

Geometric Progression

Important Terms

1. A **sequence** is an arrangement of numbers in a definite order according to some rule.
2. The various numbers occurring in a sequence are called its **terms**. We denote the terms of a sequence by $a_1, a_2, a_3 \dots$ etc. Here, the subscripts denote the positions of the terms. In general, the number at the n th place is called the n th term of the sequence and is denoted by a_n . The n th term is also called the **general term** of the sequence.
3. A sequence having a finite number of terms is called a **finite sequence**.
4. A sequence which do not have a last term and which extends indefinitely is known as an **infinite sequence**.

Geometric Progression

A sequence is said to be in **geometric progression** or G.P., if the ratio of any term to its preceding term is same throughout. Constant Ratio is **common ratio** denoted by ' r '.

General Term of a G.P.

The general term of a G.P. is given by

$$t_n = ar^{n-1}$$

where ' a ' is the first term and ' r ' is the common ratio.

Properties of G.P.

1. The ratio between the consecutive terms of a G.P. is always the same.

$$\Rightarrow \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \dots\dots\dots$$

2. r^{th} term from the beginning $\times r^{\text{th}}$ term from the end = constant = First term $\times$ Last term

3. If a, b and c are in G.P.,

$$\Rightarrow \frac{b}{a} = \frac{a}{c} \Rightarrow b^2 = ac$$

4. In a G.P. if the terms at equal distances are taken, these terms are also in G.P.
5. If each term of a G.P. be multiplied or divided by the same non-zero number, the resulting series is also a G.P.
6. The series obtained by taking the reciprocals of the terms of a G.P. is also a G.P.
7. If each term of a G.P. is raised to the same non-zero number, the resulting series is also a G.P.
8. If the corresponding terms of two different G.P.s are multiplied/divided together, the resulting series, so obtained, is also a G.P.

Sum of n terms of a G.P.

$$S_n = a + ar + ar^2 + \dots + ar^{n-1}$$

Case I: $r = 1$

$$S_n = a + a + a + \dots + \text{to } n \text{ terms} = na$$

Case II: $r < 1$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

Case III: $r > 1$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

Sum of Infinite terms of a G.P.

$$\text{Sum of infinite terms in G.P.} = \frac{a}{1 - r}, \text{ if } |r| < 1$$

Geometric Mean Between Numbers a and b

If a and b are two positive numbers then a, G, b are in G.P.

$$\Rightarrow G^2 = ab$$

$$\Rightarrow G = \sqrt{ab}$$

Three or More Terms in G.P.

Number of terms is three: $\frac{a}{r}, a, ar$

Number of terms is four: $\frac{a}{r^3}, \frac{a}{r}, a, ar^3$

Number of terms is five: $\frac{a}{r^4}, \frac{a}{r^2}, a, ar, ar^2$

and so on.