ICSE 2024 EXAMINATION

MATHEMATICS

SAMPLE PAPER - 4

Time Allowed : 2½ hours ***Max. Marks : 80***

General Instructions :

Attempt all questions from Section A and any four questions from Section B.
All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.
Omission of essential working will result in loss of marks.
The intended marks for questions or parts of questions are given in brackets [ ]
Mathematical tables are provided.

SECTION - A (40 Marks)

(Attempt *all* questions from this Section)

Question 1 : Choose the correct answers to the questions from the given options: [15]

(i) Dividend is always paid on:
(a) the face value of a share (b) the market value of a share
(c) the amount invested (d) none of these

(ii) The roots of the quadratic equation $x^2 - 5x + 5 = 0$ are :
(a) real and equal (b) real and unequal (c) rational (d) imaginary

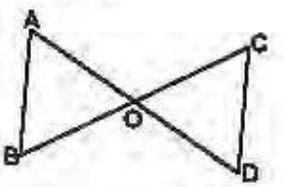
(iii) The remainder when $x^3 - 2x^2 - 5x + 6$ is divided by $(x + 2)$ is :
(a) –1 (b) 1 (c) 2 (d) 0

(iv) For a GP with first term a, common ratio r and last term l, the nth term from the end is:
(a) lr^{n-1} (b) $\frac{r^{n-1}}{l}$ (c) $\frac{r^{n-1}}{l}$ (d) $\frac{l}{r^{n-1}}$

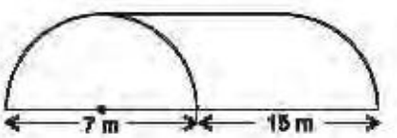
(v) 30th term of the AP 10, 7, 4, ... is :
(a) 97 (b) 77 (c) –77 (d) –87

(vi) The reflection of the point A (4, –1) in the line $x = 2$ is:
(a) (0, –1) (b) (8, –1) (c) (0, 1) (d) (–1, 0)

(vii) In the figure, if AB ∥ CD, then :
(a) ΔAOB ~ ΔCOD
(b) ΔAOB ~ ΔDOC
(c) ΔAOB ~ ΔODC
(d) none of these



(viii) A shed of a workshop is of the given shape. The volume of the air that the shed can hold is:
(a) 200 m^3
(b) 288.75 cm^3
(c) 300 m^3
(d) 307.25 m^3



(ix) If $8 - x \geq 6 - 2x$, $x \in N$, then the solution set is :
(a) {–2, –1, 0, 1, ...} (b) {1, 2, 3, ...} (c) {0, 1, 2, 3, ...} (d) {–1, 0, 1, 2, ...}

(x) A book has pages numbered from 1 to 85. What is the probability that the sum of the digits of the page number is 8, if a page is chosen at random?

(a) $\frac{6}{85}$ (b) $\frac{7}{85}$ (c) $\frac{9}{85}$ (d) $\frac{8}{85}$

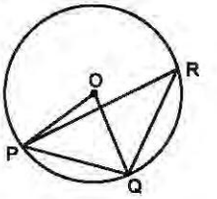
(xi) The order of a column matrix is of the form :

(a) m × 1 (b) 1 × m (c) m × 2 (d) 2 × 2

(xii) Two vertices of ΔABC are A(–1, 4) and B(5, 2) and its centroid is G(0, –3). The coordinates of C are :

(a) (4, 3) (b) (4, 15) (c) (–4, –15) (d) (–15, –4)

(xiii) In the given figure, O is the centre of a circle. If the length of chord PQ is equal to the radius of the circle, then ∠PRQ is :



(a) 15° (b) 30°

(c) 45° (d) 60°

(xiv) In a size transformation, if the scale factor k is equal to 1, then it is:

(a) an enlargement (b) a reduction

(c) an identity transformation (d) none of these

(xv) Daily wages of factory workers are recorded as :

Daily wages (in ₹)	131 – 136	137 – 142	143 – 148	149 – 154	155 – 160
No. of workers	5	27	20	18	12

The lower limit of the modal class is :

(a) ₹137 (b) ₹143 (c) ₹136.5 (d) ₹142.5

Question 2 :

(i) Anuprabha has a recurring deposit account in a bank for $3\frac{1}{2}$ years. If the bank pays the interest at the rate of 12% p.a. and Anuprabha gets ₹3061.80 on maturity, find the value of monthly instalment. [4]

(ii) ₹8000 and ₹10,000 were invested in ₹100 shares giving dividend 12% and 8% respectively. The dividends are collected and all the shares are sold at a loss of 2% and 3% respectively on the investment, find: [4]

(a) the dividend collected (b) the total sale proceeds

(c) gain% on the whole transaction

(iii) Prove that : $\frac{\cos\theta}{\text{cosec}\,\theta+1}+\frac{\cos\theta}{\text{cosec}\,\theta-1} = 2\tan\theta$ [4]

Question 3 :

(i) Find three numbers in GP whose sum is 52 and the sum of whose products in pairs is 624. [4]

(ii) A (2, –4), B(3, 3) and C(–1, 5) are the vertices of ΔABC. Find the equation of the altitude of the triangle through C. [4]

(iii) Use graph paper for this question. (Take 2 cm = 1 unit along both x and y axis). ABCD is a quadrilateral whose vertices are A(2, 2), B(2, –2), C(0, –1) and D(0, 1). [5]

(a) Reflect quadrilateral ABCD on the y-axis and name it as A′B′CD.

(b) Write down the coordinates of A′ and B′.

(c) Name two points which are invariant under the above reflection.

(d) Name the polygon A′B′CD.

SECTION - B (40 Marks)

(Attempt *any four* questions from this Section)

Question 4 :

(i) A dealer marks a juicer-mixer for ₹2150. A customer requests the dealer to reduce the price so that he has to pay ₹2124 including GST. If the rate of GST is 18%, how much reduction is needed in the price of the juicer-mixer? [3]

(ii) Solve the following equation using quadratic formula : $6x^2 + (12 - 8a)\,x - 16a = 0$ [3]

(iii) The daily profits in rupees of 100 shops in a departmental store are distributed as follows: [4]

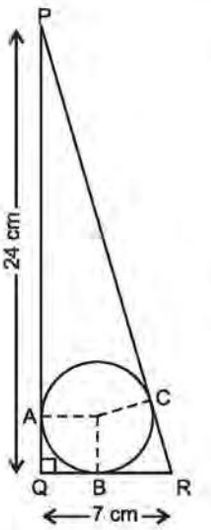
Profit per shop (in ₹)	0–100	100–200	200–300	300–400	400–500	500–600
Number of shops	12	18	27	20	17	6

Draw a histogram of the data given above on a graph paper and estimate the mode.

Question 5 :

(i) If $A = \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 14 & 3 \\ 2 & 4 \end{bmatrix}$, find a matrix C such that AC = B. [3]

(ii) In ΔPQR, ∠PQR = 90°, PQ = 24 cm and QR = 7 cm. Find the radius of the inscribed circle. [3]



(iii) Determine the value of k such that $(x - 2)$ is a factor of the polynomial $x^3 + kx^2 - 5x - 6$. [4]

Question 6 :

(i) Find the equation of the line parallel to $2x + 5y - 9 = 0$ and passing through the mid-point of the line segment joining A (2, 7) and B (−4, 1). [3]

(ii) The sides of a triangular plot of land in a map were 6 cm, 8 cm and 10 cm. If the scale of the map was 1:1000. find the actual area of the plot in m^2. [3]

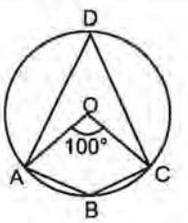
(iii) The 10th term of an AP is 52 and 16th term is 82. Find its general term. [4]

Question 7 :

(i) What is the probability that an ordinary year has 53 Sundays? [3]

(ii) A spherical metallic ball of radius 3 cm is melted and recast into three spherical balls. The radii of two of these balls are 2.5 cm and 2 cm respectively. Find the radius of the third ball. [3]

(iii) In the figure, O is the centre of the circle and ∠AOC = 100°.
Calculate ∠ADC and ∠ABC. [4]



Question 8 :

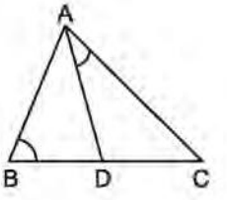
(i) Solve the following inequation and represent the solution set on the number line :
$-\frac{2}{3} < 1 + \frac{x}{3} \leq \frac{2}{3}, x \in R$ [3]

(ii) Find the mean of the following distribution. [3]

x	5	6	7	8	9
f	3	7	5	9	1

(iii) In ΔABC, D is a point on BC such that ∠ABC = ∠CAD, AB = 20 cm, AD = 10 cm and AC = 14 cm. Find: [4]

(a) DC (b) BD (c) ar(ΔADC) : ar(ΔABC)



Question 9 :

(i) Construct $\angle ABC = 120°$, where AB = BC = 5 cm. Mark two points D, E which satisfy both the following conditions.

(a) equidistant from BA and BC

(b) at a distance of 5 cm from B, point E is on the side of reflex $\angle ABC$. Join AE and EC. Describe the figures AECD, ABD and ABE. [4]

(ii) Use graph paper for this question. [6]

The following table shows the weight in gm of a sample of 100 potatoes taken from a large consignment.

Weight (in gm)	50–60	60–70	70–80	80–90	90–100	100–110	110–120	120–130
Frequency	8	10	12	16	18	14	12	10

(a) Calculate the cumulative frequencies.

(b) Draw the cumulative frequency curve and from it determine the median weight of the potatoes.

Question 10 :

(i) Using properties of proportion find $x : y$, given : $\frac{x^2+2x}{2x+4} = \frac{y^2+3y}{3y+9}$ [3]

(ii) Construct a regular hexagon of side 2.8 cm. Inscribe a circle in it. [3]

(iii) The angle of elevation of an aeroplane from a point P on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the aeroplane. [4]

ANSWERS

1. (i) (a) (ii) (b) (iii) (d) (iv) (d) (v) (c) (vi) (a) (vii) (b) (viii) (b) (ix) (b)
(x) (c) (xi) (a) (xii) (c) (xiii) (b) (xiv) (c) (xv) (c)

2. (i) ₹60 (ii) (a) ₹1760 (b) ₹17540 (c) $7\frac{2}{9}$ %

3. (i) 4, 12, 36 or 36, 12, 4 (ii) $x + 7y - 34 = 0$ (iii) (b) (–2, 2), (–2, –2) (c) C and D (d) Isosceles trapezium

4. (i) ₹350 (ii) $\frac{4}{3}a$, –2 (iii) ₹255

5. (i) $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$ (ii) 3 cm (iii) 2 **6.** (i) $2x + 5y = 18$ (ii) 2400 m^2 (iii) $2 + 5n$

7. (i) $\frac{1}{7}$ (ii) 1.5 cm (iii) 50°, 130°

8. (i) $\{x: -5 < x \leq -1, x \in R\}$ (ii) 6.92 (iii) (a) 7 cm (b) 14 cm (c) $\frac{1}{4}$

9. (i) kite, equilateral triangle, isosceles triangle (ii) (b) 93 g **10.** (i) 2 : 3 (iii) 720 km/h

SOLUTION

Time Allowed : 2½ hours *Max. Marks* : **80**

General Instructions :

Attempt all questions from ***Section A*** *and any four questions from* ***Section B****.*
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Mathematical tables are provided.

SECTION - A (40 Marks)

(Attempt *all* questions from this Section)

Question 1 : Choose the correct answers to the questions from the given options: **[15]**

(i) Dividend is always paid on:

(a) the face value of a share **(b) the market value of a share**

(c) the amount invested **(d) none of these**

(ii) The roots of the quadratic equation $x^2 - 5x + 5 = 0$ are :

(a) real and equal **(b) real and unequal** **(c) rational** **(d) imaginary**

(iii) The remainder when $x^3 - 2x^2 - 5x + 6$ is divided by $(x + 2)$ is :

(a) –1 **(b) 1** **(c) 2** **(d) 0**

(iv) For a GP with first term a, common ratio r and last term l, the nth term from the end is:

(a) lr^{n-1} **(b) $\frac{r^{n-1}}{l}$** **(c) $\frac{r^{n+1}}{l}$** **(d) $\frac{l}{r^{n-1}}$**

(v) 30th term of the AP 10, 7, 4, ... is :

(a) 97 **(b) 77** **(c) –77** **(d) –87**

(vi) The reflection of the point A (4, –1) in the line $x = 2$ is:

(a) (0, –1) **(b) (8, –1)** **(c) (0, 1)** **(d) (–1, 0)**

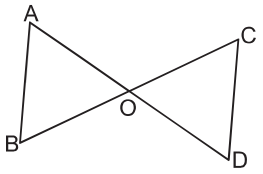
(vii) In the figure, if AB || CD, then :

(a) $\Delta AOB \sim \Delta COD$

(b) $\Delta AOB \sim \Delta DOC$

(c) $\Delta AOB \sim \Delta ODC$

(d) none of these



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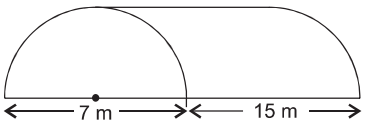


(a) 200 m^3

(b) 288.75 cm^3

(c) 300 m^3

(d) 307.25 m^3

(ix) If $8 - x \geq 6 - 2x$, $x \in N$, then the solution set is :

(a) {–2, –1, 0, 1, ...} **(b) {1, 2, 3, ...}** **(c) {0, 1, 2, 3, ...}** **(d) {–1, 0, 1, 2, ...}**

(x) A book has pages numbered from 1 to 85. What is the probability that the sum of the digits of the page number is 8, if a page is chosen at random?

(a) $\frac{6}{85}$ **(b) $\frac{7}{85}$** **(c) $\frac{9}{85}$** **(d) $\frac{8}{85}$**

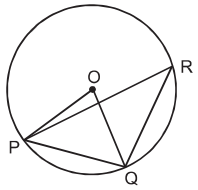
(xi) The order of a column matrix is of the form :

(a) $m \times 1$ **(b) $1 \times m$** **(c) $m \times 2$** **(d) 2×2**

(xii) Two vertices of ΔABC are A(–1, 4) and B(5, 2) and its centroid is G(0, –3). The coordinates of C are :

(a) (4, 3) **(b) (4, 15)** **(c) (–4, –15)** **(d) (–15, –4)**

(xiii) In the given figure, O is the centre of a circle. If the length of chord PQ is equal to the radius of the circle, then $\angle PRQ$ is :



(a) 15° **(b) 30°** **(c) 45°** **(d) 60°**

(xiv) In a size transformation, if the scale factor k is equal to 1, then it is:

(a) an enlargement **(b) a reduction**

(c) an identity transformation **(d) none of these**

(xv) Assertion (A) : Daily wages of the workers of a factory are as below :

Daily wages (in ₹)	131 – 136	137 – 142	143 – 148	149 – 154	155 – 160
No. of workers	5	27	20	18	12

The lower limit of the modal class of the above data is 137.

Reason (R) : The observation which occurs maximum number of times is called the mode of the data.

(a) A is true, R is false **(b) A is false, R is true** **(c) both A and R are true** **(d) both A and R are false**

Solution :

(i) (a) Dividend is always paid on face value of a share.

(ii) (b) Here $D = (-5)^2 - 4 \times 1 \times 5 = 25 - 20 = 5$. Here, $5 > 0$.
So roots are real and unequal.

(iii) (d) Required remainder is
$(-2)^3 - 2\times (-2)^2 - 5 \times (-2) + 6 = -8 - 8 + 10 + 6 = 0$

(iv) (d)

(v) (c) $T_{30} = 10 + (30 - 1) \times (-3) = 10 - 87 = -77$

(vi) (a) Required point is $(-4 + 2 \times 2, -1)$, i.e., $(0, -1)$.

(vii) (b) $\angle A = \angle D$ [Alternate angles]
$\angle AOB = \angle DOC$ [Vertically opposite angles]
$\Rightarrow \Delta AOB \sim \Delta DOC$ [AA similarity]

(viii) (b) Required volume $= \frac{1}{2}\pi r^2 h$

$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 15 \text{ m}^3 = 288.75 \text{ m}^3$.

(ix) (b) Here $8 - x \geq 6 - 2x$, $x \in N$
$\Rightarrow x \geq -2$, $x \in N$
$\Rightarrow$ Solution set is {1, 2, 3, ...}

(x) (c) Favourable number of outcomes = 9 {8, 17, 26, 35, 44, 53, 62, 71, 80}
$\therefore$ Required probability $= \frac{9}{85}$

(xi) (a) A column matrix has only 1 column.

(xii) (c) Here, $\left(\frac{-1+5+x}{3}, \frac{4+2+y}{3}\right) = (0, -3)$

$\Rightarrow 4 + x = 0$ and $6 + y = -9$

$\Rightarrow x = -4$ and $y = -15$

So, required co-ordinates are (–4, –15).

(xiii) (b) ΔPOQ is equilateral as OP = OQ = PQ

So, $\angle POQ = 60°$

$\therefore \angle PRQ = \frac{1}{2} \angle POQ = 30°$ [Angle at the centre is double the angle at the remaining part of the circle.]

(xiv) (c)

(xv) (b) Here the maximum frequency is 27, which falls in the class 137 – 142. But if we convert the classes into exclusive form, it becomes 136.5 – 142.5. So, the lower limit of modal class is 136.5.

So, its lower limit is 136.5.

Question 2 :

(i) Anuprabha has a recurring deposit account in a bank for $3\frac{1}{2}$ years. If the bank pays the interest at the rate of 12% p.a. and Anuprabha gets ₹3061.82 on maturity, find the value of monthly instalment. [4]

(ii) ₹8000 and ₹10,000 were invested in ₹100 shares giving dividend 12% and 8% respectively. The dividends are collected and all the shares are sold at a loss of 2% and 3% respectively on the investment, find: [4]

(a) the dividend collected

(b) the total sale proceeds

(c) gain% on the whole transaction

(iii) Prove that : $\frac{\cos\theta}{\text{cosec}\theta + 1} + \frac{\cos\theta}{\text{cosec}\theta - 1} = 2 \tan\theta$. [4]

Solution :

(i) Here, $n = 3\frac{1}{2}$ years = 42 months, R = 12% p.a.

SI = ₹(3061.80 – 42P)

Now, $S.I = P \times \frac{n(n+1)}{2} \times \frac{R}{100} \times \frac{1}{12}$

$\Rightarrow 3061.80 - 42P = P \times \frac{42 \times 43}{2} \times \frac{12}{100} \times \frac{1}{12} \Rightarrow 306180 - 4200P = 903P \Rightarrow 5103P = 306180$

$\Rightarrow P = \frac{306180}{5103} = 60.$

Hence, the monthly instalment = ₹60 **Ans.**

(ii) (a) Total dividend collected = 12% of ₹8000 + 8% of ₹10,000 = ₹1760 **Ans.**

(b) Total sale proceeds = 98% of ₹8000 + 97% of ₹10,000

= ₹17540. **Ans.**

(c) Actual gain = ₹(17540 + 1760) – ₹18000 = ₹1300

$\text{Gain \%} = \frac{13000 \times 100}{18000}\% = 7\frac{2}{9}$ **Ans.**

(iii) $LHS = \frac{\cos\theta}{\text{cosec}\,\theta + 1} + \frac{\cos\theta}{\text{cosec}\,\theta - 1}$

$= \frac{\cos\theta[\text{cosec}\,\theta - 1 + \text{cosec}\,\theta + 1]}{(\text{cosec}\,\theta - 1)(\text{cosec}\,\theta + 1)}$

$$= \frac{2\cos\theta \cdot \operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1} = \frac{2\cos\theta . \operatorname{cosec}\theta}{1 + \cot^2\theta - 1}$$

$$= \frac{2\cos\theta}{\sin\theta} \times \tan^2\theta$$

$= 2.\cot\theta \cdot \tan\theta \cdot \tan\theta = 2 \tan\theta =$ **RHS Proved.**

Question 3 :

(i) Find three numbers in GP whose sum is 52 and the sum of whose products in pairs is 624. [4]

(ii) A (2, –4), B(3, 3) and C(–1, 5) are the vertices of ΔABC. Find the equation of the altitude of the triangle through C. [4]

(iii) Use graph paper for this question. (Take 2 cm = 1 unit along both *x* and *y* axis). ABCD is a quadrilateral whose vertices are A(2, 2), B(2, –2), C(0, –1) and D(0, 1). [5]

(a) Reflect quadrilateral ABCD on the *y*-axis and name it as A′B′CD.

(b) Write down the coordinates of A′ and B′.

(c) Name two points which are invariant under the above reflection.

(d) Name the polygon A′B′CD.

Solution :

(i) Let the numbers be a, ar, ar^2.

Then, $a + ar + ar^2 = 52 = a(1 + r + r^2) = 52$ …(1)

Also, $a^2r + a^2r^3 + a^2r^2 = 624$

$\Rightarrow a^2r(1 + r + r^2) = 624$

$\Rightarrow ar = \frac{624}{52} = 12$ [From (1)]

$\Rightarrow a = \frac{12}{r}$

Now, from (1), $\frac{12}{r}(1 + r + r^2) = 52 \Rightarrow 12r^2 - 40r + 12 = 0$

$\Rightarrow 3r^2 - 10r + 3 = 0 \Rightarrow (3r - 1)(r - 1) = 0 \Rightarrow r = \frac{1}{3}$ or 3

So, $a = 36$ or 4

Hence, the numbers are 4, 12, 36 or 36, 12, 4. **Ans.**

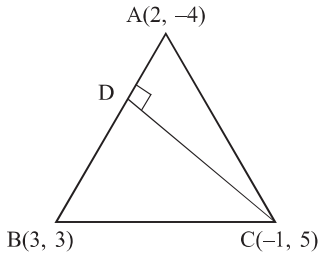
(ii) Gradient of AB $= \frac{3 + 4}{3 - 2} = \frac{7}{1} = 7$

Gradient of altitude (DC) through C $= -\frac{1}{7}$. [$\because m_1 \times m_2 = -1$]

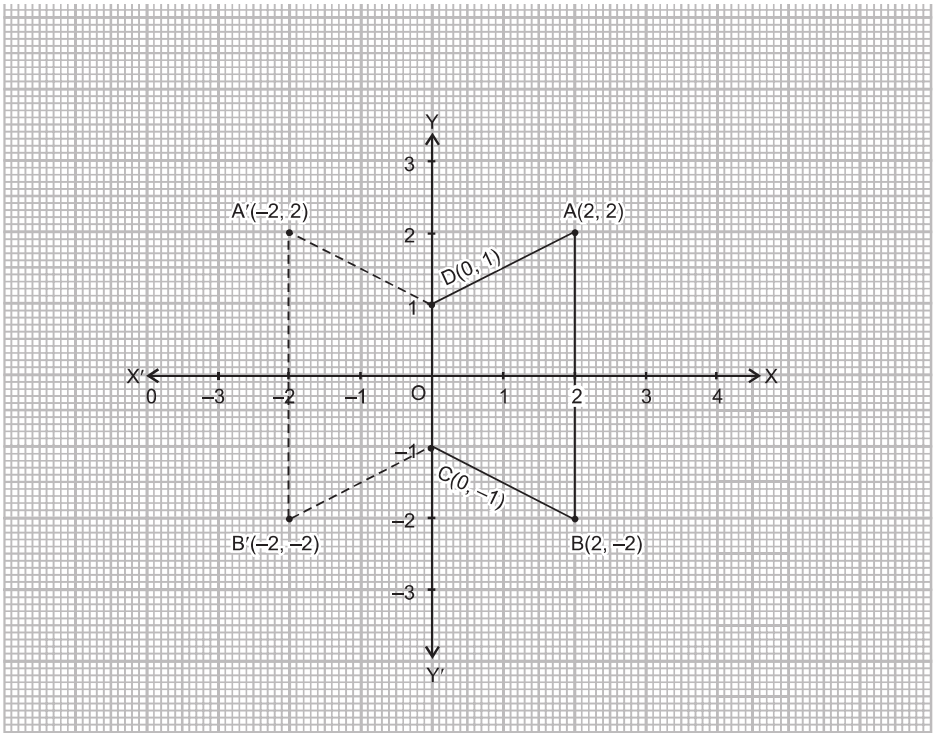
Now, DC also passes through (–1, 5).

So, required equation is

$y - 5 = -\frac{1}{7}(x + 1) \Rightarrow 7y - 35 = -x - 1 \Rightarrow x + 7y = 34$ **Ans.**



(iii) The quadrilateral ABCD has been drawn as below :



(a) The quadrilateral ABCD has been reflected on the y-axis to get the quadrilateral A′B′CD, as shown above. **Ans.**

(b) Coordinates of A′ and B′ are (–2, 2) and (–2, –2) respectively. **Ans.**

(c) From the graph, we see that the point C(0, –1) and D(0, 1) are invariant under the above reflection. **Ans.**

(d) We see that A′B′ || DC and A′D = B′C. So, A′B′CD is an isosceles trapezium. **Ans.**

SECTION - B (40 Marks)

(Attempt *any four* questions from this Section)

Question 4 :

(i) A dealer marks a juicer-mixer for ₹2150. A customer requests the dealer to reduce the price so that he has to pay ₹2124 including GST. If the rate of GST is 18%, how much reduction is needed in the price of the juicer-mixer? [3]

(ii) Solve the following equation using quadratic formula : $6x^2 + (12 - 8a)\,x - 16a = 0$ [3]

(iii) The daily profits in rupees of 100 shops in a departmental store are distributed as follows: [4]

Profit per shop (in ₹)	0–100	100–200	200–300	300–400	400–500	500–600
Number of shops	12	18	27	20	17	6

Draw a histogram of the data given above on a graph paper and estimate the mode.

Solution :

(i) Let the required deduction be ₹x.

Then, 118% of $(2150 - x) = 2124$

$\Rightarrow 2150 \times 118 - 118x = 212400 \Rightarrow 118x = 2150 \times 118 - 212400$

$\Rightarrow x = \dfrac{2150 \times 118 - 212400}{118} = 350.$

Hence, desired reduction needed is ₹350 **Ans.**

(ii) We have, $6x^2 + (12 - 8a)x - 16a = 0$

$\Rightarrow \quad 3x^2 + (6 - 4a)x - 8a = 0$

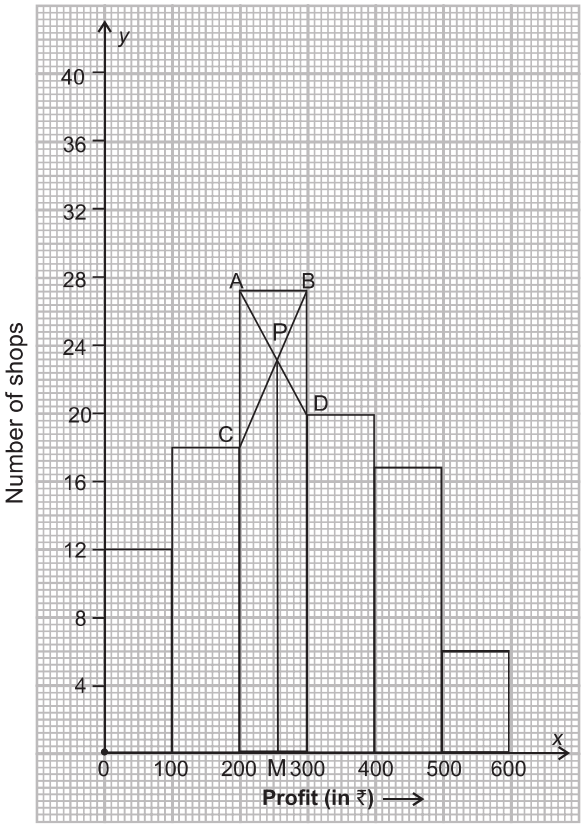
$$\therefore \quad x = \frac{-(6-4a) \pm \sqrt{(6-4a)^2 - 4 \times 3 \times (-8a)}}{2 \times 3}$$

$$= \frac{(-6+4a) \pm \sqrt{36 + 16a^2 - 48a + 96a}}{6} \quad = \frac{(-6+4a) \pm (6+4a)}{6}$$

$$= \frac{-6+4a+6+4a}{6} \text{ or } \frac{-6+4a-6-4a}{6}$$

$\Rightarrow x \quad = \frac{8a}{6} = \frac{4}{3}\, a$ or $\frac{-12}{6} = -2$. **Ans.**

(iii) The required histogram is shown below :

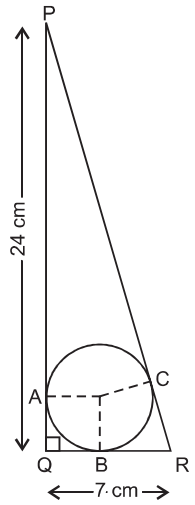


Mark the upper corners of the highest rectangle and then corners of the adjacent rectangle as A, B, C and D as shown above. Join AD and BC to intersect at P. From P draw PM $\perp$ x-axis, meeting the x-axis at M. Abscissa of M is 255. So, mode of the data is ₹255. **Ans.**

Question 5 :

(i) If A = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 14 & 3 \\ 2 & 4 \end{bmatrix}$, find a matrix C such that AC = B. **[3]**

(ii) In ΔPQR, $\angle PQR = 90°$, PQ = 24 cm and QR = 7 cm. Find the radius of the inscribed circle. **[3]**



(iii) Determine the value of k such that $(x - 2)$ is a factor of the polynomial $x^3 + kx^2 - 5x - 6$. **[4]**

Solution :

(i) Let $C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then, $AC = \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3a+2c & 3b+2d \\ -a+c & -b+d \end{bmatrix}$

But $AC = B \Rightarrow \begin{bmatrix} 3a+2c & 3b+2d \\ -a+c & -b+d \end{bmatrix} = \begin{bmatrix} 14 & 3 \\ 2 & 4 \end{bmatrix}$

$\Rightarrow 3a + 2c = 14, -a + c = 2$

$3b + 2d = 3, -b + d = 4$

Solving these equations, we get

$a = 2, b = -1, c = 4, d = 3$

Hence, the required matrix is $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$ **Ans.**

(ii) We know that the tangents drawn from an exterior point to a circle are equal.

So, PA = PC ... (i)

QA = QB ... (ii)

RB = RC .... (iii)

Also, $\angle OAQ = \angle OBQ = 90°$. [Radius through the point of contact is perpendicular to the tangent]

Also, $\angle Q = 90°$.

So, OAQB is a square.

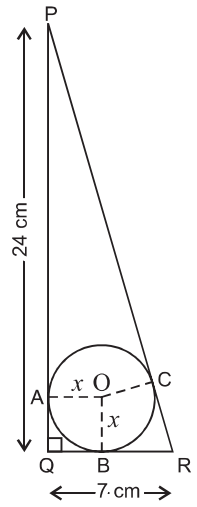
$PR^2 = PQ^2 + QR^2 = 24^2 + 7^2 = 576 + 49 = 625 \Rightarrow PR = 25$ cm

Now, PA = PC

$\Rightarrow 24 - x = 25 - CR \Rightarrow 24 - x = 25 - RB$

$\Rightarrow 24 - x = 25 - 7 + x \Rightarrow 6 = 2x \Rightarrow x = 3$

Hence, radius of the inscribed circle = 3 cm. **Ans.**



(iii) Since, $(x - 2)$ is a factor of $x^3 + kx^2 - 5x - 6$, so

$2^3 + k \times 2^2 - 5 \times 2 - 6 = 0$

$\Rightarrow 8 + 4k - 10 - 6 = 0$

$\Rightarrow 4k = 8 \Rightarrow k = 2$. **Ans.**

Question 6 :

(i) Find the equation of the line parallel to $2x + 5y - 9 = 0$ and passing through the mid-point of the line segment joining A (2, 7) and B (–4, 1). [3]

(ii) The sides of a triangular plot of land in a map were 6 cm, 8 cm and 10 cm. If the scale of the map was 1:1000. find the actual area of the plot in m^2. [3]

(iii) The 10th term of an AP is 52 and 16th term is 82. Find its general term. [4]

Solution :

(i) Coordinates of the mid-point of A(2, 7) and B (–4, 1) are $\left(\frac{2-4}{2}, \frac{7+1}{2}\right)$ or (–1, 4).

Gradient of the line $2x + 5y - 9 = 0 \Rightarrow y = \frac{-2x}{5} + \frac{9}{5}$ is $\frac{-2}{5}$.

So, gradient of the required line is $\frac{-2}{5}$.

So, equation of the required line is $y - 4 = \frac{-2}{5}(x + 1)$ [As it passes through (–1, 4)]

$\Rightarrow 5y - 20 = -2x - 2$

$\Rightarrow 2x + 5y = 18$ **Ans.**

(ii) Here, $6^2 + 8^2 = 10^2$. It means it is a right triangle

So, area of the triangle on the map $= \frac{1}{2} \times 6 \times 8 \text{ cm}^2 = 24 \text{ cm}^2$

Scale factor (R) $= \frac{1}{1000}$

Actual area $= \left(\frac{1}{k}\right)^2 \times$ area of the map $= (1000)^2 \times 24 \text{ cm}^2 = \frac{(1000)^2 \times 24}{10000} \text{ m}^2 = 2400 \text{ m}^2$ **Ans.**

(iii) $T_{10} = a + 9d = 52$ and $T_{16} = a + 15d = 82$

Now, $a + 15d = 82$

$a + 9d = 52$

$\Rightarrow \quad 6d = 30 \Rightarrow d = 5$

$a + 9 \times 5 = 52 \Rightarrow a = 52 - 45 = 7$

$\therefore T_n = a + (n - 1)d = 7 + (n - 1) \times 5 = 2 + 5n$ **Ans.**

Question 7 :

(i) Construct $\angle ABC = 120^o$, where AB = BC = 5 cm. Mark two points D, E which satisfy both the following conditions.

(a) equidistant from BA and BC

(b) at a distance of 5 cm from B, point E is on the side of reflex $\angle ABC$. Join AE and EC. Describe the figures AECD, ABD and ABE. **[5]**

(ii) Use graph paper for this question. **[5]**

The following table shows the weight in gm of a sample of 100 potatoes taken from a large consignment.

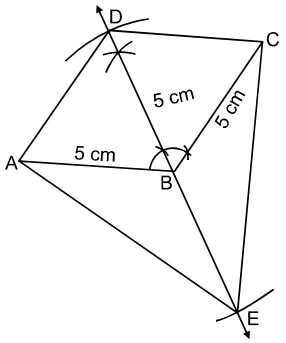
Weight (in gm)	50–60	60–70	70–80	80–90	90–100	100–110	110–120	120–130
Frequency	8	10	12	16	18	14	12	10

(a) Calculate the cumulative frequencies.

(b) Draw the cumulative frequency curve and from it determine the median weight of the potatoes.

Solution :

(i) AECD is a kite as AE = CE and AD = CD, ABD is an equilateral triangle ABE is an isosceles triangle as BA = BE. **Ans.**

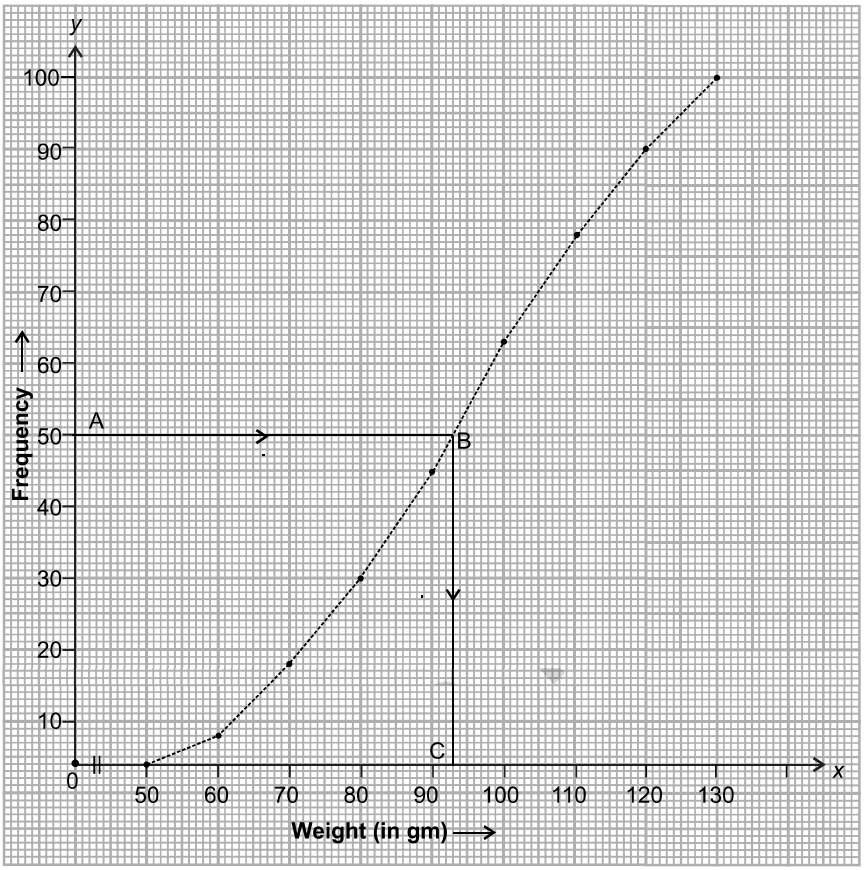


(ii) (a) The cumulative frequency table may be prepared as below :

Weight (in gm)	Frequency	Cumulative frequency
50 – 60	8	8
60 – 70	10	18
70 – 80	12	30
80 – 90	16	46
90 – 100	18	64
100 – 110	14	78
110 – 120	12	90
120 – 130	10	100

(b) Now, we take, weight along x-axis and frequency along y-axis. Plot the points (50, 0), (60, 8), (70, 18), (80, 30), (90, 46)), (100, 64), (110, 78), (120, 90) and (130, 100)

Join these points by a free hand curve to get the ogive.



Here $n = 100 \Rightarrow \frac{n}{2} = 50$

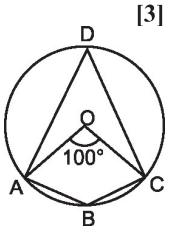
On the graph paper take a point A on y-axis representing 50. Through A, draw a horizontal line meeting the ogive at B. From B, draw BC $\perp$ x-axis, meeting the x-axis at C. The abscissa of C is 93. Hence, median weight is 93 gm. **Ans.**

Question 8 :

(i) What is the probability that an ordinary year has 53 Sundays? [3]

(ii) A spherical metallic ball of radius 3 cm is melted and recast into three spherical balls. The radii of two of these balls are 2.5 cm and 2 cm respectively. Find the radius of the third ball. [3]

(iii) In the figure, O is the centre of the circle and $\angle AOC = 100°$. Calculate $\angle ADC$ and $\angle ABC$. [4]



Solution :

(i) An ordinary year has, 52 weeks and 1 day. This 1 day can be Monday, Tuesday, Wednesday, Thursday, Friday, Saturday and Sunday. i.e, 7 days

So, required probability = $\frac{1}{7}$. **Ans.**

(ii) Let the radius of the third ball be r.

Then, $\frac{4}{3} \times \pi \times 3^3 = \frac{4}{3}\pi \times 2.5^3 + \frac{4}{3}\pi \times 2^3 + \frac{4}{3}\pi \times r^3$

$\Rightarrow 2.5^3 + 2^3 + r^3 = 3^3$

$\Rightarrow r^3 = 27 - 8 - 15.625 = 3.375$

$\Rightarrow r = 1.5$

Hence, radius of the third ball is 1.5 cm. **Ans.**

(iii) $\angle ADC = \frac{1}{2} \times \angle AOC$ [Angle at the centre is twice the angle at the remaining part of the circle]

$\Rightarrow \angle ADC = \frac{100°}{2} = 50°$

Since, ABCD is a cyclic quadrilateral,

So, $\angle ABC + \angle ADC = 180°$

$\Rightarrow \angle ABC = 180° - 50° = 130°$. **Ans.**

Question 9 :

(i) Solve the following inequation and represent the solution set on the number line :

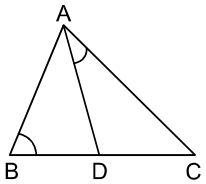
$$-\frac{2}{3} < 1 + \frac{x}{3} \leq \frac{2}{3},\ x \in R$$ **[3]**

(ii) Find the mean of the following distribution. **[3]**

x	5	6	7	8	9
f	3	7	5	9	1

(iii) In ΔABC, D is a point on BC such that $\angle ABC = \angle CAD$, AB = 20 cm, AD = 10 cm and AC = 14 cm. Find: **[4]**

(a) DC (b) BD (c) ar(ΔADC) : ar(ΔABC)



Solution :

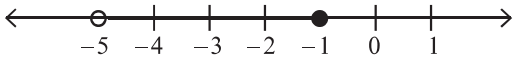
(i) We have $-\frac{2}{3} < 1 + \frac{x}{3}$ and $1 + \frac{x}{3} \leq \frac{2}{3}$, $x \in R$

$\Rightarrow \frac{x}{3} > -1 - \frac{2}{3}$ and $\frac{x}{3} \leq \frac{2}{3} - 1$, $x \in R$

$\Rightarrow x > -5$ and $x \leq -1$, $x \in R$

$\therefore$ Solution set is $\{x : -5 < x \leq -1, x \in R\}$

Graph of the solution set is shown below :



(ii) We may prepare a table as below :

x	f	fx
5	3	15
6	7	42
7	5	35
8	9	72
9	1	9
	$\Sigma f = 25$	$\Sigma fx = 173$

Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{173}{25} = 6.92$

(iii) First proof by AA similarity that $\Delta ABC \sim \Delta DAC$

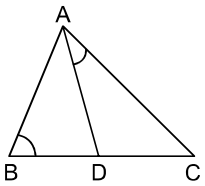
(a) So, $\frac{AB}{AD} = \frac{BC}{AC} = \frac{AC}{CD}$

$\Rightarrow \frac{AB}{AD} = \frac{AC}{CD} \Rightarrow \frac{20}{10} = \frac{14}{CD} \Rightarrow CD = 7$ cm. **Ans.**

(b) $\frac{AB}{AD} = \frac{BC}{AC} \Rightarrow \frac{20}{10} = \frac{BC}{14} \Rightarrow BC = 28$ cm

So, BD = BC − CD = (28−7) cm = 14 cm. **Ans.**

(c) $\frac{ar(\Delta ADC)}{ar(\Delta ABC)} = \frac{AD^2}{AB^2} = \frac{100}{400} = \frac{1}{4}$ **Ans.**



Question 10 :

(i) Using properties of proportion find $x : y$, given : $\frac{x^2+2x}{2x+4} = \frac{y^2+3y}{3y+9}$ **[3]**

(ii) Construct a regular hexagon of side 2.8 cm. Inscribe a circle in it. **[3]**

(iii) The angle of elevation of an aeroplane from a point P on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the aeroplane. **[4]**

Solution :

(i) $\frac{x^2+2x}{2x+4} = \frac{y^2+3y}{3y+9}$

Applying componendo and dividendo, we have,

$\frac{x^2+2x+2x+4}{x^2+2x-2x-4} = \frac{y^2+3y+3y+9}{y^2+3y-3y-9}$

$\Rightarrow \frac{x^2+4x+4}{x^2-4} = \frac{y^2+6y+9}{y^2-9}$

$\Rightarrow \frac{(x+2)^2}{(x+2)(x-2)} = \frac{(y+3)^2}{(y+3)(y-3)}$

$\Rightarrow \frac{x+2}{x-2} = \frac{y+3}{y-3}$

Again applying componendo and dividendo, we have

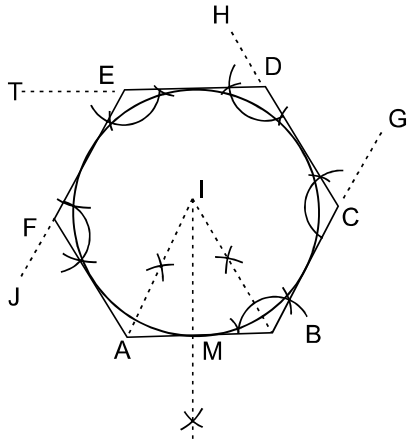
$\frac{x+2+x-2}{x+2-x+2} = \frac{y+3+y-3}{y+3-y+3}$

$\Rightarrow \frac{2x}{4} = \frac{2y}{6}$

$\Rightarrow \frac{x}{y} = \frac{2}{3}$ or $x : y = 2 : 3$ **Ans.**

(ii) **Steps of construction**

1. Draw AB = 2.8 cm, make $\angle ABG = 120°$
2. Along BG. set off BC = 2.8 cm.
3. Make $\angle BCH = 120°$
4. Along CH, set off CD = 2.8 cm
5. Make $\angle CDT = 120°$
6. Along DE, set off DE = 2.8 cm
7. Make $\angle DEJ = 120°$
8. Along EJ, set off EF = 2.8 cm
9. Join AF. Then ABCDEF is the required hexagon.
10. Draw the bisectors BI and AI of $\angle B$ and $\angle A$ to intersect at I.
11. From I, draw IM $\perp$ AB
12. With I as centre and IM as radius, draw the required circle.

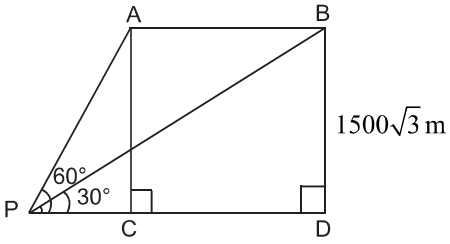


(iii) Let P be the point of observation and A and B be the two positions of the aeroplane.

In ΔAPC, $\tan 60° = \frac{AC}{PC}$

$\Rightarrow \sqrt{3} = \frac{1500\sqrt{3}}{PC}$

$\Rightarrow$ PC = $\frac{1500\sqrt{3}}{\sqrt{3}}$ = 15000 m ...(1)



In ΔPBD

$\tan 30° = \frac{BD}{PD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{1500 + CD}$ [From (1)]

$\Rightarrow$ 1500 + CD = 4500

$\Rightarrow$ CD = 4500 – 1500 = 3000 m

CD = AB $\Rightarrow$ Distance travelled by the aeroplane in 15 seconds = 3000 m.

$\therefore$ Speed of the aeroplane = $\frac{3000}{15}$ m/sec. = 200 $\times$ $\frac{18}{5}$ km/h = 720/km/h. **Ans.**

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