

Exponents (Including Laws of Exponents)

EXERCISE 5 (A)

Question 1.

Find the value of:

(i) 6^2

(ii) 7^3

(iii) 4^4

(iv) 5^5

(v) 8^3

(vi) 7^5

Solution:

(i) $6^2 = 6 \times 6 = 36$

(ii) $7^3 = 7 \times 7 \times 7 = 343$

(iii) $4^4 = 4 \times 4 \times 4 \times 4 = 256$

(iv) $5^5 = 5 \times 5 \times 5 \times 5 \times 5 = 3125$

(v) $8^3 = 8 \times 8 \times 8 = 512$

(vi) $7^5 = 7 \times 7 \times 7 \times 7 \times 7 = 16807$

Question 2.

Evaluate:

(i) $2^3 \times 4^2$

(ii) $2^3 \times 5^2$

(iii) $3^3 \times 5^2$

(iv) $2^2 \times 3^3$

(v) $3^2 \times 5^2$

(vi) $5^3 \times 2^4$

(vii) $3^2 \times 4^2$

(ix) $(5 \times 4)^2$

Solution:

(i) $2^3 \times 4^2$
 $= 2 \times 2 \times 2 \times 4 \times 4$
 $= 8 \times 16$
 $= 128$

(ii) $2^3 \times 5^2$
 $= 2 \times 2 \times 2 \times 5 \times 5$
 $= 8 \times 25$
 $= 200$

(iii) $3^3 \times 5^2$
 $= 3 \times 3 \times 3 \times 5 \times 5$
 $= 27 \times 25$

$$= 675$$

$$\text{(iv)} 2^2 \times 3^3$$

$$= 2 \times 2 \times 3 \times 3 \times 3$$

$$= 4 \times 27$$

$$= 108$$

$$\text{(v)} 3^2 \times 5^3$$

$$= 3 \times 3 \times 5 \times 5 \times 5$$

$$= 9 \times 125$$

$$= 1125$$

$$\text{(vi)} 5^3 \times 2^4$$

$$= 5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 2$$

$$= 125 \times 16$$

$$= 2000$$

$$\text{(vii)} 3^2 \times 4^2$$

$$= 3 \times 3 \times 4 \times 4$$

$$= 9 \times 16$$

$$= 144$$

$$\text{(viii)} (4 \times 3)^3$$

$$= 4 \times 4 \times 4 \times 3 \times 3 \times 3$$

$$= 64 \times 27$$

$$= 1728$$

$$\text{(ix)} (5 \times 4)^2$$

$$= 5 \times 5 \times 4 \times 4$$

$$= 25 \times 16$$

$$= 400$$

Question 3.

Evaluate:

$$(i) \left(\frac{3}{4}\right)^4$$

$$(ii) \left(-\frac{5}{6}\right)^5$$

$$(iii) \left(\frac{-3}{-5}\right)^3$$

Solution:

$$(i) \left(\frac{3}{4}\right)^4 = \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right)$$
$$= \frac{3 \times 3 \times 3 \times 3}{4 \times 4 \times 4 \times 4} = \frac{81}{256}$$

$$(ii) \left(-\frac{5}{6}\right)^5$$
$$= \left(-\frac{5}{6}\right) \times \left(-\frac{5}{6}\right) \times \left(-\frac{5}{6}\right) \times \left(-\frac{5}{6}\right) \times \left(-\frac{5}{6}\right)$$
$$= \frac{(-5) \times (-5) \times (-5) \times (-5) \times (-5)}{6 \times 6 \times 6 \times 6 \times 6}$$
$$= -\frac{3125}{776}$$

$$(iii) \left(\frac{-3}{-5}\right)^3 = \left(\frac{-3}{-5}\right) \times \left(\frac{-3}{-5}\right) \times \left(\frac{-3}{-5}\right)$$
$$= \frac{(-3) \times (-3) \times (-3)}{(-5) \times (-5) \times (-5)}$$
$$= \frac{27}{125}$$

Question 4.

Evaluate :

$$(i) \left(\frac{2}{3}\right)^3 \times \left(\frac{3}{4}\right)^2 \quad (ii) \left(-\frac{3}{4}\right)^3 \times \left(\frac{2}{3}\right)^4$$

$$(iii) \left(\frac{3}{5}\right)^2 \times \left(-\frac{2}{3}\right)^3$$

Solution:

$$(i) \left(\frac{2}{3}\right)^3 \times \left(\frac{3}{4}\right)^2$$

$$= \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right)$$

$$= \frac{8}{27} \times \frac{9}{16} = \frac{1}{6}$$

$$(ii) \left(-\frac{3}{4}\right)^3 \times \left(\frac{2}{3}\right)^4$$

$$= \left(-\frac{3}{4}\right) \times \left(-\frac{3}{4}\right) \times \left(-\frac{3}{4}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right)$$

$$\times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right)$$

$$= \frac{-27}{64} \times \frac{16}{81} = -\frac{1}{2}$$

$$(iii) \left(\frac{3}{5}\right)^2 \times \left(-\frac{2}{3}\right)^3$$

$$= \left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right)$$

$$= \frac{9}{25} \times \left(-\frac{8}{27}\right)$$

$$= -\frac{8}{75}$$

Question 5.

Which is greater :

- (i) 2^3 or 3^2
- (ii) 2^5 or 5^2
- (iii) 4^3 or 3^4
- (iv) 5^4 or 4^5

Solution:

- (i) 2^3 or 3^2

Since, $2^3 = 2 \times 2 \times 2 = 8$

and, $3^2 = 3 \times 3 = 9$

$\therefore 9$ is greater than $8 \Rightarrow 3^2 > 2^3$

- (ii) 2^5 or 5^2

Since, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

and, $5^2 = 5 \times 5 = 25$

$\therefore 32$ is greater than $25 \Rightarrow 2^5 > 5^2$

- (iii) 4^3 or 3^4

Since, $4^3 = 4 \times 4 \times 4 = 64$

and, $3^4 = 3 \times 3 \times 3 \times 3 = 81$

$\therefore 81$ is greater than $64 \Rightarrow 3^4 > 4^3$

- (iv) 5^4 or 4^5

Since, $5^4 = 5 \times 5 \times 5 \times 5 = 625$

and, $4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024$

$\therefore 1024$ is greater than $625 \Rightarrow 4^5 > 5^4$

Question 6.

Express each of the following in exponential form :

- (i) 512
- (ii) 1250
- (iii) 1458
- (iv) 3600
- (v) 1350
- (vi) 1176

Solution:

(i) 512

$$\begin{array}{r|l} 2 & 512 \\ \hline 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^9$$

(ii) 1250

$$\begin{array}{r|l} 2 & 1250 \\ \hline 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$= 2 \times 5 \times 5 \times 5 \times 5 = 2 \times 5^4$$

(iii) 1458

$$\begin{array}{r|l} 2 & 1458 \\ \hline 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$= 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2 \times 3^6$$

(iv) 3600

$$\begin{array}{r|l} 2 & 3600 \\ 2 & 1800 \\ 2 & 900 \\ 2 & 450 \\ 3 & 225 \\ 3 & 75 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$$\begin{aligned} &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \\ &= 2^4 \times 3^2 \times 5^2 \end{aligned}$$

(v) 1350

$$\begin{array}{r|l} 2 & 1350 \\ 3 & 675 \\ 3 & 225 \\ 3 & 75 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$$\begin{aligned} &= 2 \times 3 \times 3 \times 3 \times 5 \times 5 \\ &= 2 \times 3^3 \times 5^2 \end{aligned}$$

(vi) 1176

$$\begin{array}{r|l} 2 & 1176 \\ 2 & 588 \\ 2 & 294 \\ 3 & 147 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array}$$

$$\begin{aligned} &= 2 \times 2 \times 2 \times 3 \times 7 \times 7 \\ &= 2^3 \times 3 \times 7^2 \end{aligned}$$

Question 7.

If $a = 2$ and $b = 3$, find the value of:

(i) $(a + b)^2$

(ii) $(b - a)^3$

(iii) $(a \times b)^a$ (iv) $(a \times b)^b$

Solution:

(i) $(a + b)^2$

$$= (2 + 3)^2 = (5)^2 = 5 \times 5 = 25$$

(ii) $(b - a)^2$

$$= (3 - 2)^2 = (1)^2$$

$$= 1 \times 1 = 1$$

(iii) $(a \times b)^a$

$$= (2 \times 3)^2 = (6)^2$$

$$= 6 \times 6 = 36$$

(iv) $(a \times b)^b$

$$= (2 \times 3)^3 = (6)^3 = 6 \times 6 \times 6 = 216$$

Question 8.

Express:

(i) 1024 as a power of 2.

(ii) 343 as a power of 7.

(iii) 729 as a power of 3.

Solution:

(i) 1024 as a power of 2.

$$\begin{array}{r|l}
 2 & 1024 \\
 \hline
 2 & 512 \\
 \hline
 2 & 256 \\
 \hline
 2 & 128 \\
 \hline
 2 & 64 \\
 \hline
 2 & 32 \\
 \hline
 2 & 16 \\
 \hline
 2 & 8 \\
 \hline
 2 & 4 \\
 \hline
 2 & 2 \\
 \hline
 & 1
 \end{array}$$

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= 2^{10}$$

(ii) 343 as a power of 7.

$$= 7 \times 7 \times 7 = 7^3$$

$$\begin{array}{r|l} 7 & 343 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

(iii) 729 as a power of 3.

$$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$$

$$\begin{array}{r|l} 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Question 9.

If $27 \times 32 = 3^x \times 2^y$; find the values of x and y.

$$27 \times 32 = 3^x \times 2^y$$

$$27 = 3^x$$

$$\begin{array}{r|l} 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Solution:

$$\begin{aligned} 27 &= 3 \times 3 \times 3 \\ &= 3^3 = 3^x \end{aligned}$$

$$\therefore x = 3$$

$$\text{Also, } 32 = 2^y$$

$$\begin{array}{r|l} 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$$\begin{aligned} 32 &= 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^5 = 2^y \end{aligned}$$

$$\therefore y = 5$$

Question 10.

If $64 \times 625 = 2^a \times 5^b$; find :

(i) the values of a and b .

(ii) $2^b \times 5^a$

Solution:

(i) the values of a and b .

$$(i) 64 \times 625 = 2^a \times 5^b$$

$$64 = 2^a$$

$$\begin{array}{r|l} 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$64 = 2^6$$

$$\therefore a = 6$$

$$\text{Also, } 625 = 5^b$$

$$\begin{array}{r|l} 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$625 = 5 \times 5 \times 5 \times 5$$

$$625 = 5^4$$

$$\therefore b = 4$$

(ii) $2^b \times 5^a$

(ii) $2^b \times 5^a$

$$= 2^4 \times 5^6$$

$$= 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$$

$$= 16 \times 15625 = 250000$$

EXERCISE 5 (B)

Question 1.

Fill in the blanks:

In $5^2 = 25$, base = and index =

If index = $3x$ and base = $2y$, the number =

Solution:

(i) In $5^2 = 25$, base = 5 and index = 2

(ii) If index = $3x$ and base = $2y$, the number = $2y^{3x}$

Question 2.

Evaluate:

(i) $2^8 \div 2^3$

(ii) $2^{3+} 2^8$

(iii) $(2^6)^0$

(iv) $(3^0)^6$

(v) $8^3 \times 8^{-5} \times 8^4$

(vi) $5^4 \times 5^3 + 5^5$

(vii) $5^4 \div 5^3 \times 5^5$

(viii) $4^4 \div 4^3 \times 4^0$

(ix) $(3^5 \times 4^7 \times 5^8)^0$

Solution:

$$= \frac{4^4}{4^3} = 4^{4-3} = 4^1 = 4$$

$$\begin{aligned} \text{(ix)} \quad (3^5 \times 4^7 \times 5^8)^0 &= 3^{5 \times 0} \times 4^{7 \times 0} \times 5^{8 \times 0} \\ &= 3^0 4^0 5^0 = 1 \times 1 \times 1 = 1 \end{aligned}$$

$$(i) 2^8 \div 2^3 = \frac{2^8}{2^3} = 2^{8-3} = 2^5$$

$$(ii) 2^3 \div 2^8 = \frac{2^3}{2^8} = 2^{3-8} = 2^{-5} = \frac{1}{2^5}$$

$$(iii) (2^6)^0 = 2^{6 \times 0} = 2^0 = 1$$

$$(iv) (3^0)^6 = 3^{0 \times 6} = 3^0 = 1$$

$$(v) 8^3 \times 8^{-5} \times 8^4 = 8^{3+4-5} = 8^{7-5} = 8^2$$

$$(vi) 5^4 \times 5^3 \div 5^5 = \frac{5^4 \times 5^3}{5^5} \\ = 5^{4+3-5} = 5^{7-5} = 5^2$$

$$(vii) 5^4 \div 5^3 \times 5^5 = \frac{5^4}{5^3} \times 5^5 = 5^{4-3+5} = 5^6$$

$$(viii) 4^4 \div 4^3 \times 4^0 = \frac{4^4}{4^3 4^0} = \frac{4^4}{4^3 \times 1}$$

Question 3.

Simplify, giving Solutions with positive index:

$$(i) 2b^6 \cdot b^3 \cdot 5b^4 \quad (ii) x^2y^3 \cdot 6x^5y \cdot 9x^3y^4$$

$$(iii) (-a^5)(a^2) \quad (iv) (-y^2)(-y^3)$$

$$(v) (-3)^2(3)^3 \quad (vi) (-4x)(-5x^2)$$

$$(vii) (5a^2b)(2ab^2)(a^3b)$$

$$(viii) x^{2a+7} \cdot x^{2a-8} \quad (ix) 3^y \cdot 3^2 \cdot 3^{-4}$$

$$(x) 2^{4a} \cdot 3^{3a} \cdot 2^{-a} \quad (xi) 4x^2y^2 \div 9x^3y^3$$

$$(xii) (10^2)^3 (x^8)^{12} \quad (xiii) (a^{10})^{10} (16)^{10}$$

$$(xiv) (n^2)^2 (-n^2)^2 \quad (xv) -(3ab)^2 (-5a^2bc^4)^2$$

$$(xvi) (-2)^2 \times (0)^3 \times (3)^3$$

$$(xvii) (2a^3)^4 (4a^2)^2$$

$$(xviii) (4x^2y^3)^3 \div (3x^2y^3)^3$$

$$(xix) \left(\frac{1}{2x}\right)^3 \times (6x)^2$$

$$(xx) \left(\frac{1}{4ab^2c}\right)^2 \div \left(\frac{3}{2a^2bc^2}\right)^4$$

$$(xxi) \frac{(5x^7)^3 \cdot (10x^2)^2}{(2x^6)^7}$$

$$(xxii) \frac{(7p^2q^9r^5)^2 (4pqr)^3}{(14p^6q^{10}r^4)^2}$$

Solution:

$$(i) 2b^6 \cdot b^3 \cdot 5b^4 \\ = 2 \times 5 \times b^{6+3+4} = 10b^{13}$$

$$(ii) x^2y^3 \cdot 6x^5y \cdot 9x^3y^4 \\ = 6 \times 9 \times x^{2+5+3} y^{3+1+4} = 54x^{10}y^8$$

$$(iii) (-a^5) (a^2) \\ = (-1 \times a)^5 \times a^2 \\ = (-1)^5 \times a^{5+2} \\ = -1 \times a^7 = -a^7$$

$$(iv) (-y^2) (-y^3) \\ = (-1 \times y)^2 \cdot (-1 \times y)^3$$

$$\begin{aligned}
 &= (-1)^2 \cdot y^2 \cdot (-1)^3 \times y^3 \\
 &= 1^{2+3} \cdot y^{2+3} \\
 &= 1^5 y^5 = y^5
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad &(-3)^2 (3)^3 \\
 &= (-1 \times 3)^2 \cdot (3)^3 \\
 &= (-1)^2 \times 3^2 \cdot 3^3 \\
 &= -1^2 \cdot 3^{2+3} = 1 \cdot 3^5 = 3^5
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad &(-4x) (-5x^2) \\
 &= (-1 \times 4 \times x) \cdot (-1 \times 5 \times x^2)^1 \\
 &= -1 \times 4 \times x \cdot -1 \times 5 \times x^2 \\
 &= -1 \times -1 \times 4 \times 5 \times x^{1+2} \\
 &= -1^{1+1} \cdot 4^1 \cdot 5^1 x^3 = 20x^3
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii)} \quad &(5a^2b) (2ab^2) (a^3b) \\
 &= 5 \cdot 2 \cdot a^{2+1+3} b^{1+2+1} = 10a^6b^4
 \end{aligned}$$

$$\begin{aligned}
 \text{(viii)} \quad &x^{2a+7} \cdot x^{2a-8} \\
 &= x^{2a+7+2a-8} = x^{4a-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ix)} \quad &3^y \cdot 3^2 \cdot 3^{-4} \\
 &= 3^y \cdot \frac{3^2}{3^4} = 3^y = \frac{3 \times 3}{3 \times 3 \times 3 \times 3} \\
 &= 3^y \times \frac{1}{3^2} = 3^{y-2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(x)} \quad &2^{4a} \cdot 2^{3a} \cdot 2^{-a} \\
 &= 2^{4a+3a-a} = 2^{7a-a} = 2^{6a}
 \end{aligned}$$

$$\begin{aligned}
 \text{(xi)} \quad &4x^2y^2 \div 9x^3y^3 \\
 &= \frac{4x^2y^2}{9x^3y^3} = \frac{4x^{2-3}y^{2-3}}{9} = \frac{4x^{-1}y^{-1}}{9} \\
 &= \frac{4}{9xy} \quad (\text{Since index should be positive})
 \end{aligned}$$

$$(xii) (10^2)^3 (x^8)^{12}$$

$$= 10^{2 \times 3} x^{8 \times 12} = 10^6 x^{96}$$

$$(xiii) (a^{10})^{10} (1^6)^{10}$$

$$= a^{10 \times 10} 1^{6 \times 10} = a^{100} 1^{60} = a^{100}$$

$$(xiv) (n^2)^2 (-n^2)^2$$

$$= n^{2 \times 2} (-n)^{2 \times 2} = n^4 \times (-n)^4$$

$$= -n^4 - 1^4 n^4$$

$$= -n^{4+4} = -n^8$$

$$(xv) -(3ab)^2 (-5a^2bc^4)^2$$

$$= -(3^2a^2b^2) \times (-1)^2 \times 5^2a^2 \times 2b^2c^4 \times 2$$

$$= -(3^2a^2b^2) (5^2a^4b^2c^8)$$

$$= -3^25^2a^{2+4}b^{2+2}c^{4+8}$$

$$= -225a^6b^4c^{12}$$

$$(xvi) (-2)^2 \times (0)^3 \times (3)^3$$

$$= 4 \times 0 \times 27 = 0$$

$$(xvii) (2a^3)^4 (4a^2)^2$$

$$= (2a^3)^4 (2^2a^2)^2$$

$$= 2^4a^{3 \times 4} \cdot 2^2 \times 2^2 a^{2 \times 2}$$

$$= 2^4a^{12} \cdot 2^4a^4$$

$$= 2^{4+4}a^{12+4}$$

$$= 2^8a^{16}$$

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times a^{16}$$

$$= 256a^{16}$$

$$(xviii) (4x^2y^3)^3 \div (3x^2y^3)^3$$

$$= \frac{4^3 x^{2 \times 3} y^{3 \times 3}}{3^3 x^{2 \times 3} y^{3 \times 3}} = \frac{4^3 x^6 y^9}{3^3 x^6 y^9} = \frac{4^3}{3^3} = \frac{64}{27}$$

$$(xix) \left(\frac{1}{2x}\right)^3 \times (6x)^2$$

$$= \frac{1^3}{2^3 \times x^3} \times 6^2 \times x^2$$

$$= \frac{1^3 \times 6^2}{2^3 \times x^{3-2}} = \frac{6^2}{2^3 x} = \frac{6 \times 6}{2 \times 2 \times 2 \times x} = \frac{9}{2x}$$

$$\begin{aligned}
 \text{(xx)} \quad & \left(\frac{1}{4ab^2c} \right)^2 \div \left(\frac{3}{2a^2bc^2} \right)^4 \\
 &= \left(\frac{1}{4ab^2c} \right)^2 \times \left(\frac{2a^2bc^2}{3} \right)^4 \\
 &= \frac{1^2}{4^2 a^2 b^{2 \times 2} c^2} \times \frac{2^4 a^{2 \times 4} b^4 c^{2 \times 4}}{3^4} \\
 &= \frac{1^2}{3^4} \times a^{8-2} b^{4-4} c^{8-2} \quad (\because 2^4 = 4^2) \\
 &= \frac{1}{3 \times 3 \times 3 \times 3} a^6 b^0 c^6 \\
 &= \frac{1}{81} a^6 c^6 \quad (\because b^0 = 1)
 \end{aligned}$$

$$\begin{aligned}
 \text{(xxi)} \quad & \frac{(5x^7)^3 \cdot (10x^2)^2}{(2x^6)^7} = \frac{5^3 x^{7 \times 3} \cdot 10^2 \cdot x^{2 \times 2}}{2^7 \cdot x^{6 \times 7}} \\
 &= 5^3 \cdot 10^2 \cdot 2^{-7} x^{21+4-42} \\
 &= \frac{5^3 \times 10^2}{2^7 x^{17}} = \frac{5 \times 5 \times 5 \times 2 \times 5 \times 2 \times 5}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \cdot x^{17}} \\
 &= \frac{5^5}{2^5 x^{17}} = \frac{3125}{32x^{17}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(xxii)} \quad & \frac{(7p^2q^9r^5)^2 (4pqr)^3}{(14p^6q^{10}r^4)^2} \\
 &= \frac{(7^2 p^{2 \times 2} q^{9 \times 2} r^{5 \times 2}) (4^3 p^3 q^3 r^3)}{14^2 p^{6 \times 2} q^{10 \times 2} r^{4 \times 2}} \\
 &= \frac{7 \times 7 p^4 q^{18} r^{10} \cdot 4 \times 4 \times 4 p^3 q^3 r^3}{2 \times 7 \times 2 \times 7 \times p^{12} q^{20} r^8} \\
 &= p^{4-12+3} q^{18-20+3} r^{10-8+3} \cdot 4 \times 4 \\
 &= 16p^{-5}qr^5 \\
 &= \frac{16qr^5}{p^5}
 \end{aligned}$$

Question 4.

Simplify and express the Solution in the positive exponent form :

$$(i) \frac{(-3)^3 \times 2^6}{6 \times 2^3} \quad (ii) \frac{(2^3)^5 \times 5^4}{4^3 \times 5^2}$$

$$(iii) \frac{36 \times (-6)^2 \times 3^6}{12^3 \times 3^5} \quad (iv) -\frac{128}{2187}$$

$$(v) \frac{a^{-7} \times b^{-7} \times c^5 \times d^4}{a^3 \times b^{-5} \times c^{-3} \times d^8}$$

$$(vi) (a^3 b^{-5})^{-2}$$

Solution:

$$(i) \frac{(-3)^3 \times 2^6}{6 \times 2^3} = \frac{(-3)^3 \times 2^6}{2 \times 3 \times 2^3} = \frac{(-3)^3 \times 2^6}{3 \times 2^{3+1}}$$

$$= -(3)^{3-1} 2^{6-4} = -(3)^2 2^2 = -3^2 2^2$$

$$(ii) \frac{(2^3)^5 \times 5^4}{4^3 \times 5^2} = \frac{2^{3 \times 5} \times 5^4}{2^3 \times 2^2 \times 5^2}$$

$$= \frac{2^{15} \times 5^4}{2^6 \times 5^2} = 2^{15-6} \times 5^{4-2}$$

$$= 2^9 \times 5^2$$

$$(iii) \frac{36 \times (-6)^2 \times 3^6}{12^3 \times 3^5} = \frac{6 \times 6 \times (-6)^2 \times 3^6}{3^3 \times 4^3 \times 3^5}$$

$$= \frac{(6)^2 (-6)^2 \times 3^{6-3-5}}{4^3} = \frac{(6)^2 (-6)^2 3^{-2}}{4^3}$$

$$= \frac{6^2 (-6)^2}{3^2 \times 4^3} = \frac{6 \times 6 \times -6 \times -6}{3 \times 3 \times 4 \times 4 \times 4}$$

$$= \frac{9}{4} = \left(\frac{3}{2}\right)^2$$

$$(iv) -\frac{128}{2187}$$

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

$$= -\frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3} = -\frac{2^7}{3^7}$$

$$\begin{aligned} \text{(v)} \quad & \frac{a^{-7} \times b^{-7} \times c^5 \times d^4}{a^3 \times b^{-5} \times c^{-3} \times d^8} \\ &= a^{-7-3} \times b^{-7+5} \times c^{5-(-3)} \times d^{4-8} \\ &= a^{-10} \times b^{-2} \times c^8 \times d^{-4} \\ &= \frac{c^8}{a^{10} \times b^2 \times d^4} \end{aligned}$$

$$\begin{aligned} \text{(vi)} \quad & (a^3 b^{-5})^{-2} = a^{3 \times -2} b^{-5 \times -2} \\ &= a^{-6} b^{10} = \frac{b^{10}}{a^6} \end{aligned}$$

Question 5.

Evaluate

$$\text{(i)} \quad 6^{-2} \div (4^{-2} \times 3^{-2})$$

$$\text{(ii)} \quad \left[\left(\frac{5}{6} \right)^2 \times \frac{9}{4} \right] \div \left[\left(-\frac{3}{2} \right) \times \frac{125}{216} \right]$$

$$\text{(iii)} \quad 5^3 \times 3^2 + (17)^0 \times 7^3$$

$$\text{(iv)} \quad 2^5 \times 15^0 + (-3)^3 - \left(\frac{2}{7} \right)^{-2}$$

$$\text{(v)} \quad (2^2)^0 + 2^{-4} \div 2^{-6} + \left(\frac{1}{2} \right)^{-3}$$

$$\text{(vi)} \quad 5^n \times 25^{n-1} \div (5^{n-1} \times 25^{n-1})$$

Solution:

$$(i) 6^{-2} \div (4^{-2} \times 3^{-2})$$

$$= \left(\frac{1}{6}\right)^2 \div \left(\frac{1}{4}\right)^2 \times \left(\frac{1}{3}\right)^2$$

$$= \frac{1}{36} \div \frac{1}{16} \times \frac{1}{9}$$

$$= \frac{1}{36} \div \frac{1}{144}$$
$$\begin{array}{r} 36 \overline{)144} 4 \\ \underline{144} \\ 0 \end{array}$$

$$= \frac{1}{36} \times \frac{144}{1} = 4$$

$$(ii) \left[\left(\frac{5}{6}\right)^2 \times \frac{9}{4} \right] \div \left[\left(-\frac{3^2}{2}\right) \times \frac{125}{216} \right]$$

$$= \left[\left(\frac{5 \times 5}{6 \times 6}\right) \times \frac{9}{4} \right] \div \left[\left(\frac{-3 \times -3}{2 \times 2}\right) \times \frac{125}{216} \right]$$

$$= \left[\frac{25}{36} \times \frac{9}{4} \right] \div \left[\frac{9}{4} \times \frac{125}{216} \right]$$

$$= \left[\frac{25}{4} \times \frac{1}{4} \right] \div \left[\frac{1}{4} \times \frac{125}{24} \right]$$

$$= \left[\frac{25}{16} \right] \div \left[\frac{125}{96} \right]$$

$$= \frac{25}{16} \times \frac{96}{125}$$

$$= \frac{1}{1} \times \frac{6}{5} = 1\frac{1}{5}$$

$$(iii) 5^3 \times 3^2 + (17)^0 \times 7^3$$

$$= 5 \times 5 \times 5 \times 3 \times 3 + (17)^0 \times 7 \times 7 \times 7$$

$$(\because a^0 = 1)$$

$$= 125 \times 9 + 1 \times 343$$

$$= 1125 + 343 = 1468$$

$$(iv) 2^5 \times 15^0 + (-3)^3 - \left(\frac{2}{7}\right)^{-2}$$

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 1 + (-3) \times (-3) \times$$

$$(-3) - \left(\frac{7}{2}\right) \times \left(\frac{7}{2}\right)$$

$$= 32 \times 1 - 27 - \frac{49}{4} \quad (\because a^0 = 1)$$

$$= \frac{32 \times 4}{1 \times 4} - \frac{27 \times 4}{1 \times 4} - \frac{49}{4 \times 1} \quad (\because \text{LCM} = 4)$$

$$= \frac{128 - 108 - 49}{4} = \frac{-29}{4} = -7\frac{1}{4}$$

$$(v) (2^2)^0 + 2^{-4} \div 2^{-6} + \left(\frac{1}{2}\right)^{-3}$$

$$(4)^0 + \left(\frac{1}{2}\right)^4 \div \left(\frac{1}{2}\right)^6 + \left(\frac{2}{1}\right)^3 \quad (\because a^0 = 1)$$

$$1 + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \div$$

$$\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{2}{1} \times \frac{2}{1} \times \frac{2}{1}\right)$$

$$1 + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 2 \times 2 \times 2 \times 2 \times 2 \times 2\right) + 8$$

$$= 1 + 4 + 8 = 13$$

$$(vi) 5^n \times 25^{n-1} \div (5^{n-1} \times 25^{n-1})$$

$$= 5^n \times 25^{n-1} \times \frac{1}{5^{n-1} \times 25^{n-1}}$$

$$= 5^n \times \frac{1}{5^{n-1}} = 5^{n-n+1} = 5^1$$

Question 6.

If $m^2 = -2$ and $n = 2$; find the values of:

(i) $m + n^2 - 2mn$

(ii) $m^n + n^m$

(iii) $6m^{-3} + 4n^2$

(iv) $2n^3 - 3m$

Solution:

(i) $m^2 + n^2 - 2mn$

$m = -2, n = 2$

$= (-2)^2 + (2)^2 - 2(-2)(2)$

$= 4 + 4 - (-8)$

$$\begin{array}{r|l} 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$= 8 + 8 = 16 = (2^4)$

(ii) $m^n + n^m$

$m = -2, n = 2$

$= (-2)^2 + (2)^{-2}$

$= 4 + \frac{1}{2} \times \frac{1}{2}$

$= \frac{4 \times 4}{1 \times 4} + \frac{1}{4}$

$= \frac{16+1}{4} = \frac{17}{4} = 4\frac{1}{4}$

(iii) $6m^{-3} + 4n^2$

$m = -2, n = 2$

$= 6(-2)^{-3} + 4(2)^2$

$= 6 \times \frac{1}{-2} \times \frac{1}{-2} \times \frac{1}{-2} + 4 \times 2 \times 2$

$= \frac{-3}{4} + 16$

$= \frac{-3+16 \times 4}{4} = \frac{-3+64}{4} = \frac{61}{4} = 15\frac{1}{4}$

(iv) $2n^3 - 3m$

$m = -2, n = 2$

$= 2(2)^3 - 3(-2)$

$= 2 \times (2 \times 2 \times 2) - 3 \times (-2)$

$= 16 - 3 \times (-2)$

$= 16 + 6 = 22$