ICSE 2024 EXAMINATION

MATHEMATICS

SAMPLE PAPER - 3

Time Allowed : 2½ hours ***Max. Marks : 80***

General Instructions :

Attempt all questions from Section A and any four questions from Section B.
All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.
Omission of essential working will result in loss of marks.
The intended marks for questions or parts of questions are given in brackets [ ]
Mathematical tables are provided.

SECTION - A (40 Marks)

(Attempt *all* questions from this Section)

Question 1 : Choose the correct answers to the questions from the given options: [15]

(i) Manish has some shares of 20 of a company paying 12% dividend. If his annual income is ₹9600, then the number of shares he possesses is :
(a) 4000 (b) 5000 (c) 5200 (d) 5600

(ii) The roots of the equation $2x^2 - 9x + 7 = 0$ are :
(a) equal and rational (b) unequal and rational (c) irrational (d) not real

(iii) $(x - 1)$, $(x + 1)$ and $(x - 3)$ are factors of $f(x) = x^3 - ax^2 + bx + 3$. Then the value of $f(-1)$ is :
(a) 1 (b) 0 (c) –1 (d) 8

(iv) The nth term of the GP 81, 27, 3, ... is:
(a) $\frac{81}{3^n}$ (b) $\frac{3^{n-1}}{81}$ (c) $\frac{81}{3^{n-1}}$ (d) $\frac{81}{3^{n-1}}$

(v) Which term of the AP 21, 42, 63, ... is 210?
(a) 9th (b) 10th (c) 11th (d) 12th

(vi) Which of the following points is invariant with respect to the line $x = 3$?
(a) (3, 2) (b) (–3, 2) (c) (–2, –3) (d) (2, –3)

(vii) If $\Delta ABC \sim \Delta DFE$, $\angle A = 40°$, $\angle C = 60°$, AB = 6 cm, AC = 5 cm and DF = 12 cm, then :
(a) DE = 8 cm, $\angle F = 60°$ (b) DE = 10 cm, $\angle F = 80°$
(c) EF = 10 cm, $\angle D = 80°$ (d) EF = 7 cm, $\angle D = 50°$

(viii) A cylindrical pencil sharpened at one edge is the combination of:
(a) a cone and a cylinder (b) a hemisphere and a cylinder
(c) a sphere and a cylinder (d) none of these

(ix) The solution set for $2x - 1 \geq 5 - x$, $x \in R$ is :
(a) $\{x \geq 2, x \in R\}$ (b) $\{x \leq 2, x \in R\}$ (c) $\{x \geq -2, x \in R\}$ (d) $\{x \leq -2, x \in R\}$

(x) A bag contains 3 red balls, 5 white balls and 7 black balls. The probability that a ball drawn from the bag at random is neither red nor black is:
(a) $\frac{1}{5}$ (b) $\frac{1}{3}$ (c) $\frac{7}{15}$ (d) $\frac{8}{15}$

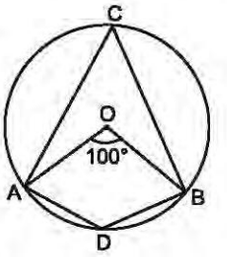
(xi) The transpose of the matrix $\begin{bmatrix} 5 & -4 & 1 \\ -1 & 6 & 0 \end{bmatrix}$ is :
(a) $\begin{bmatrix} -1 & 6 & 0 \\ 5 & -4 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & -1 \\ -4 & 6 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 5 \\ 6 & -4 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 1 \\ -4 & -6 \\ -1 & 0 \end{bmatrix}$

(xii) Centre of a circle is at (–1, 3) and one end of the diameter has coordinates (2, 5). The coordinates of the other end are:

(a) (–4, 1) (b) (1, –4) (c) (4, –1) (d) (4, 1)

(xiii) In the figure, O is the centre of the circle. $\angle AOB = 100°$, then $\angle ADB$ =

(a) 100°
(b) 110°
(c) 120°
(d) 130°



(xiv) ΔABC is enlarged by a scale factor 1.2. If the area of ΔABC is 12.5 cm^2, then the area of its image is:

(a) 18 cm^2 (b) 20 cm^2 (c) 24 cm^2 (d) 26 cm^2

(xv) The median class for the given distribution is:

Class Interval	1 – 5	6 – 10	11 – 15	16 – 20
Cumulative frequency	2	6	11	18

Question 2 :

(i) Manoj opened a recurring deposit account with Punjab National Bank and deposited ₹ 500 per month for 3 years. The bank paid him ₹20,220 on maturity. Find the rate of interest paid by the bank. [4]

(ii) Sachin invests ₹8500 in 10% ₹100 shares at ₹170. He sells the shares when the price of each share rises by ₹30. He invests the proceeds in 12% ₹100 shares at ₹125. Find: [4]

(a) the sale proceeds (b) the number of ₹125 shares he buys

(c) the change in his annual income

(iii) The ratio of the sum of first three terms to that of the first 6 terms of a GP is 125 : 152. Find the common ratio. [4]

Question 3 :

(i) On a map drawn to a scale of 1 : 25,000, a triangular plot LMN of land has the following measurements. [4]
LM = 6 cm, MN = 8 cm and $\angle LMN = 90°$. Calculate

(a) the actual length of MN and LN in km. (b) the actual area of plot in sq. km.

(ii) Find the equation of the line passing through (–2, – 4) and perpendicular to the line $3x - y + 5 = 0$. [4]

(iii) Use graph paper for this question. [5]

(a) Plot the points A (3, 5) and B (–2, – 4). Use 1 cm = 1 unit on both the axes.

(b) A′ is the image of A when reflected in the x-axis. Write down the coordinates of A′ and plot it on the graph paper.

(c) B′ is the image of B when reflected in the y-axis followed by reflection in the origin. Write down the coordinates of B′ and plot it on the graph paper.

(d) Write down the geometrical name of the figure AA′BB′.

SECTION - B (40 Marks)

(Attempt *any four* questions from this Section)

Question 4 :

(i) A shopkeeper sells an item for ₹1475 with GST = 18%. A customer willing to buy this item requests the shopkeeper to reduce the price of the item so that he pays only ₹1475, including GST. If the shopkeeper agrees to this, how much reduction is needed? [3]

(ii) Solve the following quadratic equation using formula: $5x(x+2) = 3$ [3]

(iii) In a school, the weekly pocket money of 50 students is as follows: [4]

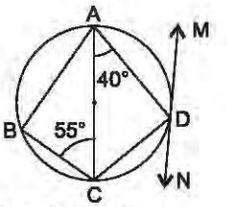
Weekly pocket money (in ₹)	40–50	50–60	60–70	70–80	80–90	90–100
Number of students	2	8	12	14	8	6

Draw a histogram for the above data and locate mode from it.

Question 5 :

(i) Find the 2 × 2 matrix X which satisfies the equation $\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$ [3]

(ii) In the given figure, ABCD is a cyclic quadrilateral, AC is a diameter of the circle and MN is a tangent to the circle at D. If $\angle CAD = 40°$ and $\angle ACB = 55°$, determine $\angle ADM$ and $\angle BAD$. [3]



(iii) Check whether $(2y + 1)$ is a factor of $g(y) = 4y^3 + 4y^2 - y - 1$. [4]

Question 6 :

(i) Find the co-ordinates of the points of trisection of the line segment joining the points (3, –3) and (6, –9). [3]

(ii) Prove the following identity : $\frac{1}{\sin\theta+\cos\theta} + \frac{1}{\sin\theta-\cos\theta} = \frac{2\sin\theta}{1-2\cos^2\theta}$ [3]

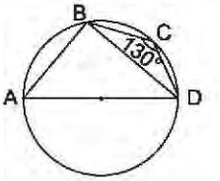
(iii) Find the sum of the first 45 terms of an AP in which the third term is 6 and the seventh term is 22. [4]

Question 7 :

(i) A bag contains 8 red, 6 white and 4 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is :
(a) red or white (b) not black (c) neither white nor black. [3]

(ii) A solid consisting of a right circular cone, standing on a hemisphere, is placed upright in a right circular cylinder full of water and touches the bottom. Find the volume of water left in the cylinder having given that the radius of the cylinder is 3 cm and its height is 6 cm, the radius of the hemisphere is 2 cm and the height of the cone is 4 cm. Give your answer to nearest cubic centimetre. [3]

(iii) In the figure, AD is the diameter of the circle. If $\angle BCD = 130°$, calculate :
(a) $\angle DAB$ (b) $\angle ADB$ [4]



Question 8 :

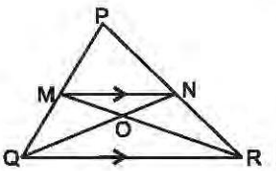
(i) Solve the inequation : $12 + \frac{5}{6} \le 5 + 3x$, $x \in R$
Represent the solution on a number line. [3]

(ii) Find the mean of the following distribution: [3]

Class	0–20	20–40	40–60	60–80	80–100	100–120	120–140
Frequency	4	15	23	12	16	8	2

(iii) In ΔPQR, MN is parallel to QR and $\frac{PM}{MQ} = \frac{2}{3}$

(a) Find $\frac{MN}{QR}$ (b) Prove that ΔOMN and ΔORQ are similar. [4]



Question 9 :

(i) Using ruler and compasses only, construct a ΔABC in which AB = 6 cm, BC = 3.5 cm and CA = 4.6 cm. [4]
(a) Draw the locus of a point, which moves so that it is always 3 cm from B.
(b) Draw the locus of a point which moves so that it is equidistant from BC and CA.
(c) Mark the point of intersection of the two loci obstained above as P. Measure PC.

(ii) Attempt this question on a graph paper. [6]

Class - Interval	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	12	20	30	38	24	16	12	8

Taking a suitable scale draw an ogive for the above data. From the ogive estimate:
(a) the median (b) the upper and lower quartile

Question 10 :

(i) Given that $\frac{a^3+3ab^2}{b^3+3a^2b} = \frac{63}{62}$. Using componendo and dividendo, find $a : b$. [3]

(ii) Using ruler and compasses only, construct an isosceles ΔABC having base = 4 cm, vertical angle = 45° and median through vertex equal to 2.8 cm. Draw the incircle of the triangle. [3]

(iii) The angles of depression of two ships A and B as observed from the top of a lighthouse 60 m high are 60° and 45° respectively. If the two ships are on the opposite sides of the lighthouse, find the distance between the two ships. Give your answer correct to the nearest whole number. [4]

ANSWERS

1. (i) (a) (ii) (b) (iii) (b) (iv) (c) (v) (b) (vi) (a) (vii) (b) (viii) (a) (ix) (a)
(x) (b) (xi) (b) (xii) (a) (xiii) (d) (xiv) (a) (xv) (c)

2. (i) 8% p.a. (ii) (a) ₹10,000 (b) 80 (c) ₹460 (iii) $\frac{3}{5}$

3. (i) (a) 2 km, 2.5 km (b) 1.5 km^2 (ii) $x + 3y + 14 = 0$ (iii) (b) (3, –5) (c) (–2, 4) (d) Isosceles trapezium

4. (i) ₹225 (ii) $x = 0.26$ or –2.26 (iii) ₹72

5. (i) $\begin{bmatrix} -17 & -16 \\ -12 & -5 \end{bmatrix}$ (ii) 40°, 75° (iii) yes

6. (i) (4, –5), (5, –7) (iii) 3870

7. (i) (a) $\frac{7}{9}$ (b) $\frac{7}{9}$ (c) $\frac{4}{9}$ (ii) 136 cm^2 (iii) (a) 50° (b) 40°

8. (i) $\{x : x \geq 3.2, x \in R\}$ (ii) 63.25 (iii) (a) $\frac{2}{3}$

9. (ii) (a) 35 (b) 48, 23 **10.** (i) 3 : 2 (iii) 95 m

SOLUTION

Time Allowed : 2½ hours *Max. Marks* : 80

General Instructions :

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Mathematical tables are provided.

SECTION - A (40 Marks)

(Attempt all questions from this Section)

Question 1 : Choose the correct answers to the questions from the given options: **[15]**

(i) Manish has some shares of 20 of a company paying 12% dividend. If his annual income is ₹9600, then the number of shares he possesses is :

(a) 4000 **(b) 5000** **(c) 5200** **(d) 5600**

(ii) The roots of the equation $2x^2 - 9x + 7 = 0$ are :

(a) equal and rational **(b) unequal and rational** **(c) irrational** **(d) not real**

(iii) $(x - 1)$, $(x + 1)$ and $(x - 3)$ are factors of $f(x) = x^3 - ax^2 + bx + 3$. Then the value of $f(-1)$ is :

(a) 1 **(b) 0** **(c) –1** **(d) 8**

(iv) The nth term of the GP 81, 27, 3, … is:

(a) $\frac{81}{3^n}$ **(b) $\frac{3^{n-1}}{81}$** **(c) $\frac{81}{3^{n-1}}$** **(d) $\frac{81}{3^n - 1}$**

(v) Which term of the AP 21, 42, 63, ... is 210?

(a) 9th **(b) 10th** **(c) 11th** **(d) 12th**

(vi) Which of the following points is invariant with respect to the line $x = 3$?

(a) (3, 2) **(b) (–3, 2)** **(c) (–2, –3)** **(d) (2, –3)**

(vii) If $\Delta ABC \sim \Delta DFE$, $\angle A = 40°$, $\angle C = 60°$, AB = 6 cm, AC = 5 cm and DF = 12 cm, then :

(a) DE = 8 cm, $\angle F = 60°$ **(b) DE = 10 cm, $\angle F = 80°$**

(c) EF = 10 cm, $\angle D = 80°$ **(d) EF = 7 cm, $\angle D = 50°$**

(viii) A cylindrical pencil sharpened at one edge is the combination of:

(a) a cone and a cylinder **(b) a hemisphere and a cylinder**

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(ix) The solution set for $2x - 1 \geq 5 - x$, $x \in R$ is :

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(x) A bag contains 3 red balls, 5 white balls and 7 black balls. The probability that a ball drawn from the bag at random is neither red nor black is:

(a) $\frac{1}{5}$ **(b) $\frac{1}{3}$** **(c) $\frac{7}{15}$** **(d) $\frac{8}{15}$**

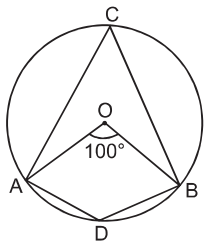
(xi) The transpose of the matrix $\begin{bmatrix} 5 & -4 & 1 \\ -1 & 6 & 0 \end{bmatrix}$ is :

(a) $\begin{bmatrix} -1 & 6 & 0 \\ 5 & -4 & 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 5 & -1 \\ -4 & 6 \\ 1 & 0 \end{bmatrix}$ **(c)** $\begin{bmatrix} -1 & 5 \\ 6 & -4 \\ 0 & 1 \end{bmatrix}$ **(d)** $\begin{bmatrix} 5 & 1 \\ -4 & -6 \\ -1 & 0 \end{bmatrix}$

(xii) Centre of a circle is at (–1, 3) and one end of the diameter has coordinates (2, 5). The coordinates of the other end are:

(a) (–4, 1) **(b) (1, –4)** **(c) (4, –1)** **(d) (4, 1)**

(xiii) In the figure, O is the centre of the circle. $\angle AOB = 100°$, then $\angle ADB$ =



(a) 100°

(b) 110°

(c) 120°

(d) 130°

(xiv) ΔABC is enlarged by a scale factor 1.2. If the area of ΔABC is 12.5 cm^2, then the area of its image is:

(a) 18 cm^2 **(b) 20 cm^2** **(c) 24 cm^2** **(d) 26 cm^2**

(xv) Assertion (A) : The median class for the given data is 11-15.

Class	1 – 5	6 – 10	11 – 15	16 – 20
Cumulative frequency	2	6	11	18

Reason (R) : Mean, median and mode of a data can never be the same.

(a) A is true, R is false **(b) A is false, R is true** **(c) both A and R are true** **(d) both A and R are false**

Solution :

(i) (a) Let Manish bought x shares.
Then 12% of $x \times 20 = 9600$
$\Rightarrow x = \dfrac{9600 \times 100}{12 \times 20} = 4{,}000.$

(ii) (b) $D = (-9)^2 - 4 \times 2 \times 7 = 81 - 56 = 25 > 0$
So, roots are real, unequal and rational

(iii) (b)

(iv) (c) $T_n = 81 \times \left(\dfrac{1}{3}\right)^{n-1} = \dfrac{81}{3^{n-1}}$

(v) (b) $210 = 21 + (n - 1) \times 21$
$\Rightarrow n - 1 = 9 \Rightarrow n = 10.$

(vi) (a) Image of (3, 2), with respect to the line $x = 3$ is
(–3 + 2 × 3, 2), i.e., (3, 2).

(vii) (b) $\dfrac{AB}{DF} = \dfrac{AC}{DE} \Rightarrow \dfrac{6}{12} = \dfrac{5}{DE} \Rightarrow DE = 10$ cm
$\angle B = 180^\circ - (40^\circ + 60^\circ) = 80^\circ$
So, $\angle F = 80^\circ$

(viii) (a)

(ix) (a) $2x - 1 \geq 5 - x,\ x \in R$
$\Rightarrow x \geq 2,\ x \in R$

(x) (b) Required probability $= \dfrac{5}{15} = \dfrac{1}{3}$.

(xi) (b)

(xii) (a) $\dfrac{2 + x}{2} = -1 \Rightarrow x = -4$

$\dfrac{5 + y}{2} = 3 \Rightarrow y = 1$

So, required coordinates are (–4, 1)

(xiii) (d) $\angle ACB = \frac{1}{2} \times 100^\circ = 50^\circ$

$\angle ADB = 180^\circ - 50^\circ = 130^\circ$.

(xiv) (a) Required area $= (1.2)^2 \times 12.5 \text{ cm}^2 = 18 \text{ cm}^2$.

(xv) (d) Here, $n = 18 \Rightarrow \frac{n}{2} = 9$, 9 falls in the class 11 – 15.

But after converting it into exclusive form, we get 10.5 – 15.5 as the median class.

Question 2 :

(i) Manoj opened a recurring deposit account with Punjab National Bank and deposited ₹ 500 per month for 3 years. The bank paid him ₹20,220 on maturity. Find the rate of interest paid by the bank. [4]

(ii) Sachin invests ₹8500 in 10% ₹100 shares at ₹170. He sells the shares when the price of each share rises by ₹30. He invests the proceeds in 12% ₹100 shares at ₹125. Find: [4]

(a) the sale proceeds

(b) the number of ₹125 shares he buys

(c) the change in his annual income

(iii) The ratio of the sum of first three terms to that of the first 6 terms of a GP is 125 : 152. Find the common ratio. [4]

Solution :

(i) Maturity value $= Pn + \frac{Pn(n+1)}{2\times12} \times \frac{r}{100}$

$\Rightarrow 20220 = 500\times36 + \frac{500\times36\times37}{2\times12} \times \frac{r}{100}$

$\Rightarrow 20220 = \frac{36000 + 15\times37\times r}{2}$

$\Rightarrow 40440 - 36000 = 15 \times 37 \times r \Rightarrow r = \frac{4440}{15\times37}$

$\Rightarrow r = 8\%$. **Ans.**

(ii) (a) Number of shares of first kind bought by Sachin $= \frac{₹8500}{₹170} = 50$

So, sale proceeds $= ₹(170 + 30) \times 50 = ₹10{,}000$ **Ans.**

(b) Number of ₹125 shares bought $= \frac{₹10{,}000}{₹125} = 80$ **Ans.**

(c) Annual income from ₹170 shares = 10% of $(50 \times ₹100) = ₹500$

Annual income from ₹125 shares

$= 12\%$ of $(80 \times ₹100) = ₹960$

Change in annual income $= ₹(960 - 500) = ₹460$. **Ans.**

(iii) $\frac{a(r^3-1)}{r-1} \times \frac{r-1}{a(r^6-1)} = \frac{125}{152}$

$\Rightarrow 152r^3 - 152 = 125r^6 - 125 \Rightarrow 125r^6 - 152r^3 + 27 = 0$

$\Rightarrow 125\,t^2 - 152t + 27 = 0$ [Put $r^3 = t$%]

$\Rightarrow 125\,t^2 - 125t - 27t + 27 = 0$

$\Rightarrow 125t\,(t-1) - 27\,(t-1) = 0$

$\Rightarrow (t-1)\,(125t - 27) = 0 \Rightarrow t = 1$ or $t = \frac{27}{125}$

$\Rightarrow r^3 = 1$ or $r^3 = \frac{27}{125} \Rightarrow r = 1$ or $r = \frac{3}{5}$

$\Rightarrow r = \frac{3}{5}$ [Rejecting $r = 1$, as it does not satisfy the given condition] **Ans.**

Question 3 :

(i) On a map drawn to a scale of 1 : 25,000, a triangular plot LMN of land has the following measurements. [4]
LM = 6 cm, MN = 8 cm and ∠LMN = 90°. Calculate

(a) the actual length of MN and LN in km. **(b) the actual area of plot in sq. km.**

(ii) Find the equation of the line passing through (–2, – 4) and perpendicular to the line $3x - y + 5 = 0$. [4]

(iii) Use graph paper for this question. [5]

(a) Plot the points A (3, 5) and B (–2, – 4). Use 1 cm = 1 unit on both the axes.

(b) A′ is the image of A when reflected in the x-axis. Write down the coordinates of A′ and plot it on the graph paper.

(c) B′ is the image of B when reflected in the y-axis followed by reflection in the origin. Write down the coordinates of B′ and plot it on the graph paper.

(d) Write down the geometrical name of the figure AA′BB′.

Solution :

(i) Here, scale factor $k = \frac{1}{25,000}$.

(a) $LN^2 = LM^2 + MN^2 \Rightarrow LN^2 = 36 + 64 = 100$

$\Rightarrow LN = 10$ cm

Actual length of MN $= \frac{1}{k} \times$ MN on the map

$= 25,000 \times 8$ cm $= 2$ km **Ans.**

Actual length of LN $= \frac{1}{k} \times$ LN on the map

$= 25,000 \times 10$ cm $= 2.5$ km **Ans.**

(b) Actual area $= \frac{1}{k^2} \times$ area of the map

$= (25,000)^2 \times \frac{1}{2} \times 6 \times 8$ cm^2

$= (25,000)^2 \times \frac{24}{(1,00,000)^2}$ km$^2 = 1.5$ km^2 **Ans.**

(ii) We have, $3x - y + 5 = 0$

$\Rightarrow -y = -3x - 5 \Rightarrow y = 3x + 5$

$\therefore m_1 = 3$

$\Rightarrow m_1 \times m_2 = -1 \Rightarrow 3 \times m_2 = -1$

$\Rightarrow m_2 = -\frac{1}{3}$

Equation of line which passes through

(–2, – 4) with gradient $-\frac{1}{3}$ is

$(y - y_1) = m(x - x_1) \Rightarrow (y + 4) = -\frac{1}{3}(x + 2)$

$\Rightarrow 3y + 12 = -x - 2 \Rightarrow x + 3y = -14$

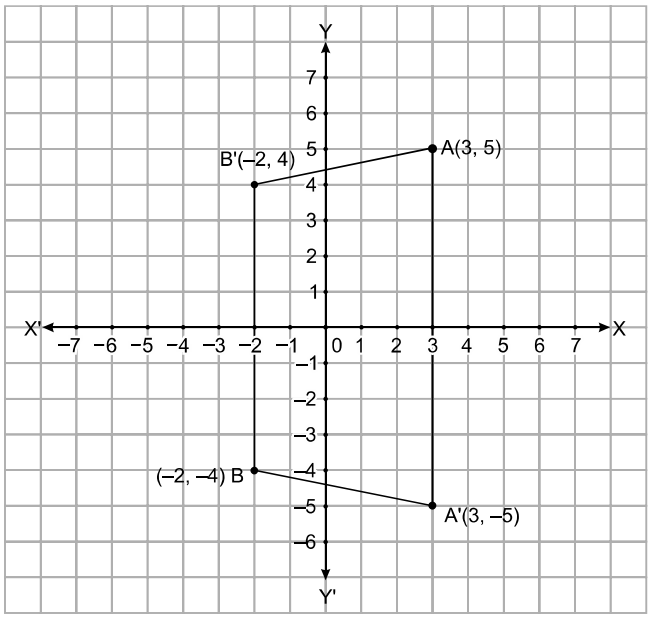
$\therefore x + 3y + 14 = 0$. **Ans.**

(iii) (a) The given points have been plotted. **Ans.**

(b) Coordinates of A' are (3, –5) **Ans.**

(c) Coordinates of B' are (–2, 4) **Ans.**

(d) Isosceles trapezium. **Ans.**



SECTION - B (40 Marks)

(Attempt *any four* questions from this Section)

Question 4 :

(i) A shopkeeper sells an item for ₹1475 with GST = 18%. A customer willing to buy this item requests the shopkeeper to reduce the price of the item so that he pays only ₹1475, including GST. If the shopkeeper agrees to this, how much reduction is needed? [3]

(ii) Solve the following quadratic equation using formula: $5x(x + 2) = 3$ [3]

(iii) In a school, the weekly pocket money of 50 students is as follows: [4]

Weekly pocket money (in ₹)	40–50	50–60	60–70	70–80	80–90	90–100
Number of students	2	8	12	14	8	6

Draw a histogram for the above data and locate mode from it.

Solution :

(i) Let the CP of an item be x.

x + 18% of x = 1475 $\Rightarrow$ $x + \frac{18}{100}x = 1475$ $\Rightarrow$ $\frac{118x}{100} = 1475$ $\Rightarrow$ $x = \frac{1475 \times 100}{118}$ $\Rightarrow$ $x = 1250$

Reduction = ₹1475 – ₹1250 = ₹225. **Ans.**

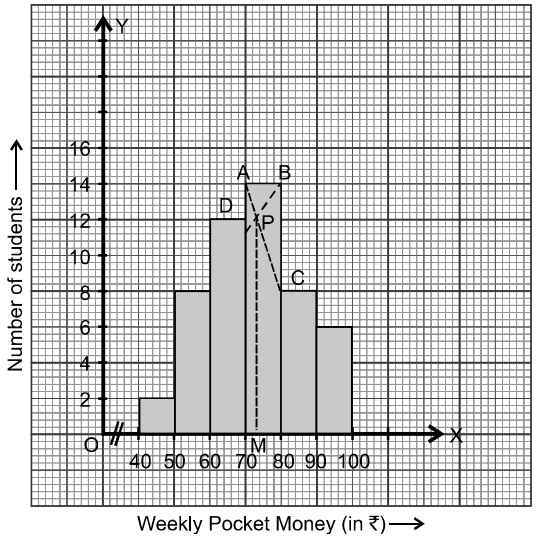
(ii) $5x(x + 2) = 3 \Rightarrow 5x^2 + 10x - 3 = 0$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{(10)^2 - 4 \times 5 \times (-3)}}{2 \times 5}$$

$$= \frac{-10 \pm \sqrt{100 + 60}}{10} = \frac{-10 \pm \sqrt{160}}{10} = \frac{-10 \pm 4\sqrt{10}}{10} = \frac{-10 \pm 12.64}{10}$$

$$\Rightarrow x = \frac{-10 + 12.6}{10} = \frac{2.6}{10} = 0.26 \text{ or } x = \frac{-10 - 12.6}{10} = -2.26$$ **Ans.**

(iii)



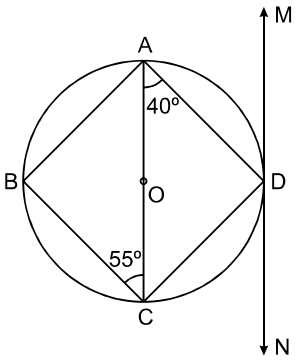
Mark the upper corners of the highest rectangle and then corners of the adjoining rectangles as A, B, C, D. Join AC and BD to intersect at point M. Draw PM $\perp$ x-axis. Abscissa of M is 72.

So, mode = ₹72. **Ans.**

Question 5 :

(i) Find the 2 × 2 matrix X which satisfies the equation $\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$ **[3]**

(ii) In the given figure, ABCD is a cyclic quadrilateral, AC is a diameter of the circle and MN is a tangent to the circle at D. If ∠CAD = 40° and ∠ACB = 55°, determine ∠ADM and ∠BAD. **[3]**



(iii) Check whether (2y + 1) is a factor of $g(y) = 4y^3 + 4y^2 - y - 1$. **[4]**

Solution :

(i) $\begin{bmatrix}3 & 7\\ 2 & 4\end{bmatrix}\begin{bmatrix}0 & 2\\ 5 & 3\end{bmatrix}+2X=\begin{bmatrix}1 & -5\\ -4 & 6\end{bmatrix}$

$\Rightarrow \begin{bmatrix}3\times0+7\times5 & 3\times2+7\times3\\ 2\times0+4\times5 & 2\times2+4\times3\end{bmatrix}+2X=\begin{bmatrix}1 & -5\\ -4 & 6\end{bmatrix}$

$\Rightarrow \begin{bmatrix}0+35 & 6+21\\ 0+20 & 4+12\end{bmatrix}+2X=\begin{bmatrix}1 & -5\\ -4 & 6\end{bmatrix}$

$\Rightarrow \begin{bmatrix}35 & 27\\ 20 & 16\end{bmatrix}+2X=\begin{bmatrix}1 & -5\\ -4 & 6\end{bmatrix}$

$\Rightarrow 2X=\begin{bmatrix}1 & -5\\ -4 & 6\end{bmatrix}-\begin{bmatrix}35 & 27\\ 20 & 16\end{bmatrix}$

$\Rightarrow 2X=\begin{bmatrix}-34 & -32\\ -24 & -10\end{bmatrix} \Rightarrow X=\begin{bmatrix}-17 & -16\\ -12 & -5\end{bmatrix}$ **Ans.**

(ii) $\angle CBA = \frac{1}{2}\angle AOC$

$\angle CBA = 90°$ and $\angle CDA = 90°$

In ΔABC, we have

$\angle ABC + \angle ACB + \angle BAC = 180° \Rightarrow 90° + 55° + \angle BAC = 180° \Rightarrow 145° + \angle BAC = 180°$

$\Rightarrow \angle BAC = 180° - 145° \Rightarrow \angle BAC = 35°$

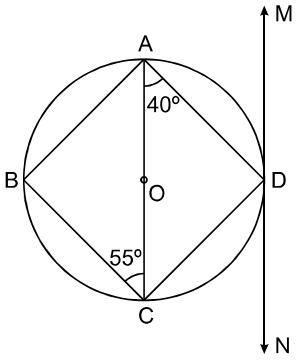
$\angle BAD = 35° + 40° = 75°$

$\angle BAD + \angle BCD = 180°$

$\Rightarrow \angle ACD = 50°$

$\Rightarrow \angle ADM = \angle ACD$ [Angles in the alternate segment]

$\Rightarrow \angle BAD = 25°$ **Ans.**



(iii) We have, $g(y) = 4y^3 + 4y^2 - y - 1$

$\Rightarrow g\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3+4\left(-\frac{1}{2}\right)^2-\left(-\frac{1}{2}\right)-1 = 4\times\left(-\frac{1}{8}\right)+4\times\frac{1}{4}+\frac{1}{2}-1 = -\frac{1}{2}+1+\frac{1}{2}-1=0$

Thus, $2y + 1$ is a factor of $g(y)$. **Ans.**

Question 6 :

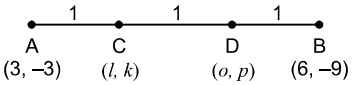
(i) Find the co-ordinates of the points of trisection of the line segment joining the points (3, –3) and (6, –9). **[3]**

(ii) Prove the following identity : $\frac{1}{\sin\theta+\cos\theta}+\frac{1}{\sin\theta-\cos\theta}=\frac{2\sin\theta}{1-2\cos^2\theta}$ **[3]**

(iii) Find the sum of the first 45 terms of an AP in which the third term is 6 and the seventh term is 22. **[4]**

Solution :

(i)



$$l = \frac{mx_2 + nx_1}{m+n} = \frac{1\times 6 + 2\times 3}{1+2} = \frac{6+6}{3} = \frac{12}{3} = 4.$$

$$k = \frac{my_2 + ny_1}{m+n} = \frac{1\times(-9) + 2\times(-3)}{1+2} = \frac{-9-6}{3} = \frac{-15}{3} = -5.$$

$\therefore$ So, C(4, –5)

$$o = \frac{2\times 6 + 1\times 3}{2+1} = \frac{12+3}{3} = \frac{15}{3} = 5.$$

$$p = \frac{2\times -9 + 1\times -3}{2+1} = \frac{-18-3}{3} = \frac{-21}{3} = -7.$$

$\therefore$ So, D(5, –7)

Hence, the coordinates, of the points of trisection are (4, –5) and (5, –7) **Ans.**

(ii) $\text{LHS} = \frac{1}{\sin\theta+\cos\theta} + \frac{1}{\sin\theta-\cos\theta} = \frac{\sin\theta-\cos\theta+\sin\theta+\cos\theta}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}$

$= \frac{2\sin\theta}{\sin^2\theta-\cos^2\theta} = \frac{2\sin\theta}{1-\cos^2\theta-\cos^2\theta} = \frac{2\sin\theta}{1-2\cos^2\theta} = \text{RHS}$ **Proved.**

(iii) We have, $a_3 = 6 \Rightarrow a + 2d = 6$ ...(1)

And $a_7 = 22 \Rightarrow a + 6d = 22$ ...(2)

(1) – (2)

$a + 2d = 6$

$\underline{a + 6d = 22}$

$-4d = -16 \Rightarrow d = 4$

So, $a + 2 \times 4 = 6 \Rightarrow a = 6 - 8 \Rightarrow a = -2$

$S_n = \frac{n}{2}[2a + (n-1)\times d]$

$\Rightarrow S_{45} = \frac{45}{2}[2\times(-2) + (45-1)\times 4] = \frac{45}{2}[-4 + 44\times 4] = \frac{45}{2}[-4+176] = \frac{45}{2}\times 172 = 45 \times 86$

$\therefore S_{45} = 3870.$ **Ans.**

Question 7 :

(i) Using ruler and compasses only, construct a ΔABC in which AB = 6 cm, BC = 3.5 cm and CA = 4.6 cm. [5]

(a) Draw the locus of a point, which moves so that it is always 3 cm from B.

(b) Draw the locus of a point which moves so that it is equidistant from BC and CA.

(c) Mark the point of intersection of the two loci obtained above as P. Measure PC.

(ii) Attempt this question on a graph paper. [5]

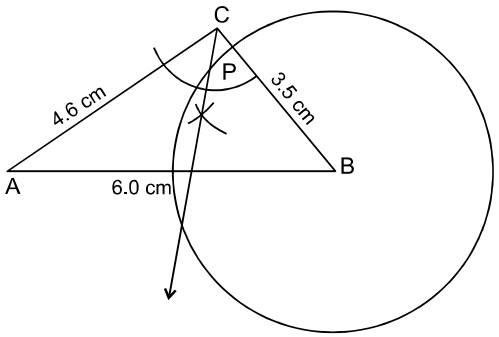
Class-Interval	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	12	20	30	38	24	16	12	8

Taking a suitable scale draw an ogive for the above data. From the ogive estimate:

(a) the median (b) the upper and lower quartile

Solution :

(i) From the figure, we get PC = 0.7 cm.

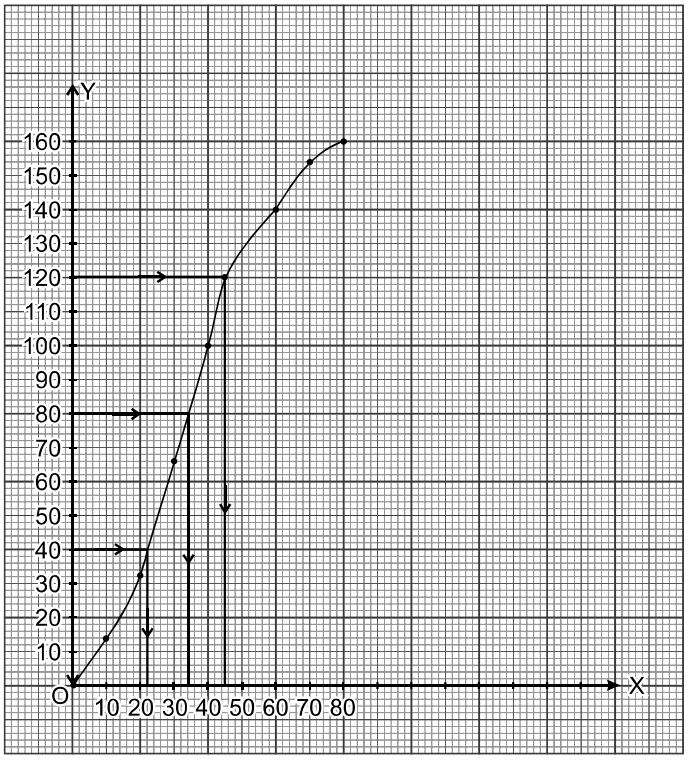


(ii)

Class interval	Frequency	*c. f.*
0 – 10	12	12
10 – 20	20	32
20 – 30	30	62
30 – 40	38	100
40 – 50	24	124
50 – 60	16	140
60 – 70	12	152
70 – 80	8	160

On a graph paper, plot the points (0, 0), (10, 12), (20, 32), (30, 62), (40, 100), (50, 124), (60, 140), (70, 152), and (80, 160). Join these points by a free hand curve to get the ogive.

(a) Here N = 160 $\Rightarrow \frac{N}{2}$ = 80, from the graph median = 35. **Ans.**



(b) Upper quartile = 48 **Ans.**

Lower quartile = 23 **Ans.**

Question 8 :

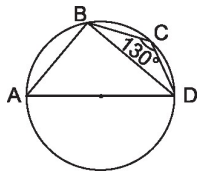
(i) A bag contains 8 red, 6 white and 4 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is :

(a) red or white (b) not black (c) neither white nor black. **[3]**

(ii) A solid consisting of a right circular cone, standing on a hemisphere, is placed upright in a right circular cylinder full of water and touches the bottom. Find the volume of water left in the cylinder having given that the radius of the cylinder is 3 cm and its height is 6 cm, the radius of the hemisphere is 2 cm and the height of the cone is 4 cm. Give your answer to nearest cubic centimetre. **[3]**

(iii) In the figure, AD is the diameter of the circle. If ∠BCD = 130°, calculate :

(a) ∠DAB (b) ∠ADB **[4]**



Solution :

(i) Total number of balls = 8 + 6 + 4 = 18

(a) P(red or white) = $\frac{14}{18} = \frac{7}{9}$ **Ans.**

(b) P(not black) = $\frac{14}{18} = \frac{7}{9}$ **Ans.**

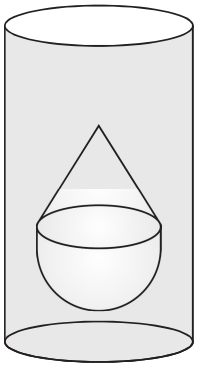
(c) P(neither white not black) = $\frac{8}{18} = \frac{4}{9}$. **Ans.**

(ii) Radius of the hemisphere = 2 cm

Height of cone = 4 cm

Radius of cylinder = 3 cm

Height of cylinder = 6 cm



Volume of water left in cylinder = volume of cylinder – [volume of cone + volume of hemisphere]

$= \pi r^2 h - \left[\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right]$

$= \pi\left[3^2 \times 6 - \frac{1}{3}\times 2\times 2\times 4 - \frac{2}{3}\times 2^3\right]$

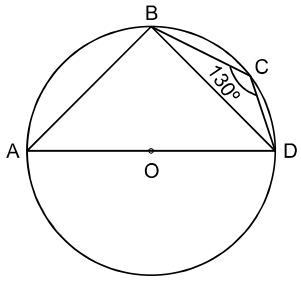
$= \pi\left[9\times 6 - \frac{16}{3} - \frac{16}{3}\right] = \pi\left[54 - \frac{32}{3}\right] = \pi\left[\frac{162-32}{3}\right]$

$= \frac{22}{7}\times\frac{130}{3} = 136 \text{ cm}^3$. **Ans.**

(iii) (a) ∠DAB + ∠BCD = 180°

$\Rightarrow$ ∠DAB + 130° = 180°

$\Rightarrow$ ∠DAB = 180° – 130° $\Rightarrow$ ∠DAB = 50° **Ans.**



(b) ∠ABD = $\frac{1}{2}$ ∠AOD = $\frac{1}{2}\times 180° = 90°$

$\Rightarrow$ ∠ABD = 90°

In ΔABD,

$\Rightarrow$ ∠ABD + ∠BAD + ∠ADB = 180° $\Rightarrow$ 90° + 50° + ∠ADB = 180°

$\Rightarrow$ 140° + ∠ADB = 180° $\Rightarrow$ ∠ADB = 180° – 140° $\Rightarrow$ ∠ADB = 40° **Ans.**

Question 9 :

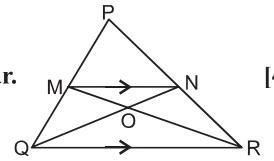
(i) Solve the inequation : $12 + \frac{5}{6}x \le 5 + 3x$, $x \in \mathbf{R}$

Represent the solution on a number line. **[3]**

(ii) Find the mean of the following distribution: **[3]**

Class	0–20	20–40	40–60	60–80	80–100	100–120	120–140
Frequency	4	15	23	12	16	8	2

(iii) In ΔPQR, MN is parallel to QR and $\frac{PM}{MQ} = \frac{2}{3}$



(a) Find $\frac{MN}{QR}$ **(b) Prove that ΔOMN and ΔORQ are similar.** **[4]**

Solution :

(i) $12+\frac{5}{6}x \le 5+3x$, $x \in R \Rightarrow$ $72 + 5x \le 30 + 18x$, $x \in R$

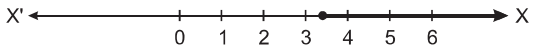
$\Rightarrow 5x - 18x \le 30 - 72$, $x \in R$

$\Rightarrow -13x \le -42$, $x \in R$

$\Rightarrow x \le \frac{42}{13}$, $x \in R$

$\Rightarrow x \le 3.2$, $x \in R$

$\therefore$ Solution set = $\{x \in R : x \ge 3.2\}$



(ii)

Class	Frequency (f)	x	$f \times x$
0 – 20	4	10	40
20 – 40	15	30	450
40 – 60	23	50	1150
60 – 80	12	70	840
80 – 100	16	90	1440
100 – 120	8	110	880
120 – 140	2	130	260
Total	80		5060

Mean = $\frac{\Sigma fx}{\Sigma f} = \frac{5060}{80} = 63.25$.

(iii) MN || QR

(a) In ΔPMN and ΔPQR

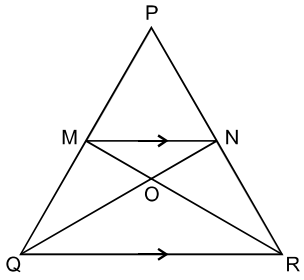
∠PMN = ∠PQR [Corres. ∠s]

∠PNM = ∠PRQ [Corres. ∠s]

So, ΔPMN ~ ΔPQR [By AA similarity axiom]

$\Rightarrow \frac{PM}{MQ} = \frac{MN}{QR}$

$\Rightarrow \frac{2}{3} = \frac{MN}{QR}$ **Ans.**



(b) In ΔOMN and ΔORQ

∠MON = ∠QOR [Vert. opp. ∠s]

∠OMN = ∠ORQ [Alt. int. ∠s]

So, ΔOMN ~ ΔORQ [By AA similarity axiom] **Proved.**

Question 10 :

(i) Given that $\frac{a^3+3ab^2}{b^3+3a^2b} = \frac{63}{62}$. Using componendo and dividendo, find $a : b$. **[3]**

(ii) Using ruler and compasses only, construct an isosceles ΔABC having base AB = 4 cm and median through vertex C is equal to 2.8 cm. Draw the incircle of the triangle. **[3]**

(iii) The angles of depression of two ships A and B as observed from the top of a lighthouse 60 m high are 60° and 45° respectively. If the two ships are on the opposite sides of the lighthouse, find the distance between the two ships. Give your answer correct to the nearest whole number. **[4]**

Solution :

(i) $\dfrac{a^3 + 3ab^2}{b^3 + 3a^2b} = \dfrac{63}{62}$

Using componendo and dividendo, we get

$$\frac{a^3 + 3ab^2 + b^3 + 3a^2b}{a^3 + 3ab^2 - b^3 - 3a^2b} = \frac{63+62}{63-62}$$

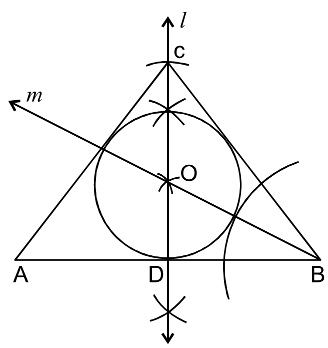
$$\Rightarrow \frac{(a+b)^3}{(a-b)^3} = 125 \Rightarrow \frac{a+b}{a-b} = 5$$

Again using componendo and dividendo, we get

$$\frac{a+b+a-b}{a+b-a+b} = \frac{5+1}{5-1}$$

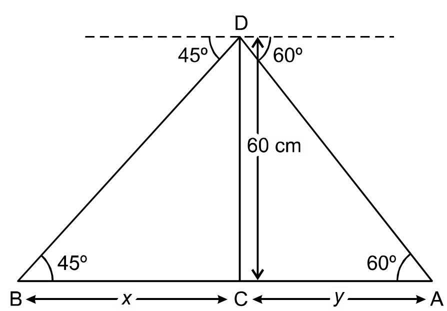
$$\Rightarrow \frac{2a}{2b} = \frac{6}{4} \Rightarrow \frac{a}{b} = \frac{3}{2} \Rightarrow a : b = 3 : 2.$$ **Ans.**

(ii)



(iii) CD be the light house which is 60 cm and A and B are two ships which are on the opposite sides of light house.
$\angle DBC = 45°$ and $\angle DAB = 60°$

BC = x and AC = y



Now, in ΔBCD, $\tan 45° = \dfrac{CD}{BC} \Rightarrow 1 = \dfrac{60}{x} \Rightarrow x = 60$ cm

In ΔBAC, $\tan 60° = \dfrac{DC}{AC}$

$$\Rightarrow \sqrt{3} = \frac{60}{y} \Rightarrow y = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

$\therefore$ Distance between two ships = $x + y = 60 + 20\sqrt{3} = 60 + 34.64$

$= 94.64$ m $= 95$ m. **Ans.**

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