

ICSE 2024 EXAMINATION

MATHEMATICS

SAMPLE PAPER - 6

Time Allowed : 2½ hours

Max. Marks : 80

General Instructions :

Attempt all questions from Section A and any four questions from Section B.
All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.
Omission of essential working will result in loss of marks.
The intended marks for questions or parts of questions are given in brackets []
Mathematical tables are provided.

SECTION - A (40 Marks)

(Attempt all questions from this Section)

Question 1 : Choose the correct answers to the questions from the given options:

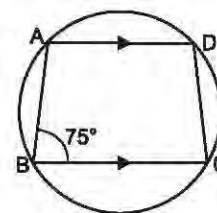
[15]

- (i) For a transaction of ₹80,000 in Delhi, if GST rate is 18%, then SGST is :
(a) ₹7,200 (b) ₹14,400 (c) ₹6,400 (d) nil
- (ii) The discriminant of the quadratic equation $x^2 - 2x + 1 = 0$:
(a) $= 0$ (b) > 0 (c) < 0 (d) none of these
- (iii) The value of m so that $(x + 6)$ is a factor of $x^3 + 5x^2 - 4x + m$, is :
(a) 7 (b) -3 (c) 6 (d) 12
- (iv) If a share of ₹100 is selling at ₹120, then it is said to be at :
(a) a discount of ₹20 (b) a premium of ₹20 (c) par (d) none of these
- (v) Which term of the AP 72, 68, 64, ... is 0?
(a) 15 (b) 18 (c) 19 (d) 20
- (vi) The progression $a_1, a_2, a_3, \dots, a_n$ forms an GP only if :
(a) $\frac{a_n}{a_{n-1}} = \text{constant}$ (b) $a_n - a_{n-1} = \text{constant}$ (c) $a_n \times a_{n-1} = \text{constant}$ (d) $\frac{a_{n-1}}{a_n} = \text{constant}$
- (vii) If in triangle ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar when :
(a) $\angle B = \angle E$ (b) $\angle A = \angle E$ (c) $\angle B = \angle D$ (d) $\angle A = \angle F$
- (viii) If the curved surface area of a cylinder of height 14 cm is 88 sq cm, then the diameter of the cylinder is:
(a) 2 cm (b) 4 cm (c) 1 cm (d) 3 cm
- (ix) If $4x - 2 \geq 8 - x$, $x \in \mathbb{N}$, then the solution set is :
(a) $\{2, 3, 4, 5, \dots\}$ (b) $\{1, 2, 3, 4, \dots\}$ (c) $\{0, 1, 2, 3, \dots\}$ (d) $\{2, 3, 4, 5, 6\}$
- (x) The probability of getting a sum of 13, when a pair of dice is rolled is:
(a) 0 (b) $\frac{1}{12}$ (c) $\frac{1}{13}$ (d) $\frac{1}{11}$
- (xi) Order of matrix P is 2×1 and that of Q is also 2×1 . The order of the matrix $2P - Q$ is:
(a) 2×2 (b) 4×1 (c) 2×1 (d) 3×2
- (xii) The coordinates of a point which divides a line segment joining points $(-3, 4)$ and $(7, -6)$ in the ratio 1 : 2 internally, are :
(a) $(-1, -3)$ (b) $(1, 3)$ (c) $(\frac{1}{3}, \frac{2}{3})$ (d) $(\frac{1}{3}, \frac{-2}{3})$

(xiii) In the given figure, ABCD is a cyclic trapezium such that $AD \parallel BC$.

If $\angle ABC = 75^\circ$, then $\angle BCD$ is:

- (a) 35°
- (b) 55°
- (c) 65°
- (d) 75°



(xiv) In a size transformation, the resulting figure is called:

- (a) object
- (b) image
- (c) pre-image
- (d) reduction

(xv) The median of the data $-11, 4, 9, -8, 0, 5, -1$, is:

- (a) 0
- (b) -1
- (c) -2
- (d) 4

Question 2 :

- (i) Manish deposits ₹2000 per month in a Recurring Deposit Account for $1\frac{1}{2}$ years at 8% p.a. Find the amount he will receive at the time of maturity. [4]
- (ii) What least number must be added to each of the numbers 5, 11, 19 and 37 so that the resulting numbers are proportional? [4]
- (iii) If $2 \tan \theta = 5$, find the value of $\frac{3 \sin \theta - 4 \cos \theta}{\sin \theta + 4 \cos \theta}$. [4]

Question 3 :

- (i) The total surface area of a cylinder of radius 5 cm is 660 cm^2 . Find the height of the cylinder. [4]
- (ii) Harikishan invested ₹8,000 in 7% ₹100 shares at ₹80. After a year he sold these shares at ₹75 each and invested the proceeds (including his dividend) in 18% ₹25 shares at ₹41 each. Find : [4]
 - (a) his dividend for the first year
 - (b) his annual income in the second year
 - (c) the percentage increase in his return on his original investment
- (iii) Use a graph paper for this question. [5]
 - (a) Plot the points A(0, 5), B(2, 5), C(5, 2), D(5, -2), E(2, -5) and F(0, -5).
 - (b) Reflect the points B, C, D and E on the y-axis and name them respectively as B', C', D' and E'.
 - (c) Write the coordinates of B', C', D' and E'.
 - (d) Name the figure formed by BCDEE'D'C'B'.

SECTION - B (40 Marks)

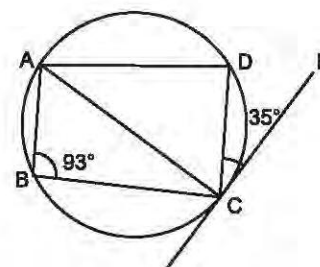
(Attempt any four questions from this Section)

Question 4 :

- (i) A dealer in Mumbai sold a refrigerator to a consumer in Mumbai for ₹21,500. If the rate of GST is 18%, find: [3]
 - (a) IGST
 - (b) CGST
 - (c) SGST
- (ii) If the roots of the equation $2x^2 - 2cx + ab = 0$ be real and distinct, prove that the roots of $x^2 - 2(a + b)x + (a^2 + b^2 + c^2) = 0$ will be imaginary. [3]
- (iii) The 2nd and 5th terms of a GP are $-\frac{1}{2}$ and $\frac{1}{16}$ respectively. Find the sum of first 8 terms of the GP. [4]

Question 5 :

- (i) Find x and y if : $3 \begin{bmatrix} 5 & -6 \\ 4 & x \end{bmatrix} - \begin{bmatrix} 6 & y \\ 0 & 6 \end{bmatrix} = 3 \begin{bmatrix} 3 & -2 \\ 4 & 0 \end{bmatrix}$ [3]
- (ii) In the given figure, CE is a tangent to the circle at point C. ABCD is a cyclic quadrilateral. If $\angle ABC = 93^\circ$ and $\angle DCE = 35^\circ$, find : [3]
 - (a) $\angle ADC$
 - (b) $\angle CAD$
 - (c) $\angle ACD$
- (iii) Using factor theorem, show that $(x - 2)$ is a factor of $2x^3 - 5x^2 + 4x - 4$. [4]

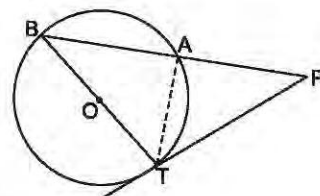


Question 6 :

- (i) Write down the equation of the line whose gradient is $\frac{3}{2}$ and which passes through P, where P divides the line segment joining A (-2, 6) and B (3, -4) in the ratio 2 : 3. [3]
- (ii) The surface area of a solid is 5 m^2 , while the surface area of its model is 20 cm^2 . Find [3]
- (a) the scale factor
- (b) the volume of the solid if the volume of the model is 100 cm^3 .
- (iii) How many terms of the AP 7, 11, 15, 19, 23, must be taken to get the sum 250? [4]

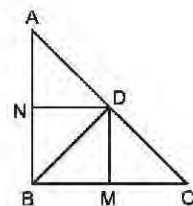
Question 7 :

- (i) A large firm employs 4250 employees. One person is chosen at random. What is the probability that the person's birthday is on Monday in the year 2016? [3]
- (ii) A cylindrical can whose base is horizontal and of radius 3.5 cm contains sufficient water so that when a sphere is placed in the can, the water just covers the sphere. Given that the sphere just fits into the can, calculate : [3]
- (a) the total surface area of the can in contact with water when the sphere is in it.
- (b) the depth of water in the can before the sphere was put into the can.
- (iii) In the given figure, PAB is a secant and PT a tangent to the circle with centre O. If $\angle ATP = 40^\circ$, PA = 9 cm and AB = 7 cm. [4]
- Find:
- (a) $\angle APT$ (b) length of PT



Question 8 :

- (i) Solve the following inequation and represent your solution on the real number line: [3]
- $$-5\frac{1}{2} - x \leq \frac{1}{2} - 3x \leq 3\frac{1}{2} - x, x \in \mathbb{R}$$
- (ii) Calculate the mean daily wage of a worker from the following table : [3]
- | | | | | | |
|--------------------|-------|-------|-------|-------|-------|
| Daily wages (in ₹) | 40-45 | 45-50 | 50-55 | 55-60 | 60-65 |
| No. of workers | 2 | 3 | 7 | 12 | 6 |
- (iii) In the figure, ABC is a right triangle with $\angle ABC = 90^\circ$, $BD \perp AC$, $DM \perp BC$ and $DN \perp AB$. Prove that [4]
- (a) $DM^2 = DN \times MC$ (b) $DN^2 = DM \times AN$



Question 9 :

- (i) Draw an ogive for the following frequency distribution : [6]
- | | | | | | | | |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|------------|
| Class | 6500-7000 | 7000-7500 | 7500-8000 | 8000-8500 | 8500-9000 | 9000-9500 | 9500-10000 |
| Frequency | 10 | 18 | 22 | 25 | 17 | 10 | 8 |
- From the ogive find the median.
- (ii) Draw a line segment AB = 10 cm. Mark C, the mid-point of AB. Draw and describe the locus of a point which is [4]
- (i) 2 cm from AB (ii) 4 cm from C. Mark the points E, F, G, H which satisfy both the above conditions.
- (a) Describe the figure EFGH. (b) What kind of triangle is ECF?

Question 10 :

- (i) Draw a regular hexagon of side 3.5 cm. Circumscribe a circle to it. [3]
- (ii) Solve for x, using the properties of proportion: $\frac{3x + \sqrt{9x^2 - 5}}{3x - \sqrt{9x^2 - 5}} = 5$ [3]
- (iii) A boy standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60° . When he moves 20 m back from the bank, he finds the angle to be 30° . Find the height of the tree and the breadth of the river. [4]

ANSWERS

1. (i) (a) (ii) (a) (iii) (d) (iv) (b) (v) (c) (vi) (d) (vii) (c) (viii) (a) (ix) (a)
(x) (a) (xi) (c) (xii) (c) (xiii) (d) (xiv) (b) (xv) (a)
2. (i) ₹38280 (ii) 2 (iii) $\frac{7}{13}$
3. (i) 16 cm (ii) (a) ₹700 (b) ₹900 (c) 2.5% (iii) (c) $(-2, 5), (-5, 2), (-5, -2), (-2, -5)$ (d) Regular octagon
4. (i) (a) 0 (b) ₹1935 (c) ₹1935 (iii) $\frac{85}{128}$
5. (i) 2, -12 (ii) (a) 87° (b) 35° (c) 58°
6. (i) $3x - 2y + 4 = 0$ (ii) (a) 50 (b) 12.5 m^3 (iii) 10
7. (i) $\frac{1}{7}$ (ii) (a) 192.5 cm^2 (b) $\frac{7}{3} \text{ cm}$ (iii) (a) 50° (b) 12 cm
8. (i) $\{x : \frac{-3}{2} \leq x \leq 3, x \in \mathbb{R}\}$ (ii) ₹55.33
9. (i) 8100 (ii) (a) Rectangle (b) Equilateral 10. (ii) 1 (iii) 17.32 m, 10 m

SOLUTION

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SECTION - A (40 Marks) (Attempt all questions from this Section)

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- (v) Which term of the AP 72, 68, 64, ... is 0?
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(a) 2 cm (b) 4 cm (c) 1 cm (d) 3 cm
- (ix) If $4x - 2 \geq 8 - x$, $x \in \mathbb{N}$, then the solution set is :
(a) {2, 3, 4, 5, ...} (b) {1, 2, 3, 4, ...} (c) {0, 1, 2, 3, ...} (d) {2, 3, 4, 5, 6}
- (x) The probability of getting a sum of 13, when a pair of dice is rolled is:
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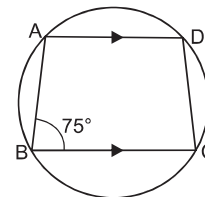
(xii) The coordinates of a point which divides a line segment joining points $(-3, 4)$ and $(7, -6)$ in the ratio $1 : 2$ internally, are :

- (a) $(-1, -3)$ (b) $(1, 3)$ (c) $\left(\frac{1}{3}, \frac{2}{3}\right)$ (d) $\left(-\frac{1}{3}, -\frac{2}{3}\right)$

(xiii) In the given figure, ABCD is a cyclic trapezium such that $AD \parallel BC$.

If $\angle ABC = 75^\circ$, then $\angle BCD$ is:

- (a) 35°
(b) 55°
(c) 65°
(d) 75°



(xiv) In a size transformation, the resulting figure is called:

- (a) object (b) image (c) pre-image (d) reduction

(xv) Assertion (A) : The median of the data $-11, 4, 9, -8, 0, 5, -1$ is -1 .

Reason (R) : For calculating the median of a data we only consider positive observations.

- (a) A is true, R is false (b) A is false, R is true (c) both A and R are true (d) both A and R are false

Solution :

- (i) (a) $\text{SGST} = 9\% \text{ of } ₹80,000 = ₹7,200$
(ii) (a) Discriminant $= (-2)^2 - 4 \times 1 \times 1 = 4 - 4 = 0$
(iii) (d) Since $(x + 6)$ is a factor of $x^3 + 5x^2 - 4x + m$, so, $(-6)^3 + 5 \times (-6)^2 - 4 \times (-6) + m = 0$
 $\Rightarrow -216 + 180 + 24 + m = 0 \Rightarrow m = 12$
(iv) (b)
(v) (c) Here, $T_n = 0 \Rightarrow 72 + (n - 1)(-4) = 0 \Rightarrow n - 1 = 18 \Rightarrow n = 19$.
(vi) (d)
(vii) (c)
(viii) (a) Here $2\pi rh = 88 \Rightarrow r = \frac{88 \times 7}{2 \times 22 \times 14} = 1 \Rightarrow \text{Diameter} = 2 \text{ cm}$.
(ix) (a) $4x - 2 + x \geq 8 = 5x \geq 10 = x \geq 2, x \in \mathbb{N}$
So, solution set is $\{2, 3, 4, \dots\}$
(x) (a) It is an impossible event.
(xi) (c) On adding and subtracting two matrices, the order does not change.
(xii) (c) Required coordinates are $\left(\frac{1 \times 7 + 2 \times (-3)}{1 + 2}, \frac{1 \times (-6) + 2 \times 4}{1 + 2}\right)$, i.e; $\left(\frac{1}{3}, \frac{2}{3}\right)$
(xiii) (d) $\angle B + \angle D = 180^\circ \Rightarrow \angle D = 105^\circ$
 $\angle BCD + \angle ADC = 180^\circ$ [Co-interior angles]
 $\Rightarrow \angle BCD = 75^\circ$.
(xiv) (b)
(xv) (d) Arranging the given data in ascending order, we get : $-11, -8, -1, 0, 4, 5, 9$ [$n = 7$ (odd)]
So, median $= \left(\frac{7+1}{2}\right)^{\text{th}}$ observation
 $= 4^{\text{th}}$ observation $= 0$

Question 2 :

- (i) Manish deposits ₹2000 per month in a Recurring Deposit Account for $1\frac{1}{2}$ years at 8% p.a. Find the amount he will receive at the time of maturity. [4]
(ii) What least number must be added to each of the numbers 5, 11, 19 and 37 so that the resulting numbers are proportional? [4]
(iii) If $2 \tan \theta = 5$, find the value of $\frac{3 \sin \theta - 4 \cos \theta}{\sin \theta + 4 \cos \theta}$. [4]

Solution :

- (i) Here $P = ₹ 2000$, $n = 18$ months, $R = 8\%$ p.a.

$$\text{Then S.I.} = P \left(\frac{n(n+1)}{2} \times \frac{1}{12} \times \frac{R}{100} \right) = ₹ \left(2000 \times \frac{18 \times 19}{2} \times \frac{1}{12} \times \frac{8}{100} \right) = ₹ 2280$$

$$\text{Amount} = ₹ (2000 \times 18 + 2280) = ₹ 38,280$$

Hence, Manish will receive ₹ 38,280 on maturity. **Ans.**

(ii) Let the required number be x .

Then, $5 + x$, $11 + x$, $19 + x$ and $37 + x$ are in proportion.

$$\Rightarrow (5 + x) : (11 + x) : : (19 + x) : (37 + x)$$

$$\Rightarrow \frac{5+x}{11+x} = \frac{19+x}{37+x} \Rightarrow 185 + 5x + 37x + x^2 = 209 + 11x + 19x + x^2$$

$$\Rightarrow 42x - 30x = 209 - 185 \Rightarrow 12x = 24 \Rightarrow x = 2$$

Hence, the required number is 2. **Ans.**

(iii) We have, $\frac{3 \sin \theta - 4 \cos \theta}{\sin \theta + 4 \cos \theta}$

$$\begin{aligned} \text{Dividing Nr. and Dr. by } \cos \theta \text{ we get, } \frac{3 \tan \theta - 4}{\tan \theta + 4} &= \frac{3 \left(\frac{5}{2} \right) - 4}{\frac{5}{2} + 4} \left[\tan \theta = \frac{5}{2}, \text{ given} \right] \\ &= \frac{(15-8)/2}{(5+8)/2} = \frac{7}{13}. \quad \text{Ans.} \end{aligned}$$

Question 3 :

(i) The total surface area of a cylinder of radius 5 cm is 660 cm². Find the height of the cylinder. [4]

(ii) Harikishan invested ₹8,000 in 7% ₹100 shares at ₹80. After a year he sold these shares at ₹75 each and invested the proceeds (including his dividend) in 18% ₹25 shares at ₹41 each. Find : [4]

(a) his dividend for the first year

(b) his annual income in the second year

(c) the percentage increase in his return on his original investment

(iii) Use a graph paper for this question. [5]

(a) Plot the points A(0, 5), B(2, 5), C(5, 2), D(5, -2), E(2, -5) and F(0, -5).

(b) Reflect the points B, C, D and E on the y-axis and name them respectively as B', C', D' and E'.

(c) Write the coordinates of B', C', D' and E'.

(d) Name the figure formed by BCDEE'D'C'B'.

(e) Name a line of symmetry for the figure formed.

Solution :

(i) We have, $r = 5$ cm, $h = ?$

$$\text{Total surface area} = 660 \text{ cm}^2$$

$$\therefore \text{Total surface area} = 2\pi r (r + h)$$

$$\Rightarrow 660 = 2 \times \frac{22}{7} \times 5 (5 + h)$$

$$\Rightarrow 5 + h = \frac{660 \times 7}{2 \times 22 \times 5} = 21 \Rightarrow h = 21 - 5 = 16.$$

Hence, height of the cylinder = 16 cm. **Ans.**

(ii) To investment = ₹8000

Market value of a 1 share = ₹80

$$\text{Number of shares bought} = \frac{₹8000}{₹80} = 100$$

Total face value of 100 shares = ₹100 × 100

$$(a) \text{ So, dividend for 1st year} = \frac{₹100 \times 100 \times 7}{100} = ₹700. \quad \text{Ans.}$$

(b) Amount invested in second kind of shares = ₹(75 × 100 + 700) = ₹8200.

$$\text{Number of shares bought} = \frac{₹8200}{41} = 200.$$

$$\text{Face value of 200 shares} = ₹200 \times 25 = ₹5000.$$

$$\text{So, annual dividend} = \frac{₹5000 \times 18}{100} = ₹900 \text{ Ans.}$$

(c) Increase in income = ₹(900 – 700) = ₹200

$$\text{Required percentage increase} = \frac{200}{8000} \times 100\% = 2.5\% \text{ Ans.}$$

- (iii) (a) The given points have been plotted.
 (b) The points B, C, D, E have been reflected as shown.
 (c) Coordinates of the image point B' of point B on reflection in y-axis = (–2, 5)

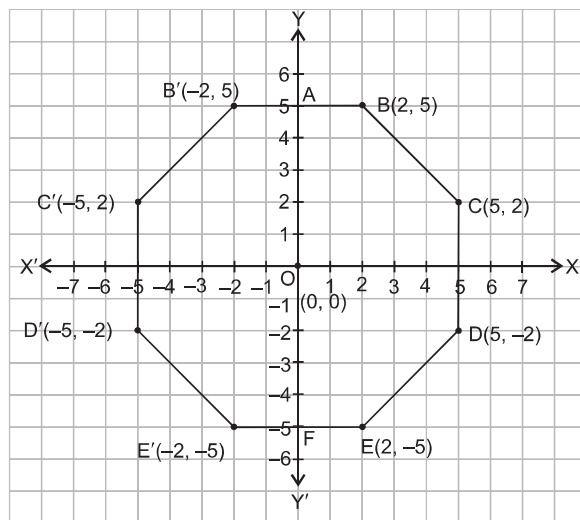
Coordinates of the image point C' of point C on reflection in y-axis = (–5, 2)

Coordinates of the image point D' of point D on reflection in y-axis = (–5, –2)

Coordinates of the image point E' of point E on reflection in y-axis = (–2, –5)

(d) the figure BCDEE'D'C'B' is a regular octagon.

(e) y-axis or $x = 0$ is the line of symmetry of the above figure.



SECTION - B (40 Marks)

(Attempt *any four* questions from this Section)

Question 4 :

- (i) A dealer in Mumbai sold a refrigerator to a consumer in Mumbai for ₹21,500. If the rate of GST is 18%, find: [3]
 (a) IGST (b) CGST (c) SGST
- (ii) If the roots of the equation $2x^2 - 2cx + ab = 0$ be real and distinct, prove that the roots of $x^2 - 2(a + b)x + (a^2 + b^2 + c^2) = 0$ will be imaginary. [3]
- (iii) The 2nd and 5th terms of a GP are $-\frac{1}{2}$ and $\frac{1}{16}$ respectively. Find the sum of first 8 terms of the GP. [4]

Solution :

(i) (a) Since the refrigerator is sold within the same state, so IGST = 0. Ans.

(b) GST rate = 18%

$$\Rightarrow \text{CGST rate} = 9\% \text{ and SGST rate} = 9\%$$

$$\therefore \text{CGST} = 9\% \text{ of } ₹21,500 = ₹1935 \text{ Ans.}$$

(c) SGST = 9% of ₹21,500 = ₹1935 Ans.

(ii) Discriminant of the equation $2x^2 - 2cx + ab = 0$ is given by $D = 4c^2 - 8ab$

$$\text{But, } D > 0 \Rightarrow 4c^2 - 8ab > 0$$

$$\Rightarrow c^2 > 2ab$$

Now, the discriminant of $x^2 - 2(a + b)x + (a^2 + b^2 + c^2) = 0$ is given by

$$D = 4(a + b)^2 - 4 \times 1(a^2 + b^2 + c^2)$$

$$= 4a^2 + 4b^2 + 8ab - 4a^2 - 4b^2 - 4c^2$$

$$= 8ab - 4c^2 = 4(2ab - c^2)$$

$$\Rightarrow D = 4(2ab - c^2) < 0 \text{ as } c^2 > 2ab$$

Since $D < 0$, so roots are imaginary. **Proved**

$$(iii) ar = \frac{-1}{2} \text{ and } ar^4 = \frac{1}{16}$$

$$\Rightarrow r^3 = \frac{1}{16} \times \frac{2}{-1} = -\frac{1}{8} \Rightarrow r = -\frac{1}{2}$$

$$a = \frac{-1}{2} \times \frac{-2}{1} = 1$$

$$S_8 = \frac{1 \times \left[1 - \left(-\frac{1}{2} \right)^8 \right]}{\left(1 + \frac{1}{2} \right)} = \frac{2}{3} \times \frac{255}{256} = \frac{85}{128} \text{ Ans.}$$

Question 5 :

$$(i) \text{ Find } x \text{ and } y \text{ if : } 3 \begin{bmatrix} 5 & -6 \\ 4 & x \end{bmatrix} - \begin{bmatrix} 6 & y \\ 0 & 6 \end{bmatrix} = 3 \begin{bmatrix} 3 & -2 \\ 4 & 0 \end{bmatrix}$$

[3]

(ii) In the given figure, CE is a tangent to the circle at point C. ABCD is a cyclic quadrilateral. If $\angle ABC = 93^\circ$ and $\angle DCE = 35^\circ$. [3]

Find :

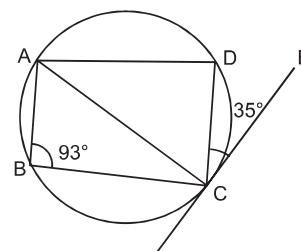
(a) $\angle ADC$

(b) $\angle CAD$

(c) $\angle ACD$

(iii) Using factor theorem, show that $(x - 2)$ is a factor of $2x^3 - 5x^2 + 4x - 4$.

[4]



Solution :

$$(i) \text{ We have, } 3 \begin{bmatrix} 5 & -6 \\ 4 & x \end{bmatrix} - \begin{bmatrix} 6 & y \\ 0 & 6 \end{bmatrix} = 3 \begin{bmatrix} 3 & -2 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 15 & -18 \\ 12 & 3x \end{bmatrix} - \begin{bmatrix} 6 & y \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & -6 \\ 12 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 15-6 & -18-y \\ 12-0 & 3x-6 \end{bmatrix} = \begin{bmatrix} 9 & -6 \\ 12 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 9 & -18-y \\ 12 & 3x-6 \end{bmatrix} = \begin{bmatrix} 9 & -6 \\ 12 & 0 \end{bmatrix}$$

$$\Rightarrow -18 - y = -6 \text{ and } 3x - 6 = 0$$

$$\Rightarrow y = -18 + 6 \text{ and } x = 2 \Rightarrow x = 2 \text{ and } y = -12 \text{ Ans.}$$

(ii) (a) ABCD is a cyclic quadrilateral.

$\therefore \angle ABC + \angle ADC = 180^\circ$ [Opposite angles of a cyclic quadrilateral are supplementary]

$$\Rightarrow \angle ADC = 180^\circ - \angle ABC = 180^\circ - 93^\circ = 87^\circ \text{ Ans.}$$

(b) $\angle DCE = \angle CAD$ [Angles in the alternate segments]

$$\Rightarrow \angle CAD = 35^\circ \text{ Ans.}$$

(c) In $\triangle ACD$, $\angle ACD + \angle CAD + \angle ADC = 180^\circ$

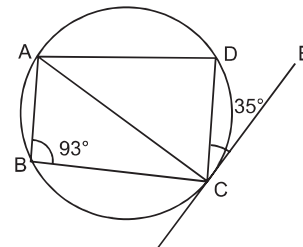
[Angle sum property of a Δ]

$$\Rightarrow \angle ACD + 35^\circ + 87^\circ = 180^\circ \Rightarrow \angle ACD = 180^\circ - 122^\circ = 58^\circ \text{ Ans.}$$

(iii) Let us find $f(2)$

$$f(2) = 2(2)^3 - 5(2)^2 + 4(2) - 4 = 16 - 20 + 8 - 4 = 0$$

Since, $f(2) = 0$, so by factor theorem $(x - 2)$ is a factor of $f(x)$. **Proved.**



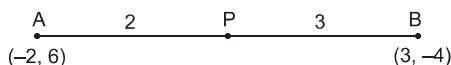
Question 6 :

(i) Write down the equation of the line whose gradient is $\frac{3}{2}$ and which passes through P, where P divides the line segment joining A $(-2, 6)$ and B $(3, -4)$ in the ratio 2 : 3. [3]

- (ii) The surface area of a solid is 5 m^2 , while the surface area of its model is 20 cm^2 . Find [3]
 (a) the scale factor
 (b) the volume of the solid if the volume of the model is 100 cm^3 .
 (iii) How many terms of the AP 7, 11, 15, 19, 23, must be taken to get the sum 250? [4]

Solution :

(i) $m = \frac{3}{2}$



P divides AB in 2 : 3

So, co-ordinates of P are $\left(\frac{2 \times 3 + 3(-2)}{5}, \frac{2(-4) + 3(6)}{5} \right)$, i.e., (0, 2)

Equation of the required line is $(y - 2) = \frac{3}{2}(x - 0) = \frac{3}{2}x$

$\Rightarrow 2y = 3x + 4 \Rightarrow 3x - 2y + 4 = 0$. **Ans.**

- (ii) (a) Let the scale factor be k .

Then, $5 \text{ m}^2 = k^2 \times 20 \text{ cm}^2$

$\Rightarrow 5 \times 100 \times 100 = k^2 \times 20$

$\Rightarrow k^2 = 5 \times 5 \times 100 \Rightarrow k = 50$ **Ans.**

- (b) $V = (50)^3 \times 100 \text{ cm}^3$.

$= \frac{125 \times 100 \times 1000}{100 \times 100 \times 100} \text{ m}^3 = 12.5 \text{ m}^3$ **Ans.**

- (iii) The given AP is 7, 11, 15, 19, 23,

Here, $a = 7, d = 11 - 7 = 4$

Let the sum of n terms of the given AP be 250.

Then, $S_n = 250 \Rightarrow \frac{n}{2} [2a + (n - 1)d] = 250 \Rightarrow \frac{n}{2} [2 \times 7 + (n - 1) \times 4] = 250$

$\Rightarrow 7n + 2n^2 - 2n = 250 \Rightarrow 2n^2 + 5n - 250 = 0 \Rightarrow 2n^2 + 25n - 20n - 250 = 0 \Rightarrow n(2n + 25) - 10(2n + 25) = 0$

$\Rightarrow (2n + 25)(n - 10) = 0 \Rightarrow n = 10 \left[\because n \neq \frac{-25}{2} \right]$

Hence, the required number of terms is 10. **Ans.**

Question 7 :

- (i) Draw an ogive for the following frequency distribution :

[5]

Class	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	9000-9500	9500-10000
Frequency	10	18	22	25	17	10	8

From the ogive find the median.

- (ii) Draw a line segment AB = 10 cm. Mark C, the mid-point of AB. Draw and describe the locus of a point which is

(i) 2 cm from AB (ii) 4 cm from C. Mark the points E, F, G, H which satisfy both the above conditions. [5]

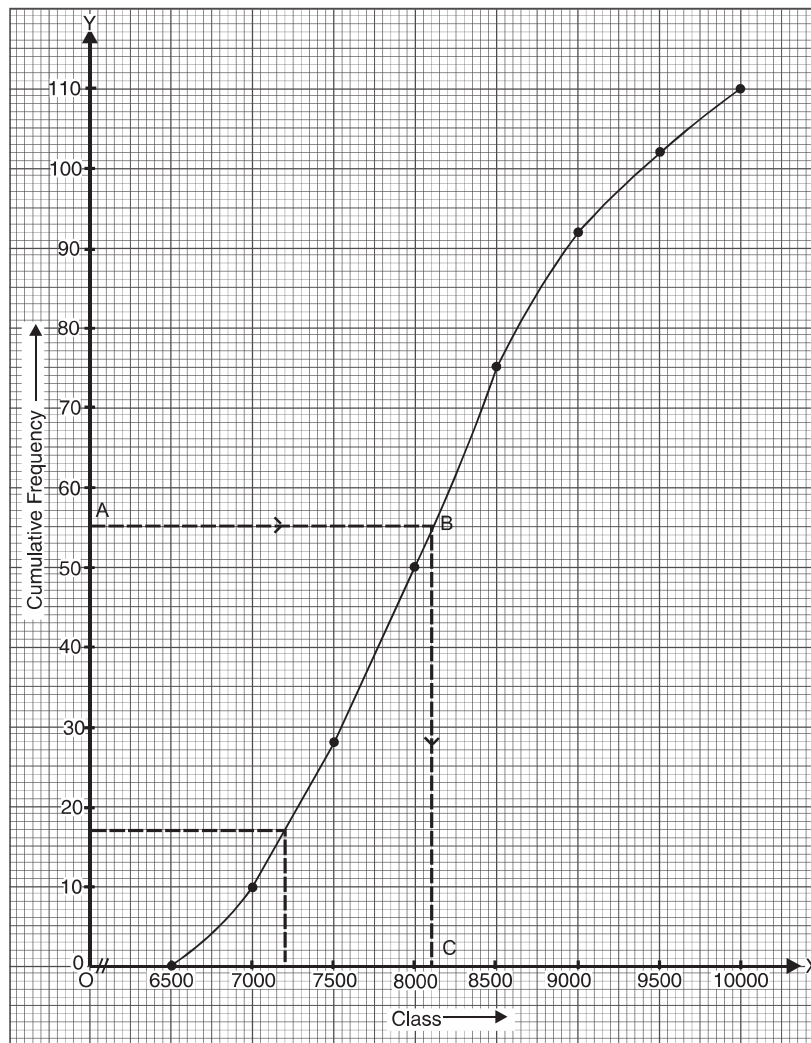
- (a) Describe the figure EFGH. (b) What kind of triangle is ECF?

Solution :

(i)

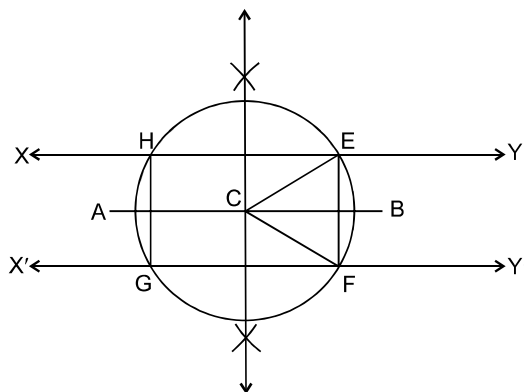
Class	Frequency	Cumulative frequency
6500-7000	10	10
7000-7500	18	28
7500-8000	22	50
8000-8500	25	75
8500-9000	17	92
9000-9500	10	102
9500-10000	8	110

Now, we plot the points (6500, 0), (7000, 10), (7500, 28), (8000, 50), (8500, 75), (9000, 92), (9500, 102) and (10000, 110). Joining these points by a free hand curve to get the ogive.



Here $N = 110 \Rightarrow \frac{N}{2} = 55$. Let A represent 55 on the y -axis. From A, draw AB parallel to x -axis, meeting the ogive at B. From B, draw $BC \perp x$ -axis, The abscissa of A is 8100. Hence, median of the data is 8100 **Ans.**

(ii)



- (a) Clearly EFGH is a rectangle.
- (b) In $\triangle CEF$, $CE = CF = 4$ cm and $EF = (2 + 2)$ cm = 4 cm
So, $\triangle CEF$ is an equilateral triangle.

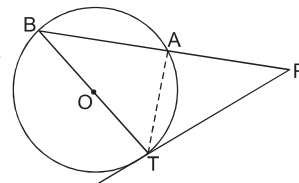
Question 8 :

- (i) A large firm employs 4250 employees. One person is chosen at random. What is the probability that the person's birthday is on Monday in the year 2016? [3]
- (ii) A cylindrical can whose base is horizontal and of radius 3.5 cm contains sufficient water so that when a sphere is placed in the can, the water just covers the sphere. Given that the sphere just fits into the can, calculate : [4]
- (a) the total surface area of the can in contact with water when the sphere is in it.
- (b) the depth of water in the can before the sphere was put into the can.

- (iii) In the given figure, PAB is a secant and PT a tangent to the circle with centre O. If $\angle ATP = 40^\circ$, $PA = 9$ cm and $AB = 7$ cm. [3]

Find:

- (a) $\angle APT$
- (b) length of PT



Solution :

- (i) There are 7 days in a week and there is only 1 Monday in a week.

$$\therefore \text{Required probability} = \frac{1}{7} \quad \text{Ans.}$$

- (ii) Radius of cylinder = Radius of sphere = 3.5 cm

Height of the cylinder $h = 2r = 7$ cm

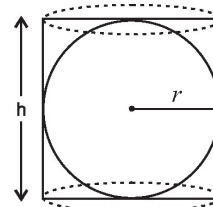
$$\begin{aligned} \text{(a) Total surface area of can in contact of water} &= 2\pi rh + \pi r^2 \\ &= \left(2 \times \frac{22}{7} \times 3.5 \times 7 + \frac{22}{7} \times 3.5 \times 3.5\right) \text{ cm}^2 \\ &= 192.5 \text{ cm}^2. \quad \text{Ans.} \end{aligned}$$

- (b) Let x be the depth of water.

$$\therefore \text{Volume of water} = \pi r^2 x$$

Volume of water + Volume of sphere = Volume of cylinder

$$\begin{aligned} \Rightarrow \pi r^2 x + \frac{4}{3} \pi r^3 &= \pi r^2 \cdot (2r) \\ \Rightarrow \frac{22}{7} \times (3.5)^2 \times x + \frac{4}{3} \times \frac{22}{7} \times (3.5)^3 &= \frac{22}{7} \times 2 \times (3.5)^3 \Rightarrow x = \frac{7}{3} \text{ cm.} \end{aligned}$$



$$\text{Hence, depth of water} = 2\frac{1}{3} \text{ cm.} \quad \text{Ans.}$$

- (iii) (a) $\angle ATP = \angle TBA$

[Angles in the alternate segments]

$$\Rightarrow \angle TBA = 40^\circ$$

$$\text{Also, } \angle BTP = 90^\circ$$

[Radius through the point of contact is perpendicular to the tangent]

$$\text{In } \triangle PTB, \angle TPB + \angle BTP + \angle TBP = 180^\circ \quad [\text{Angle sum property of a } \triangle]$$

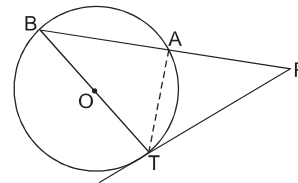
$$\Rightarrow \angle TPB + 90^\circ + 40^\circ = 180^\circ$$

$$\Rightarrow \angle TPB = 180^\circ - 130^\circ = 50^\circ$$

$$\text{Hence, } \angle APT = 50^\circ \quad \text{Ans.}$$

- (b) We know that $PT^2 = PA \times PB$

$$\Rightarrow PT^2 = 9 \times (9 + 7) \Rightarrow PT = 3 \times 4 = 12 \text{ cm} \quad \text{Ans.}$$



Question 9 :

- (i) Solve the following inequation and represent your solution on the real number line:

$$-5\frac{1}{2} - x \leq \frac{1}{2} - 3x \leq 3\frac{1}{2} - x, x \in \mathbf{R}$$

[3]

- (ii) Calculate the mean daily wage of a worker from the following table :

Daily wages (in ₹)	40–45	45–50	50–55	55–60	60–65
No. of workers	2	3	7	12	6

From the ogive find the median.

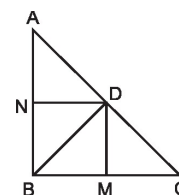
[3]

- (iii) In the figure, ABC is a right triangle with $\angle ABC = 90^\circ$,

$BD \perp AC$, $DM \perp BC$ and $DN \perp AB$. Prove that

(a) $DM^2 = DN \times MC$

(b) $DN^2 = DM \times AN$



[4]

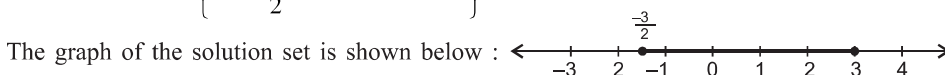
Solution :

(i) We have, $-5\frac{1}{2} - x \leq \frac{1}{2} - 3x \leq 3\frac{1}{2} - x$, $x \in \mathbb{R}$

$$\Rightarrow -5\frac{1}{2} - x \leq \frac{1}{2} - 3x \text{ and } \frac{1}{2} - 3x \leq 3\frac{1}{2} - x, x \in \mathbb{R} \Rightarrow -x + 3x \leq \frac{1}{2} + \frac{11}{2} \text{ and } \frac{1}{2} - \frac{7}{2} \leq -x + 3x, x \in \mathbb{R}$$

$$\Rightarrow 2x \leq 6 \text{ and } -3 \leq 2x, x \in \mathbb{R} \Rightarrow x \leq 3 \text{ and } -\frac{3}{2} \leq x, x \in \mathbb{R} \Rightarrow -\frac{3}{2} \leq x \leq 3, x \in \mathbb{R}$$

$$\therefore \text{Solution set} = \left\{ x : -\frac{3}{2} \leq x \leq 3, x \in \mathbb{R} \right\} \quad \text{Ans.}$$



Daily wages (in ₹)	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
40–45	2	42.5	85
45–50	3	47.5	142.5
50–55	7	52.5	367.5
55–60	12	57.5	690
60–65	6	62.5	375
	$\Sigma f_i = 30$		$\Sigma f_i x_i = 1660$

$$\text{Mean} = \bar{x} = \frac{1660}{30} = ₹ 55.33. \quad \text{Ans.}$$

- (iii) We know that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of perpendicular are similar to the whole triangle and to each other.

$$\therefore \triangle BMD \sim \triangle DMC \quad \dots \text{ (I)}$$

$$\text{and } \triangle BND \sim \triangle DNA \quad \dots \text{ (II)}$$

$$\text{From (I) we have, } \frac{DM}{BM} = \frac{MC}{DM}$$

$$\Rightarrow DM^2 = BM \times MC \quad \dots \text{ (III)}$$

$$\text{From (II) we have, } \frac{DN}{BN} = \frac{AN}{DN}$$

$$\Rightarrow DN^2 = BN \times AN \quad \dots \text{ (IV)}$$

In $\triangle BDM$ and $\triangle DBN$, we have

$$\angle BMD = \angle DNB \quad [\text{Each} = 90^\circ]$$

$$\angle MBD = \angle NDB \quad [\text{Alternate angles}]$$

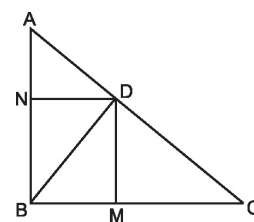
$$BD = DB \quad [\text{Common}]$$

$$\therefore \triangle BDM \cong \triangle DBN \quad [\text{ASA}]$$

$$\Rightarrow BM = DN \text{ and } DM = BN \quad [\text{CPCT}]$$

Substituting these values in (III) and (IV), we get

$$DM^2 = DN \times MC \text{ and } DN^2 = DM \times AN \quad \text{Proved}$$



Question 10 :

(i) Draw a regular hexagon of side 3.5 cm. Circumscribe a circle to it. [3]

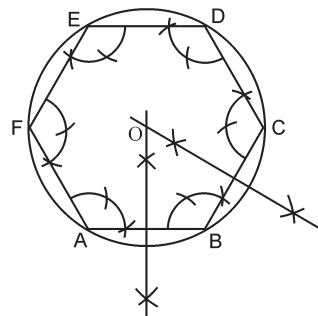
(ii) Solve for x , using the properties of proportion: $\frac{3x + \sqrt{9x^2 - 5}}{3x - \sqrt{9x^2 - 5}} = 5$ [3]

(iii) A boy standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60° . When he moves 20 m back from the bank, he finds the angle to be 30° . Find the height of the tree and the breadth of the river. [4]

Solution :

(i) Steps of Construction :

1. Draw $AB = 3.5$ cm.
2. At B, draw $\angle ABC = 120^\circ$ and $BC = 3.5$ cm.
3. At C, draw $\angle BCD = 120^\circ$ and $CD = 3.5$ cm.
4. At D, draw $\angle CDE = 120^\circ$ and $DE = 3.5$ cm.
5. At E, draw $\angle DEF = 120^\circ$ and $EF = 3.5$ cm.
6. Join AF to get the required hexagon.
7. Draw the perpendicular bisectors of AB and BC, which intersect at O.
8. With O as centre and radius equal to OA, draw the required circle.



(ii) We have, $= \frac{3x + \sqrt{9x^2 - 5}}{3x - \sqrt{9x^2 - 5}} = 5$

$$\Rightarrow \frac{3x + \sqrt{9x^2 - 5} + 3x - \sqrt{9x^2 - 5}}{3x + \sqrt{9x^2 - 5} - 3x + \sqrt{9x^2 - 5}} = \frac{5+1}{5-1} \quad [\text{Applying componendo and dividendo}]$$

$$\Rightarrow \frac{6x}{2\sqrt{9x^2 - 5}} = \frac{6}{4} \Rightarrow \sqrt{9x^2 - 5} \Rightarrow 2x \Rightarrow 9x^2 - 5 = 4x^2$$

$$\Rightarrow 5x^2 = 5 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

$\Rightarrow x = 1$ [Rejecting $x = -1$, as it does not satisfy the given equation] **Ans.**

(iii) Let $OA = h$ m be the tree, B and C be the positions of the boys such that $BC = 20$ m.

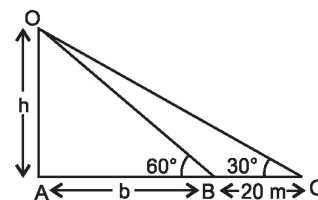
$$\text{In } \triangle OAB, \tan 60^\circ = \frac{h}{b} \Rightarrow \sqrt{3} = \frac{h}{b} \Rightarrow h = b\sqrt{3} \quad \dots(I)$$

$$\text{In } \triangle OAC, \tan 30^\circ = \frac{h}{b+20} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{b+20} \Rightarrow b+20 = h\sqrt{3} \quad \dots(II)$$

$$\Rightarrow b+20 = b\sqrt{3} \times \sqrt{3} \Rightarrow b = 10 \text{ m}$$

$$\text{Put } b = 10 \text{ in (II) we have, } h = 10\sqrt{3} = 10 \times 1.732 = 17.32 \text{ m}$$

Hence, height of the tree = 17.32 m and breadth of the river = 10 m. **Ans.**



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