii) 6.16 m²

$$(i) \quad \text{Area of circle} = 154 \text{ cm}^2$$

$$\pi r^2 = 154$$

$$r^2 = \frac{154}{\pi}$$

$$r^2 = \frac{154}{22} \times 7$$

$$r^2 = 7 \times 7$$

$$r = 7 \text{ cm}$$

$$\therefore \text{circumference} = 2\pi r$$

$$= 2 \times \frac{22}{7} \times 7$$

$$= 44 \text{ cm}$$

Hence 7 cm ; 44 cm **Ans.**

$$(ii) \quad \text{Area of circle} = 616 \text{ m}^2$$

$$\pi r^2 = 616$$

$$\frac{22}{7} r^2 = \frac{616}{100}$$

$$r^2 = \frac{616}{100} \times \frac{7}{22}$$

$$r^2 = \frac{196}{100}$$

$$r^2 = 1.96$$

$$r = \sqrt{1.96}$$

$$r = 1.4 \text{ m}$$

$$\text{Circumference} = 2\pi r$$

$$= 2 \times \frac{22}{7} \times 1.4$$

$$= 8.8 \text{ m}$$

$\therefore$ 1.4 m; 8.8 m **Ans.**

Question 3.

The circumference of a circular table is 88 m. Find its area.

Solution:

Circumference of circle = 88 m

$$2\pi r = 88 \text{ m}$$

$$2 \times \frac{22}{7} \times r = 88$$

$$r = \frac{88 \times 7}{2 \times 22}$$

$$r = 14 \text{ m}$$

$$\begin{aligned} \text{Area of circle} &= \pi r^2 \\ &= \frac{22}{7} \times 14 \times 14 \\ &= 616 \text{ m}^2 \text{ Ans.} \end{aligned}$$

Question 4.

The area of a circle is 1386 sq.cm ; find its circumference.

Solution:

$$\text{Area of circle} = 1386 \text{ cm}^2$$

$$\pi r^2 = 1386$$

$$\frac{22}{7} r^2 = 1386$$

$$r^2 = 1386 \times \frac{7}{22}$$

$$r^2 = 441$$

$$r = \sqrt{441}$$

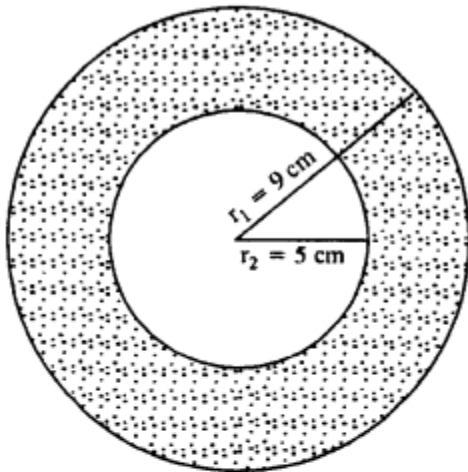
$$r = 21 \text{ cm}$$

$$\begin{aligned} \text{Circumference} &= 2\pi r \\ &= 2 \times \frac{22}{7} \times 21 \\ &= 132 \text{ m Ans.} \end{aligned}$$

Question 5.

Find the area of a flat circular ring formed by two concentric circles (circles with same centre) whose radii are 9 cm and 5 cm.

Solution:



External radius, $r_1 = 9 \text{ cm}$

Internal radius, $r_2 = 5 \text{ cm}$

$$\begin{aligned}\text{Area of ring} &= \pi r_1^2 - \pi r_2^2 \\ &= \pi(r_1^2 - r_2^2) \\ &= \pi(9^2 - 5^2) \\ &= \frac{22}{7} [81 - 25] \\ &= \frac{22}{7} \times 56 \\ &= 176 \text{ cm}^2 \text{ Ans.}\end{aligned}$$

Question 6.

Find the area of the shaded portion in each of the following diagrams :

(i)

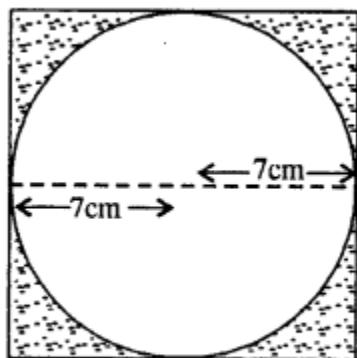


(ii)



Solution:

(i)



Radius of circle, $r = 7$ cm

$\therefore$ Side of Square $= 7 + 7 = 14$ cm

Area of circle $= \pi r^2$

$$= \frac{22}{7} \times 7 \times 7$$

$$= 154 \text{ cm}^2$$

Area of Square $= 14 \times 14$

$$= 196 \text{ cm}^2$$

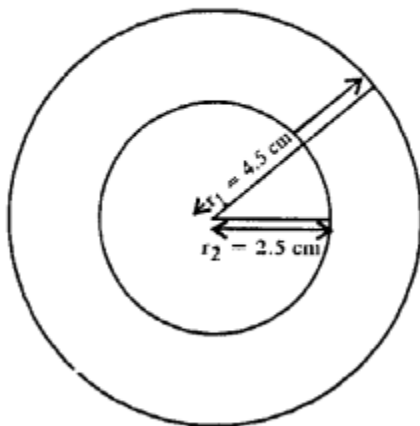
$\therefore$ Area of Shaded portion $= 196 - 154$

$$= 42 \text{ cm}^2 \text{ Ans.}$$

(ii) Radii of concentric circles are

$$r_1 = 4.5 \text{ m}$$

$$r_2 = 2.5 \text{ m}$$



$\therefore$ Area of shaded portion $= \pi r_1^2 - \pi r_2^2$

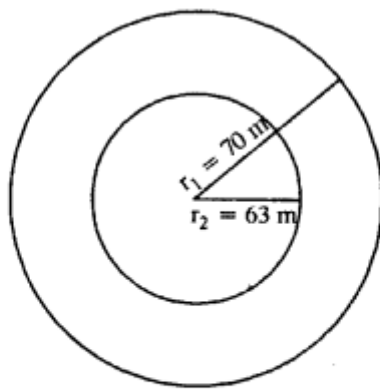
$$\begin{aligned}
 &= \pi[r_1^2 - r_2^2] = \frac{22}{7}[(4.5)^2 - (2.5)^2] \\
 &= \frac{22}{7} \times (4.5 + 2.5)(4.5 - 2.5) \\
 &= \frac{22}{7} \times 14 = 44 \text{ cm}^2 \text{ Ans.}
 \end{aligned}$$

Question 7.

The radii of the inner and outer circumferences of a circular running track are 63 m and 70 m respectively. Find :

- (i) the area of the track ;
- (it) the difference between the lengths of the two circumferences of the track.

Solution:



Outer radius, $r_1 = 70$ m

Inner radius, $r_2 = 63$ m

$$\begin{aligned}
 \therefore \text{Area of track} &= \pi r_1^2 - \pi r_2^2 \\
 &= \frac{22}{7}[(70)^2 - (63)^2] \\
 &= \frac{22}{7}(70 + 63)(70 - 63) \\
 &= \frac{22}{7} \times 133 \times 7 \\
 &= 2926 \text{ m}^2
 \end{aligned}$$

Length of outer edge *i.e.* circumference

$$\begin{aligned}
 &= 2\pi r_1 \\
 &= 2 \times \frac{22}{7} \times 70 = 440 \text{ m}
 \end{aligned}$$

Length of inner edge $= 2\pi r_2$

$$= 2 \times \frac{22}{7} \times 63 = 396 \text{ m}$$

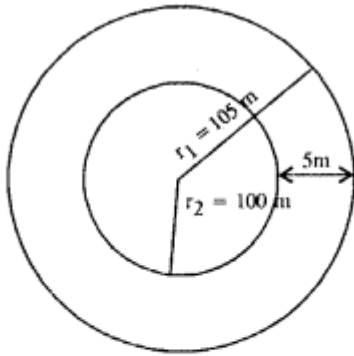
Difference between lengths of two circumferences $= 440 - 396 = 44 \text{ m}$

Hence (i) 2926 m^2 (ii) 44 m

Question 8.

A circular field of radius 105 m has a circular path of uniform width of 5 m along and inside its boundary. Find the area of the path.

Solution:



Radius of circular field, $r_1 = 105\text{ m}$

Width of path = 5 m

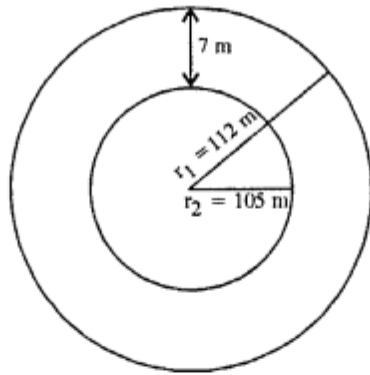
$\therefore$ Radius of inner circle, $r_2 = 105 - 5 = 100\text{ m}$

$$\begin{aligned}
 \therefore \text{Area of path} &= \pi r_1^2 - \pi r_2^2 \\
 &= \frac{22}{7} [(105)^2 - (100)^2] \\
 &= \frac{22}{7} (105 + 100) \\
 &\quad (105 - 100) \\
 &= \frac{22}{7} \times 205 \times 5 \\
 &= \frac{22550}{7} \text{ m}^2 \\
 &= 3221 \frac{3}{7} \text{ m}^2 \text{ Ans.}
 \end{aligned}$$

Question 9.

There is a path of uniform width 7 m round and outside a circular garden of diameter 210 m. Find the area of the path.

Solution:



$$\text{Diameter} = 210 \text{ m}$$

$$\text{Radius of inner circle } r_2 = 105 \text{ m}$$

$$\text{Width} = 7 \text{ m}$$

$$\text{Radius of outer circle } r_1 = 105 + 7 = 112 \text{ m}$$

$$\begin{aligned}\therefore \text{Area of path} &= \pi r_1^2 - \pi r_2^2 \\ &= \pi [r_1^2 - r_2^2] \\ &= \frac{22}{7} (r_1 + r_2)(r_1 - r_2) \\ &= \frac{22}{7} (112 + 105) \\ &\quad (112 - 105) \\ &= \frac{22}{7} \times 217 \times 7 \\ &= 4774 \text{ m}^2 \text{ Ans.}\end{aligned}$$

Question 10.

A wire, when bent in the form of a square encloses an area of 484 cm^2 . Find :

- (i) one side of the square ;
- (ii) length of the wire ;
- (iii) the largest area enclosed; if the same wire is bent to form a circle.

Solution:

$$(i) \text{ Area of Square} = 484 \text{ cm}^2$$

$$\text{Side of Square} = \sqrt{\text{Area}} = \sqrt{484} = 22 \text{ cm}$$

$$(ii) \text{ Perimeter, i.e. length of wire} = 4 \times 22 = 88 \text{ cm}$$

$$(iii) \text{ Circumference of circle} = 88 \text{ cm}$$

$$2\pi r = 88$$

$$2 \times \frac{22}{7} \times r = 88$$

$$r = \frac{88 \times 7}{2 \times 22}$$

$$r = 14 \text{ cm}$$

$$\therefore \text{The largest area enclosed} = \pi r^2$$

$$= \frac{22}{7} \times 14 \times 14$$

$$= 616 \text{ cm}^2$$

Hence (i) 22 cm (ii) 88 cm (iii) 616 cm² Ans.

Question 11.

A wire, when bent in the form of a square; encloses an area of 196 cm². If the same wire is bent to form a circle; find the area of the circle.

Solution:

Area of Square = 196 cm²

Side of Square = $\sqrt{\text{Area}} = \sqrt{196} = 14 \text{ cm}$

Perimeter of Square = 4 × 14 cm

i.e. length of wire = 56 cm

Circumference of circle = 56 cm

$$2\pi r = 56$$

$$2 \times \frac{22}{7} \times r = 56$$

$$r = \frac{56 \times 7}{2 \times 22}$$

$$r = \frac{98}{11} \text{ cm}$$

$$\therefore \text{Area of circle enclosed} = \pi r^2$$

$$= \frac{22}{7} \times \frac{98}{11} \times \frac{98}{11}$$

$$= 2744/11$$

$$249.45 \text{ cm}^2$$

Question 12.

The radius of a circular wheel is 42 cm. Find the distance travelled by it in :

(i) 1 revolution ;

(ii) 50 revolutions ;

(iii) 200 revolutions ;

Solution:

(i) Radius of wheel, $r = 42$ cm

Circumference i.e. distance travelled in 1 revolution $= 2\pi r = 2 \times \frac{22}{7} \times 42 = 264$ cm

(ii) Distance travelled in 50 revolutions $= 264 \times 50 = 13200$ cm $= 132$ m

(iii) Distance travelled in 200 revolutions $= 264 \times 200 = 52800$ cm $= 528$ m

Hence (i) 264 cm (ii) 132 m (iii) 528 m

Question 13.

The diameter of a wheel is 0.70 m. Find the distance covered by it in 500 revolutions. If the wheel takes 5 minutes to make 500 revolutions; find its speed in :

(i) m/s

(ii) km/hr.

Solution:

Diameter $= 0.70$ m

Radius, $r = 0.35$ m

Distance covered in 1 revolution, i.e. circumference $= 2\pi r = 2 \times \frac{22}{7} \times 0.35 = 2.20$ m

Distance covered in 500 revolutions $= 2.20 \times 500 = 1100$ m

Time taken $= 5$ minutes $= 5 \times 60 = 300$ sec.

$$\begin{aligned} \therefore \text{Speed in m/s} &= \frac{1100}{300} \\ &= \frac{11}{3} = 3\frac{2}{3} \text{ m/s} \end{aligned}$$

$$\text{Again, Distance} = 1100 \text{ m}$$

$$= \frac{1100}{1000}$$

$$= \frac{11}{10} \text{ km}$$

$$\text{Time} = 5 \text{ minutes}$$

$$= \frac{5}{60} \text{ hr.}$$

$$\text{Speed in km/hr} = \frac{\frac{11}{10}}{\frac{5}{60}} = \frac{11}{10} \times \frac{60}{5}$$

$$= \frac{66}{5} = 13.2 \text{ km/hr.}$$

Hence 1100 m, (i) $3\frac{2}{3}$ m/s (ii) 13.2 km/hr Ans.

Question 14.

A bicycle wheel, diameter 56 cm, is making 45 revolutions in every 10 seconds. At what speed in kilometre per hour is the bicycle travelling ?

Solution:

Sol. Diameter = 56 cm

$\therefore$ Radius, r = 28 cm

$\therefore$ Distance travelled in 1 revolution

$$\text{i.e. circumference} = 2\pi r = 2 \times \frac{22}{7} \times 28 = 176 \text{ cm}$$

$\therefore$ Distance travelled in 45 revolutions

$$= 176 \times 45 = 7920 \text{ cm} = \frac{7920}{100 \times 1000} \text{ km}$$

$$\text{Time} = 10 \text{ sec} = \frac{10}{60 \times 60} \text{ hr.}$$

$$\text{Speed} = \frac{\frac{7920}{100 \times 1000}}{\frac{10}{60 \times 60}} \text{ m}$$

$$= \frac{7920}{100 \times 1000} \times \frac{60 \times 60}{10} = \frac{28512}{1000} \text{ km/hr}$$

$$= 28.512 \text{ km/hr Ans.}$$

Question 15.

A roller has a diameter of 1.4 m. Find :

(i) its circumference ;

(ii) the number of revolutions it makes while travelling 61.6 m.

Solution:

Diameter = 1.4 m

$$r = \frac{1.4}{2} = 0.7 \text{ m}$$

$\therefore$ Circumference of roller = $2\pi r$

$$= 2 \times \frac{22}{7} \times 0.7 = 4.4 \text{ m}$$

Revolutions made in 4.4 m distance = 1

$$\text{Revolutions made in 1 m distance} = \frac{1}{4.4}$$

Revolutions made in 61.6 m distance

$$= \frac{1}{4.4} \times 61.6 = \frac{616}{44} = 14$$

Hence (i) 4.4 m (ii) 14 **Ans.**

Question 16.

Find the area of the circle, length of whose circumference is equal to the sum of the lengths of the circumferences with radii 15 cm and 13 cm.

Solution:

In a circle

Circumference = Sum of circumferences of two circle of radii 15 cm and 13 cm

Now circumference of first smaller circle = $2\pi r$

$$= 2 \times \frac{22}{7} \times 15 = \frac{660}{7} \text{ cm}$$

Circumference of second smaller circle

$$= 2 \times \frac{22}{7} \times 13 = \frac{572}{7} \text{ cm}$$

$\therefore$ Circumference of bigger circle

$$= \frac{660}{7} + \frac{572}{7} = \frac{1232}{7} \text{ cm}$$

Let R be its radius, then

$$2\pi R = \frac{1232}{7} \Rightarrow \frac{2 \times 22}{7} R = \frac{1232}{7}$$

$$\Rightarrow R = \frac{1232}{7} \times \frac{7}{44} = 28 \text{ cm}$$

$\therefore$ Area of the circle = πR^2

$$= \frac{22}{7} \times 28 \times 28 \text{ cm}^2 = 2464 \text{ cm}^2$$

Question 17.

A piece of wire of length 108 cm is bent to form a semicircular arc bounded by its diameter. Find its radius and area enclosed.

Solution:

Length of wire = 108 cm

Let r be the radius of the semicircle

$$\pi r + 2r = 108$$

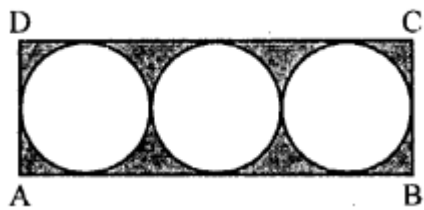
$$\Rightarrow r(\pi + 2) = 108 \Rightarrow r \left(\frac{22}{7} + 2 \right) = 108$$

$$\Rightarrow \frac{36}{7} r = 108 \Rightarrow r = \frac{108 \times 7}{36} = 21 \text{ cm}$$

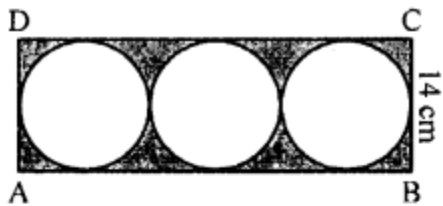
$$\begin{aligned} \text{Area} &= \frac{\pi r^2}{2} = \frac{22}{7 \times 2} \times 21 \times 21 = \frac{1386}{2} \text{ cm}^2 \\ &= 693 \text{ cm}^2 \end{aligned}$$

Question 18.

In the following figure, a rectangle ABCD enclosed three circles. If BC = 14 cm, find the area of the shaded portion (Take $\pi = \frac{22}{7}$)



In rectangle ABCD, BC = 14 cm



$$\therefore \text{Radius of each circle} = \frac{14}{2} = 7 \text{ cm}$$

$$\therefore \text{Length of rectangle} = (7 + 7 + 7) \times 2 \\ = 42 \text{ cm}$$

$$\text{Now area of rectangle} = l \times b = 42 \times 14 \\ = 588 \text{ cm}^2$$

$$\text{and area of 3 circles} = 3 \times \pi r^2$$

$$= 3 \times \frac{22}{7} \times 7 \times 7 = 462 \text{ cm}^2$$

$$\therefore \text{Area of shaded portion} \\ = 588 - 462 = 126 \text{ cm}^2$$