

Quadratic Equations

Important Concepts

1. If $p(x)$ is a quadratic polynomial, then $p(x) = 0$ is called a **quadratic equation**.
The general form of a quadratic equation, in the variable x , is $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.
2. The value of x that satisfies an equation is called the **zeroes** or **roots** of the equation.
A real number α is said to be a solution/root of the quadratic equation $ax^2 + bx + c = 0$
if $a\alpha^2 + b\alpha + c = 0$.
3. A quadratic equation has **at most two zeros**.

Nature of the roots of a quadratic equation:

The nature of the roots of a quadratic equation depends upon the value of discriminant $b^2 - 4ac$.

- i. If $b^2 - 4ac > 0$, the roots are **real** and **unequal**
- ii. If $b^2 - 4ac = 0$, the roots are **real** and **equal**
- iii. If $b^2 - 4ac < 0$, the roots are **imaginary (not real)**

If $ax^2 + bx + c$, $a \neq 0$, can be reduced to the product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.

Solution of Quadratic Equation by Factoring:

Example,

Solve the equation $\frac{9}{2}x = 5 + x^2$ by factorization:

Step 1: Clear all fractions and brackets, if necessary

$$9x = 2(5 + x^2)$$

Step 2: Transpose all the terms to the left hand side to get an equation in the form $ax^2 + bx + c = 0$

$$9x = 2x^2 + 10$$

$$\Rightarrow 2x^2 - 9x + 10 = 0$$

Step 3: Factorise the expression on the left hand side.

$$2x^2 - 9x + 10 = 0$$

$$\Rightarrow 2x^2 - 5x - 4x + 10 = 0$$

$$\Rightarrow x(2x - 5) - 2(2x - 5) = 0$$

$$\Rightarrow (x - 2)(2x - 5) = 0$$

Step 4: Put each factor equal to zero and solve

$$(x - 2)(2x - 5) = 0$$

$$\Rightarrow x - 2 = 0 \quad 2x - 5 = 0$$

$$\Rightarrow x = 2; \quad 2x = 5$$

$$\Rightarrow x = 2; \quad x = \frac{5}{2}$$

$$\text{Thus, we have, } x = 2 \text{ or } x = \frac{5}{2}$$

Solution of Quadratic Equation by Quadratic Formula:

The roots of a quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) can be calculated by using **quadratic formula**:

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \text{ where } b^2 - 4ac \geq 0$$

Equations Reducible to Quadratic Form

There are many equations which are not in the quadratic form but can be reduced to quadratic form by simplifications.

Let us solve the equation $x^4 - 2x^2 - 3 = 0$

It is clear that the above equation is not a quadratic equation.

Now assume that $x^2 = y$

Then rewrite the given quadratic equation as, $(x^2)^2 - 2(x^2) - 3 = 0$

Substituting $x^2 = y$ in the above equation, we have $y^2 - 2y - 3 = 0$

This is a quadratic equation in y .

Let us solve the quadratic equation through factorization.

$$y^2 - 2y - 3 = 0$$

$$\Rightarrow y^2 - 3y + y - 3 = 0$$

$$\Rightarrow y(y - 3) + (y - 3) = 0$$

$$\Rightarrow (y + 1)(y - 3) = 0$$

$$\Rightarrow y + 1 = 0 \text{ or } y - 3 = 0$$

$$\Rightarrow y = -1 \text{ or } y = 3$$

Applications of quadratic equation in solving real life problems

Following points can be helpful in solving word problems:

- i. Every two digit number 'xy' where x is a ten's place and y is a unit's place can be expressed as $xy = 10x + y$

- ii. Downstream: It means that the boat is running in the direction of the stream

Upstream: It means that the boat is running in the opposite direction of the stream

Thus, if

Speed of boat in still water is x km/h

And the speed of stream is y km/h

Then the speed of boat downstream will be $(x + y)$ km/h and in upstream it will be $(x - y)$ km/h.

- iii. If a person takes x days to finish a work, then his one day's work = $\frac{1}{x}$