

# ICSE 2024 EXAMINATION

## MATHEMATICS

### SAMPLE PAPER - 2

Time Allowed : 2½ hours

Max. Marks : 80

#### General Instructions :

- Attempt all questions from Section A and any four questions from Section B.*  
*All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.*  
*Omission of essential working will result in loss of marks.*  
*The intended marks for questions or parts of questions are given in brackets [ ]*  
*Mathematical tables are provided.*

#### SECTION - A (40 Marks)

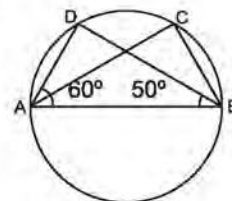
(Attempt all questions from this Section)

Question 1 : Choose the correct answers to the questions from the given options:

[15]

- (i) A shopkeeper bought an LED for ₹24,000 from a dealer. He sold it to a consumer at a profit of ₹6000. If the rate of GST is 28%, then the tax liability of the shopkeeper is :  
 (a) ₹1680 (b) ₹1800 (c) ₹2000 (d) ₹2420
- (ii) The roots of the quadratic equation  $3x^2 - 2x + 1 = 0$  are :  
 (a) real and distinct (b) rational and unequal (c) real and equal (d) not real
- (iii) On dividing  $f(x)$  by  $(2x + 3)$ , the remainder is :  
 (a) 0 (b)  $f\left(\frac{2}{3}\right)$  (c)  $f\left(\frac{-3}{2}\right)$  (d)  $f\left(\frac{-2}{3}\right)$
- (iv) Which of the following is a diagonal matrix?  
 (a)  $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$  (b)  $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 8 \end{bmatrix}$  (c)  $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 5 & 4 \\ 1 & 0 & 0 \end{bmatrix}$  (d) none of these
- (v) 10% ₹100 shares at ₹120 means annual dividend on 1 share is:  
 (a) ₹10 (b) ₹12 (c) ₹2 (d) none of these
- (vi) The reflection of the point  $P(-2, 3)$  in the  $y$ -axis is:  
 (a)  $(2, 3)$  (b)  $(2, -3)$  (c)  $(-2, -3)$  (d)  $(-2, 3)$
- (vii) If  $\triangle ABC \sim \triangle DEF$ , then  $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF}$  is :  
 (a)  $AB \times DE$  (b)  $\frac{BC}{EF}$  (c)  $\frac{AC}{DE}$  (d)  $\frac{AB}{EF}$
- (viii) The radii of two circular cylinders are in the ratio 4 : 5, and their heights are in the ratio 5 : 3. The ratio of their volumes is:  
 (a) 8 : 5 (b) 16 : 15 (c) 17 : 13 (d) 13 : 8
- (ix) The 5th term from the end of the GP 2, 6, 18, ... 13122 is:  
 (a) 54 (b) 162 (c) 486 (d) 1458
- (x) A bag contains 4 red and 8 blue marbles. A marble is drawn at random. The probability of drawing a red marble is:  
 (a)  $\frac{1}{2}$  (b)  $\frac{1}{4}$  (c)  $\frac{1}{3}$  (d)  $\frac{1}{5}$

- (xi) In a size transformation with scale factor  $k$ , the area of the given figure is
- (a)  $k \times$  area of the resulting figure (b)  $k^2 \times$  area of the resulting figure
- (c)  $\frac{1}{k} \times$  area of the resulting figure (d)  $\frac{1}{k^2} \times$  area of the resulting figure
- (xii) The point P(1, 2) divides the join of A(-2, 1) and B(7, 4) in the ratio:
- (a) 1 : 2 (b) 2 : 1 (c) 3 : 2 (d) 2 : 3
- (xiii) In the given figure, if  $\angle DAB = 60^\circ$  and  $\angle ABD = 50^\circ$ , then  $\angle ACB$  is equal to :
- (a)  $60^\circ$  (b)  $50^\circ$  (c)  $70^\circ$  (d)  $80^\circ$
- (xiv) The sum of first 10 terms of the AP whose  $n$ th term is  $T_n = 2n + 1$ , is :
- (a) 100 (b) 120 (c) 125 (d) 130
- (xv) The lower quartile of given observations 21, 14, 19, 18, 10, 15, 11 is:
- (a) 10 (b) 11 (c) 14 (d) 15



**Question 2 :**

- (i) Mrs. Bhatnagar has a 4 years time deposit account and deposits ₹650 per month. If she received ₹36,296 at the time of maturity, find the rate of interest. [4]
- (ii) If  $a, b, c$  are in continued proportion, prove that  $\frac{(a+b)^2}{(b+c)^2} = \frac{a}{c}$ . [4]
- (iii) By selling at ₹92 some  $2\frac{1}{2}\%$  shares of face value ₹100 and investing the proceeds in 5% shares of face value ₹100 selling at ₹115, a person increased his income by ₹90 a year. Find . [4]
- (a) the number of shares sold (b) the number of shares bought
- (c) the original income

**Question 3 :**

- (i) The curved surface area of a cylinder is  $5,500 \text{ cm}^2$  and the circumference of the base is 110 cm. Find the height and volume of the cylinder. [4]
- (ii) Find the equation of the straight line passing through the intersection of  $2x + 5y = 4$  with  $x$ -axis and parallel to the line  $3x - 7y = -8$ . [4]
- (iii) Use graph paper for this question. Take 1 cm = 1 unit on both axes.
- (a) Plot the points P (2, 3) and Q (3, 1).
- (b) Reflect P in  $x$ -axis to P'. Reflect P' in  $y$ -axis to P''. Write coordinates of P' and P''.
- (c) Reflect Q in  $y$ -axis to Q' and reflect Q' in the origin to Q''. Write coordinates of Q' and Q''.
- (d) Write the geometrical name of PQQ'P'. [5]

**SECTION - B (40 Marks)**

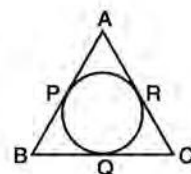
**(Attempt any four questions from this Section)**

**Question 4 :**

- (i) A dealer purchased goods for ₹16,50,000 and sold them for ₹20,00,000 within the state. If the rate of GST is 12%, find the net CGST and SGST payable by the dealer. [3]
- (ii) Solve the quadratic equation  $x^2 - 3(x + 3) = 0$ ; Give your answer correct to two significant figures. [3]
- (iii) If  $p$ th,  $q$ th and  $r$ th terms of a GP be  $a, b, c$  respectively, then prove that  $a^{q-r} \times b^{r-p} \times c^{p-q} = 1$ . [4]

**Question 5 :**

- (i) If  $A = \begin{bmatrix} 1 & -2 \\ 3 & 6 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & -4 \\ -2 & 5 \end{bmatrix}$ , find a matrix X such that  $X + A = B$ . [3]
- (ii) In the figure, the incircle of  $\triangle ABC$  touches the sides AB, BC and CA at the points P, Q, R respectively.
- Show that  $AP + BQ + CR = BP + CQ + AR$
- $= \frac{1}{2}$  (perimeter of  $\triangle ABC$ ).



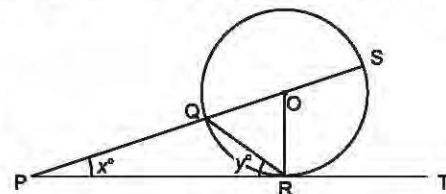
- (iii) If the expression  $x^3 + ax + b$  is divided by  $(x + 1)$  and  $(x + 2)$ , it gives the same remainder 12. Find the values of  $a$  and  $b$ . [4]

**Question 6 :**

- (i) Find the gradient and the intercept on the  $y$ -axis of the line  $3y + 2x - 12 = 0$ . [3]  
 (ii) Show that  $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$  [3]  
 (iii) Find the sum  $(-5) + (-8) + (-11) + \dots (-62)$ . [4]

**Question 7 :**

- (i) The king, queen and jack of clubs are removed from a deck of playing cards and the remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of heart. [3]  
 (ii) The sides of a rectangular plot of land in a map were 12 cm and 5 cm. The map was drawn to scale 1 : 2000. Find the (a) diagonal of the plot in m (b) area of the plot in  $m^2$ . [3]  
 (iii) In the figure, PT touches a circle with centre O at R. Diameter SQ when produced meets PT at P. If  $\angle SPR = x^\circ$ , and  $\angle QRP = y^\circ$ , show that  $x^\circ + 2y^\circ = 90^\circ$ . [4]

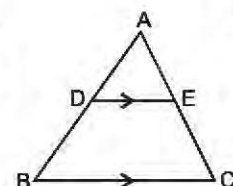


**Question 8 :**

- (i) Solve the inequation :  $8 < 5(x + 1) - 2 \leq 18$ ,  $x \in R$ . Graph the solution set. [3]  
 (ii) Find the mean of the following data : [3]

| Class     | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|-----------|------|-------|-------|-------|-------|
| Frequency | 12   | 16    | 6     | 7     | 9     |

- (iii) In the figure,  $DE \parallel BC$ . If  $AD = 3.4$  cm,  $AB = 8.5$  cm, and  $AC = 13.5$  cm, find  $AE$ . [4]



**Question 9 :**

- (i) Draw a circle of radius 4 cm and mark two chords AB and AC of the circle of length 6 cm and 5 cm respectively. [4]  
 (a) Construct the locus of points inside the circle that are equidistant from A and C.  
 (b) Construct the locus of points inside the circle that are equidistant from AB and AC.  
 (ii) Weight of 100 students is recorded below :

| Weight in kg    | 30-35 | 35-40 | 40-45 | 45-50 | 50-55 | 55-60 |
|-----------------|-------|-------|-------|-------|-------|-------|
| No. of students | 4     | 16    | 40    | 22    | 10    | 8     |

Draw an ogive and hence estimate the median. [6]

**Question 10 :**

- (i) If  $\frac{7m+2n}{7m-2n} = \frac{5}{3}$ , use properties of proportion to find,  $m : n$  [3]  
 (ii) Construct a triangle ABC in which  $AB = 6$  cm,  $BC = 4$  cm and  $AC = 5.5$  cm. Draw the incircle of the triangle. [3]  
 (iii) From the top of a building 20 m high, the angle of elevation of the top of a monument is  $45^\circ$  and the angle of depression of its foot is  $15^\circ$ . Find the height of the monument. [4]

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## ANSWERS

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1. (i) (a) (ii) (d) (iii) (c) (iv) (b) (v) (a) (vi) (a) (vii) (b) (viii) (b) (ix) (b)  
(x) (c) (xi) (d) (xii) (a) (xiii) (c) (xiv) (b) (xv) (b)
2. (i) 8% p.a. (iii) (a) 60 (b) 48 (c) ₹150
3. (i) 50 cm, 48,125 cm<sup>3</sup> (ii)  $3x - 7y = 6$  (iii) (b) (2, -3), (-2, -3) (c) (-3, 1), (3, -1) (d) Isosceles trapezium
4. (i) ₹21,000, ₹21,000 (ii) 4.9 or -1.9
5. (i)  $\begin{bmatrix} 2 & -2 \\ -5 & -1 \end{bmatrix}$  (iii) -7, 6
6. (i)  $-\frac{2}{3}$ , 4 units (iii) -670
7. (i)  $\frac{13}{49}$  (ii) (a) 260 m (b) 24,000 m<sup>2</sup>
8. (i)  $\{x : 1 \leq x \leq 3, x \in \mathbb{R}\}$  (ii) 22 (iii) 5.4 cm
9. (ii) 44 10. (i) 8 : 7 (iii) 94.64 m

# SOLUTION

Time Allowed : 2½ hours

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## SECTION - A (40 Marks)

(Attempt all questions from this Section)

Question 1 : Choose the correct answers to the questions from the given options:

[15]

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- (v) 10% ₹100 shares at ₹120 means annual dividend on 1 share is:  
(a) ₹10 (b) ₹12 (c) ₹2 (d) none of these
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- (vii) If  $\triangle ABC \sim \triangle DEF$ , then  $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF}$  is :  
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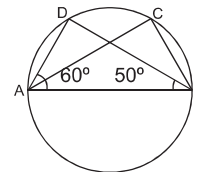
- (xi) In a size transformation with scale factor  $k$ , the area of the given figure is
- (a)  $k \times$  area of the resulting figure (b)  $k^2 \times$  area of the resulting figure
- (c)  $\frac{1}{k} \times$  area of the resulting figure (d)  $\frac{1}{k^2} \times$  area of the resulting figure

(xii) The point P(1, 2) divides the join of A(-2, 1) and B(7, 4) in the ratio:

- (a) 1 : 2 (b) 2 : 1 (c) 3 : 2 (d) 2 : 3

(xiii) In the given figure, if  $\angle DAB = 60^\circ$  and  $\angle ABD = 50^\circ$ , then  $\angle ACB$  is equal to :

- (a)  $60^\circ$  (b)  $50^\circ$   
(c)  $70^\circ$  (d)  $80^\circ$



(xiv) The sum of first 10 terms of the AP whose  $n$ th term is  $T_n = 2n + 1$ , is :

- (a) 100 (b) 120 (c) 125 (d) 130

(xv) Assertion (A) : If the median of the data 16, 18, 20,  $24 - x$ ,  $22 + 2x$ , 28, 30, 32 arranged in ascending order is 24, then the value of  $x$  is 2.

Reason (R) : To find the median of a data, we should always arrange the data in ascending order.

- (a) A is true, R is false (b) A is false, R is true (c) both A and R are true (d) both A and R are false

**Solution :**

- (i) (a) Disered tax liability = 28% of ₹6000 = ₹1680.
- (ii) (d) Here  $D = (-2)^2 - 4 \times 3 \times 1 = -8 < 0$   
So, the roots are not real.
- (iii) (c)  $2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$ . So, required remainder is  $f\left(-\frac{3}{2}\right)$
- (iv) (b)
- (v) (a) Annual dividend = 10% of ₹100 = ₹10
- (vi) (a)  $R_y(-2, -3) = (2, 3)$
- (vii) (b)
- (viii) (b) Required ratio =  $\frac{\pi \times (4x)^2 \times 5y}{\pi \times (5x)^2 \times 3y} = \frac{16 \times 5}{25 \times 3} = 16 : 15$ .
- (ix) (b) Required term =  $\frac{13122}{3^4} = 162$ .
- (x) (c) Required probability =  $\frac{4}{12} = \frac{1}{3}$
- (xi) (d)
- (xii) (a) Let the required ratio be  $m : 1$ ; Then,  $1 = \frac{m \times 7 + 1 \times (-2)}{m + 1}$   
 $\Rightarrow m + 1 = 7m - 2 \Rightarrow 6m = 3 = m = \frac{1}{2}$   
 $\therefore$  Required ratio is  $\frac{1}{2} : 1 = 1 : 2$ .
- (xiii) (c)  $\angle ADB = 180^\circ - (60^\circ + 50^\circ) = 70^\circ$   
 $\angle ACB = \angle ADB = 70^\circ$
- (xiv) (b)  $d = T_2 - T_1 = 5 - 3 = 2 \Rightarrow S_{10} = \frac{10}{2} [2 \times 3 + (10 - 1) \times 2] = 120$ .
- (xv) (a) Here,  $n = 8$ , so median =  $\frac{1}{2} \left[ \text{value of } \left(\frac{8}{2}\right)^{\text{th}} \text{ observation} + \text{value of } \left(\frac{8}{2} + 1\right)^{\text{th}} \text{ observation} \right]$   
 $= \frac{1}{2} (24 - x + 22 + 2x) = \frac{1}{2} (46 + x)$   
 $\Rightarrow 24 = \frac{1}{2} (46 + x) \Rightarrow 46 + x = 48 \Rightarrow x = 2$ .

**Question 2 :**

- (i) Mrs. Bhatnagar has a 4 years time deposit account and deposits ₹650 per month. If she received ₹36296 at the time of maturity, find the rate of interest. [4]
- (ii) If  $a, b, c$  are in continued proportion, prove that  $\frac{(a+b)^2}{(b+c)^2} = \frac{a}{c}$ . [4]

- (iii) By selling at ₹92 some  $2\frac{1}{2}\%$  shares of face value ₹100 and investing the proceeds in 5% shares of face value ₹100 selling at ₹115, a person increased his income by ₹90 a year. Find . [4]

(a) the number of shares sold  
(c) the original income

(b) the number of shares bought

**Solution :** (i)  $n = 48$  months,  $P = ₹ 650$

$$\therefore \text{S. I.} = P \times \frac{n(n+1)}{2} \times \frac{r}{100} \times \frac{1}{12} = ₹ \left\{ 650 \times \frac{48 \times 49}{2} \times \frac{r}{100} \times \frac{1}{12} \right\} = ₹ 637r$$

$\therefore$  Total amount received on maturity = ₹  $(48 \times 650 + 637r)$

But,  $48 \times 650 + 637r = 36296$

$$\Rightarrow 637r = 36296 - 31200 = 5096 \Rightarrow r = \frac{5096}{637} = 8$$

Hence, rate of interest = 8% **Ans.**

- (ii) Let  $a$ ,  $b$  and  $c$  are in continued proportion. Then

$$\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$$

$$\text{Now, LHS} = \frac{(a+b)^2}{(b+c)^2} = \frac{a^2 + b^2 + 2ab}{b^2 + c^2 + 2bc} = \frac{a^2 + ac + 2ab}{ac + c^2 + 2bc} = \frac{a(a+c+2b)}{c(a+c+2b)} = \frac{a}{c} = \text{RHS} \quad \text{Proved.}$$

- (iii) (a) Let the number of shares sold =  $x$

$$\therefore \text{Sale proceeds} = ₹92 \times x = ₹92x$$

$$\therefore \text{Number of shares bought} = \frac{92x}{115}$$

$$\therefore \text{New income} = ₹ \frac{92x}{115} \times 5 = ₹ \frac{92x}{23}$$

$$\text{Original income} = ₹ 2\frac{1}{2} \times x = ₹ \frac{5}{2}x$$

$$\text{But, } \frac{92x}{23} - \frac{5}{2}x = 90 \Rightarrow \frac{184x - 115x}{46} = 90 \Rightarrow \frac{69x}{46} = 90 \Rightarrow x = 90 \times \frac{46}{69} = 60$$

$\therefore$  Number of shares sold = 60 **Ans.**

$$(b) \text{ Number of shares bought} = \frac{92x}{115} = \frac{92 \times 60}{115} = 48 \quad \text{Ans.}$$

$$(c) \text{ Original income} = ₹ \frac{5}{2}x = ₹ \frac{5}{2} \times 60 = ₹150 \quad \text{Ans.}$$

**Question 3 :**

- (i) The curved surface area of a cylinder is  $5,500 \text{ cm}^2$  and the circumference of the base is 110 cm. Find the height and volume of the cylinder. [4]
- (ii) Find the equation of the straight line passing through the intersection of  $2x + 5y = 4$  with  $x$ -axis and parallel to the line  $3x - 7y = -8$ . [4]
- (iii) Use graph paper for this question. Take 1 cm = 1 unit on both axes.
- (a) Plot the points P (2, 3) and Q (3, 1).
- (b) Reflect P in  $x$ -axis to P'. Reflect P' in  $y$ -axis to P''. Write coordinates of P' and P''.
- (c) Reflect Q in  $y$ -axis to Q' and reflect Q' in the origin to Q''. Write coordinates of Q' and Q''.
- (d) Write the geometrical name of PQQ''P'. [5]

**Solution :**

$$(i) 2\pi r = 110 \Rightarrow r = \frac{110 \times 7}{2 \times 22} = 17.5 \text{ cm}$$

Also, curved surface area of the cylinder =  $2\pi rh$

$$\Rightarrow 5,500 = 2\pi rh \Rightarrow 5,500 = 110 \times h \Rightarrow h = \frac{5,500}{110} = 50 \text{ cm} \quad \text{Ans.}$$

$$\therefore \text{Volume of the cylinder} = \pi r^2 h = \frac{22}{7} \times 17.5 \times 17.5 \times 50 \text{ cm}^3 = 48,125 \text{ cm}^3 \quad \text{Ans.}$$

(ii) Given line is  $2x + 5y = 4$ ; Equation of x-axis is  $y = 0$ .

$$\therefore 2x + 0 = 4 \Rightarrow x = 2$$

$\Rightarrow$  Coordinates of the point of intersection of  $2x + 5y = 4$  and x-axis are (2, 0).

Let the equation of the required line be  $y = mx + c$  ....(1)

$\therefore$  (1) passes through (2, 0)

$$\therefore 0 = 2m + c \Rightarrow 2m + c = 0 \quad \dots (2)$$

Again, (1) is parallel to  $3x - 7y = -8$

$$\text{or } y = \frac{3x}{7} + \frac{8}{7}$$

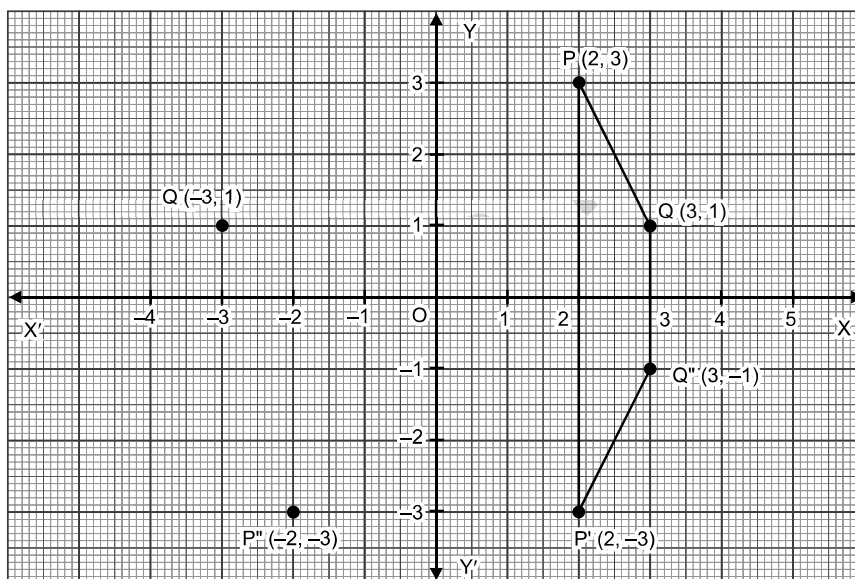
$$\therefore \text{Gradient } m \text{ of (1)} = \frac{3}{7}$$

$$\text{Putting } m = \frac{3}{7} \text{ in (2), we have, } 2 \times \frac{3}{7} + c = 0 \Rightarrow c = -\frac{6}{7}$$

$$\text{Substituting } m = \frac{3}{7} \text{ and } c = -\frac{6}{7} \text{ in (1), we get, } y = \frac{3}{7}x - \frac{6}{7}$$

$$\Rightarrow 7y = 3x - 6 \Rightarrow 3x - 7y = 6, \text{ which is the required equation. } \text{Ans.}$$

(iii)



(a) The given points have been plotted.

(b) Coordinates of  $P'$  are (2, -3) and coordinates of  $P''$  are (-2, -3) **Ans.**

(c) Coordinates of  $Q'$  and  $Q''$  are respectively (-3, 1), and (3, -1). **Ans.**

(d)  $PQQ'P'$  is an isosceles trapezium. **Ans.**

## SECTION - B (40 Marks)

(Attempt *any four* questions from this Section)

**Question 4 :**

(i) A dealer purchased goods for ₹16,50,000 and sold them for ₹20,00,000 within the state. If the rate of GST is 12%, find the net CGST and SGST payable by the dealer. [3]

(ii) Solve the quadratic equation  $x^2 - 3(x + 3) = 0$ ; Give your answer correct to two significant figures. [3]

(iii) If  $p$ th,  $q$ th and  $r$ th terms of a GP be  $a$ ,  $b$ ,  $c$  respectively, then prove that  $a^{q-r} \times b^{r-p} \times c^{p-q} = 1$ . [4]

**Solution :**

(i) Purchase price for the dealer = ₹16,50,000

$$\therefore \text{Input GST} = \text{CGST} + \text{SGST} = 6\% \text{ of ₹16,50,000} + 6\% \text{ of ₹16,50,000} = ₹99,000 + ₹99,000$$

Selling price = ₹20,00,000

$$\begin{aligned}\therefore \text{Output GST} &= \text{CGST} + \text{SGST} \\ &= 6\% \text{ of ₹20,00,000} + 6\% \text{ of ₹20,00,000} \\ &= ₹1,20,000 + ₹1,20,000\end{aligned}$$

Net CGST paid by the dealer = ₹(1,20,000 – 99,000) = ₹21,000 **Ans.**

Net SGST paid by the dealer = ₹(1,20,000 – 99,000) = ₹21,000 **Ans.**

(ii) We have,  $x^2 - 3(x + 3) = 0 \Rightarrow x^2 - 3x - 9 = 0$

$$\Rightarrow x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times (-9)}}{2 \times 1} \Rightarrow x = \frac{3 \pm \sqrt{9+36}}{2} \Rightarrow x = \frac{3 \pm 6.70}{2}$$

$$\Rightarrow x = \frac{9.70}{2} \text{ or } x = \frac{-3.70}{2} \Rightarrow x = 4.85 \text{ or } x = -1.85$$

$$\Rightarrow x = 4.9 \text{ or } x = -1.9 \text{ **Ans.**}$$

(iii) Let A be the first term and R be the common ratio of the given GP.

Then,  $T_p = a \Rightarrow AR^{p-1} = a$

$$T_q = b = AR^{q-1} = b$$

$$T_r = c = AR^{r-1} = c$$

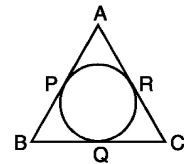
$$\begin{aligned}\text{Now, } a^{q-r} \times b^{r-p} \times c^{p-q} &= \\ &= (AR^{p-1})^{q-r} \times (AR^{q-1})^{r-p} \times (AR^{r-1})^{p-q} \\ &= A^{q-r+r-p+p-q} \times R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)} \\ &= A^0 \times R^0 = 1 \text{ Proved.}\end{aligned}$$

**Question 5 :**

(i) If  $A = \begin{bmatrix} 1 & -2 \\ 3 & 6 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & -4 \\ -2 & 5 \end{bmatrix}$ , find a matrix X such that  $X + A = B$ . [3]

(ii) In the figure, the incircle of  $\triangle ABC$  touches the sides AB, BC and CA at the points P, Q, R respectively.

$$\begin{aligned}\text{Show that } AP + BQ + CR &= BP + CQ + AR \\ &= \frac{1}{2} (\text{perimeter of } \triangle ABC).\end{aligned}$$



(iii) If the expression  $x^3 + ax + b$  is divided by  $(x + 1)$  and  $(x + 2)$ , it gives the same remainder 12. Find the values of  $a$  and  $b$ . [4]

**Solution :**

(i)  $X + A = B \Rightarrow X + A + (-A) = B + (-A)$

$$\Rightarrow X + [A + (-A)] = B + (-A)$$

$$\Rightarrow X + O = B + (-A)$$

$$\Rightarrow X = B + (-A) \Rightarrow X = \begin{bmatrix} 3 & -4 \\ -2 & 5 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ -3 & -6 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -5 & -1 \end{bmatrix} \text{ **Ans.**}$$

(ii) We know that the tangents drawn from an external point to a circle are equal.

$$\therefore AP = AR, BQ = BP \text{ and } CR = CQ$$

$$\therefore AP + BQ + CR = AR + BP + CQ$$

...(1)

$$\text{Now, perimeter of } \triangle ABC = AB + BC + CA$$

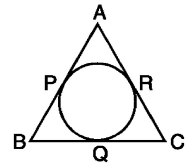
$$= AP + PB + BQ + QC + CR + AR$$

$$= (AP + BQ + CR) + (PB + QC + AR)$$

$$= 2 (AP + BQ + CR) \quad [\text{From (1)}]$$

$$\Rightarrow AP + BQ + CR = AR + BP + CQ = \frac{1}{2} (\text{perimeter of } \triangle ABC)$$

$$\Rightarrow AP + BQ + CR = BP + CQ + AR = \frac{1}{2} (\text{perimeter of } \triangle ABC) \text{ **Proved.**}$$



(iii) Let  $f(x) = x^3 + ax + b$

On dividing  $f(x)$  by  $(x + 1)$ , we have

$$f(-1) = (-1)^3 + a(-1) + b = 12$$

$$\Rightarrow -1 - a + b = 12$$

$$\Rightarrow -a + b = 13 \quad \dots (1)$$

Again dividing  $f(x)$  by  $(x + 2)$ , we have

$$f(-2) = (-2)^3 + a(-2) + b = 12$$

$$\Rightarrow -8 - 2a + b = 12$$

$$\Rightarrow -2a + b = 20 \quad \dots (2)$$

Subtracting (1) from (2), we get

$$-a = 7 \Rightarrow a = -7$$

Substituting  $a = -7$  in (1), we get

$$7 + b = 13 \Rightarrow b = 6$$

Hence,  $a = -7$ ,  $b = 6$  **Ans.**

#### Question 6 :

(i) Find the gradient and the intercept on the y-axis of the line  $3y + 2x - 12 = 0$ . [3]

(ii) Show that  $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$  [3]

(iii) Find the sum  $(-5) + (-8) + (-11) + \dots (-62)$ . [4]

**Solution :**

$$(i) \quad 3y + 2x - 12 = 0 \Rightarrow 3y = -2x + 12 \Rightarrow y = \left(-\frac{2}{3}\right)x + 4$$

Comparing it with  $y = mx + c$ , we get  $m = -\frac{2}{3}$  and  $c = 4$

Gradient (slope)  $m = -\frac{2}{3}$  and intercept on y-axis = 4 units. **Ans.**

$$(ii) \quad \text{LHS} = (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A)$$

$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right) \left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$$

$$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right) \left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$

$$= \left\{\frac{(\sin A + \cos A) - 1}{\sin A}\right\} \left\{\frac{(\sin A + \cos A) + 1}{\cos A}\right\}$$

$$= \frac{(\sin A + \cos A)^2 - 1^2}{\sin A \cos A} \quad [\because (a - b)(a + b) = a^2 - b^2]$$

$$= \frac{\sin^2 A + \cos^2 A + 2 \sin A \cos A - 1}{\sin A \cos A} = \frac{2 \sin A \cos A}{\sin A \cos A} = 2 = \text{RHS} \quad \text{Proved.}$$

(iii) Here,  $a = -5$ ,  $d = -8 - (-5) = -3$ ,  $l = -62$

Let the given AP has  $n$  terms.

Then,  $T_n = l = a + (n - 1)d$

$$\Rightarrow -62 = -5 + (n - 1)(-3)$$

$$\Rightarrow n - 1 = \frac{-62 + 5}{(-3)} = 19$$

$$\Rightarrow n = 20$$

$$\therefore \text{Sum of the given AP} = \frac{n}{2} (a + l) = \frac{20}{2} [-5 + (-62)] = 10 \times (-67) = -670. \quad \text{Ans.}$$

**Question 7 :**

- (i) Draw a circle of radius 4 cm and mark two chords AB and AC of the circle of length 6 cm and 5 cm respectively. [5]

- (a) Construct the locus of points inside the circle that are equidistant from A and C.  
(b) Construct the locus of points inside the circle that are equidistant from AB and AC.

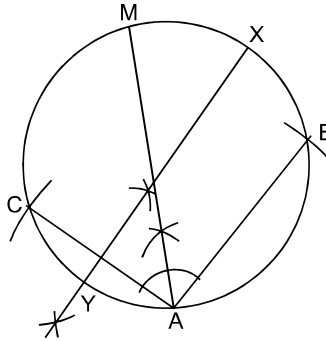
- (ii) Weight of 100 students is recorded below :

| Weight in kg    | 30–35 | 35–40 | 40–45 | 45–50 | 50–55 | 55–60 |
|-----------------|-------|-------|-------|-------|-------|-------|
| No. of students | 4     | 16    | 40    | 22    | 10    | 8     |

Draw an ogive and hence estimate the median. [5]

**Solution :**

- (i) (a) XY is the locus of points equidistant from A and C.



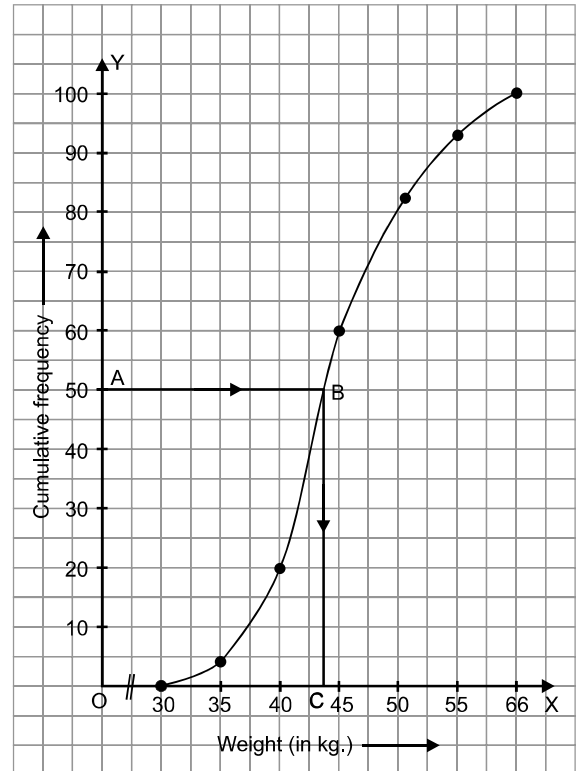
- (b) MA is the locus of points equidistant from AB and AC.

(ii)

| Weight (in kg) | No. of students | Cumulative frequency |
|----------------|-----------------|----------------------|
| 30–35          | 4               | 4                    |
| 35–40          | 16              | 20                   |
| 40–45          | 40              | 60                   |
| 45–50          | 22              | 82                   |
| 50–55          | 10              | 92                   |
| 55–60          | 8               | 100                  |
|                | N = 100         |                      |

$$N = 100 \Rightarrow \frac{N}{2} = 50$$

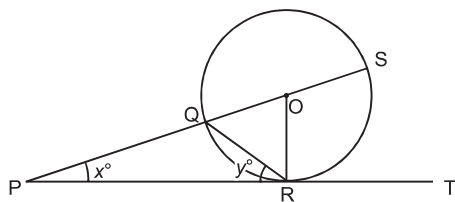
Let A denote 50 on the y-axis. From A, draw AB parallel to x-axis, which meets the ogive at B. From B, draw BC  $\perp$  x axis. The abscissa of C is 44. So, median = 44 Ans.



**Question 8 :**

- (i) The king, queen and jack of clubs are removed from a deck of playing cards and the remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of heart. [3]  
(ii) The sides of a rectangular plot of land in a map were 12 cm and 5 cm. The map was drawn to scale 1 : 2000. Find the (a) diagonal of the plot in m (b) area of the plot in m<sup>2</sup>. [3]

- (iii) In the figure, PT touches a circle with centre O at R. Diameter SQ when produced meets PT at P. If  $\angle SPR = x^\circ$  and  $\angle QRP = y^\circ$ , show that  $x^\circ + 2y^\circ = 90^\circ$ . [4]



**Solution :**

- (i) Total number of outcomes =  $52 - 3 = 49$

Number of favourable outcomes = 13

$$\therefore P(\text{a heart}) = \frac{\text{Number of favourable outcomes}}{\text{total number of outcomes}} = \frac{13}{49} \text{ Ans.}$$

- (ii) Here the scale factor,  $k = \frac{1}{2000}$

(a) Diagonal length on the map =  $\sqrt{12^2 + 5^2} = \sqrt{169} = 13 \text{ cm.}$

$$\text{Diagonal length of the plot} = \frac{1}{k} \times \text{diagonal length on the map}$$

$$= 2000 \times 13 \text{ cm} = 260 \text{ m. Ans.}$$

(b) Area of the plot =  $\frac{1}{k^2} \times \text{area on the map} = (2000)^2 \times 12 \times 5 \text{ cm}^2$

$$= \frac{(2000)^2 \times 12 \times 5}{10000} \text{ m}^2 = 24,000 \text{ m}^2 \text{ Ans.}$$

- (iii)  $\angle PRO = 90^\circ$  [Radius through the point of contact is perpendicular to the tangent]

$$\Rightarrow \angle ORQ = 90^\circ - y^\circ$$

Also,  $OR = OQ$  [Radii of the same circle]

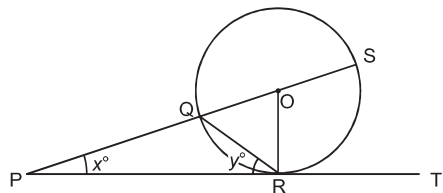
$$\Rightarrow \angle OQR = \angle ORQ = 90^\circ - y^\circ$$

In  $\triangle PQR$ ,  $\angle OQR = \angle QPR + \angle QRP$

[Exterior angle = sum of two interior opposite angles]

$$\Rightarrow 90^\circ - y^\circ = x^\circ + y^\circ$$

$$\Rightarrow x^\circ + 2y^\circ = 90^\circ \text{ Proved.}$$



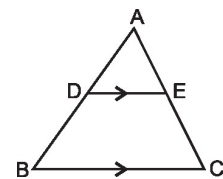
**Question 9 :**

- (i) Solve the inequation :  $8 < 5(x + 1) - 2 \leq 18$ ,  $x \in \mathbb{R}$ . Graph the solution set. [3]

- (ii) Find the mean of the following data : [3]

| Class     | 0–10 | 10–20 | 20–30 | 30–40 | 40–50 |
|-----------|------|-------|-------|-------|-------|
| Frequency | 12   | 16    | 6     | 7     | 9     |

- (iii) In the figure,  $DE \parallel BC$ . If  $AD = 3.4 \text{ cm}$ ,  $AB = 8.5 \text{ cm}$ , and  $AC = 13.5 \text{ cm}$ , find  $AE$ . [4]



**Solution :**

- (i)  $8 < 5(x + 1) - 2 \leq 18$ ,  $x \in \mathbb{R}$

$$\Rightarrow 8 < 5(x + 1) - 2 \text{ and } 5(x + 1) - 2 \leq 18, \quad x \in \mathbb{R}$$

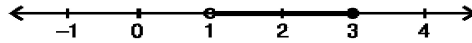
$$\Rightarrow 8 < 5x + 5 - 2 \text{ and } 5x + 5 - 2 \leq 18, \quad x \in \mathbb{R}$$

$$\Rightarrow 8 - 3 < 5x \text{ and } 5x \leq 18 - 3, \quad x \in \mathbb{R}$$

$$\Rightarrow 1 < x \text{ and } x \leq 3, x \in \mathbb{R} \Rightarrow 1 < x \leq 3, x \in \mathbb{R}$$

$$\therefore \text{Solution set} = \{x : 1 < x \leq 3, x \in \mathbb{R}\}$$

Graph of the solution set is shown below :



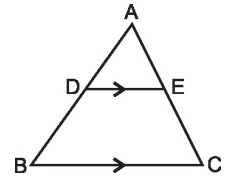
(ii)

| Class | Frequency ( $f_i$ ) | Class marks ( $x_i$ ) | $f_i x_i$               |
|-------|---------------------|-----------------------|-------------------------|
| 0–10  | 12                  | 5                     | 60                      |
| 10–20 | 16                  | 15                    | 240                     |
| 20–30 | 6                   | 25                    | 150                     |
| 30–40 | 7                   | 35                    | 245                     |
| 40–50 | 9                   | 45                    | 405                     |
|       | $\Sigma f_i = 50$   |                       | $\Sigma f_i x_i = 1100$ |

$$\text{Mean } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1100}{50} = 22 \quad \text{Ans.}$$

(iii) Since,  $DE \parallel BC$ , then by Basic Proportionality Theorem, we have

$$\begin{aligned} \frac{AD}{DB} &= \frac{AE}{EC} \\ \Rightarrow \frac{AD}{AB - AD} &= \frac{AE}{AC - AE} \\ \Rightarrow \frac{3.4}{8.5 - 3.4} &= \frac{AE}{13.5 - AE} \Rightarrow \frac{3.4}{5.1} = \frac{AE}{13.5 - AE} \\ \Rightarrow \frac{2}{3} &= \frac{AE}{13.5 - AE} \Rightarrow 3AE = 27 - 2AE \Rightarrow AE = \frac{27}{5} = 5.4 \text{ cm} \quad \text{Ans.} \end{aligned}$$



**Question 10 :**

(i) If  $\frac{7m+2n}{7m-2n} = \frac{5}{3}$ , use properties of proportion to find,  $m : n$  [3]

(ii) Construct a triangle ABC in which  $AB = 6$  cm,  $BC = 4$  cm and  $AC = 5.5$  cm. Draw the incircle of the triangle. [3]

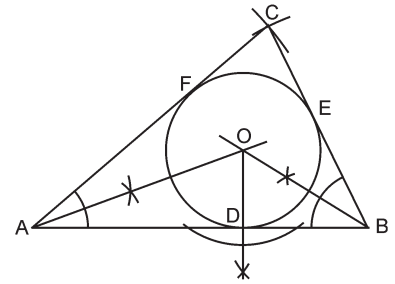
(iii) From the top of a building 20 m high, the angle of elevation of the top of a monument is  $45^\circ$  and the angle of depression of its foot is  $15^\circ$ . Find the height of the monument. [4]

**Solution :**

$$\begin{aligned} \text{(i) We have, } \frac{7m+2n}{7m-2n} &= \frac{5}{3} \\ \Rightarrow \frac{7m+2n+7m-2n}{7m+2n-7m+2n} &= \frac{5+3}{5-3} \quad [\text{Applying componendo and dividendo}] \\ \Rightarrow \frac{14m}{4n} &= \frac{8}{2} \Rightarrow \frac{7m}{2n} = \frac{4}{1} \Rightarrow \frac{m}{n} = \frac{4}{1} \times \frac{2}{7} = \frac{8}{7} \Rightarrow m : n = 8 : 7 \quad \text{Ans.} \end{aligned}$$

(ii) **Steps of Construction :**

1. Draw  $AB = 6$  cm.
2. With A and B as centres and radii equal to 5.5 cm and 4 cm respectively, draw two arcs. These arcs cut each other at C.
3. Join AC and BC to get the required triangle.
4. Draw the bisectors of  $\angle A$  and  $\angle B$ . The bisectors of these angles meet at O.
5. From O, draw perpendicular on AB, which meets AB at D.
6. With O as centre and OD as radius, draw a circle, which touches the sides AB, BC and AC at D, E and F respectively.



(iii) Let AB be the building and CD be the monument.

Let  $CB = x$  m and  $DE = h$  m.

$$\therefore AE = CB = x \text{ m}$$

$$\text{In } \triangle ADE, \frac{h}{x} = \tan 45^\circ \Rightarrow h = x$$

$$\text{In } \triangle ABC, \frac{20}{x} = \tan 15^\circ$$

$$\Rightarrow x = \frac{20}{\tan 15^\circ} = \frac{20}{0.2679} = 74.64$$

$$\Rightarrow h = 74.64$$

[From (1)]

Hence, height of the monument =  $(20 + 74.64)$  m = 94.64 m **Ans.**

— \* \* \* —

...(1)

