

Chapter 11

Perimeter and Area

Introduction to Perimeter and Area

Suppose you want to fence your garden. How would you decide how much fence is required?



In such a case, we need to calculate the perimeter of the garden. The word perimeter in which the word 'peri' means 'around' and the word 'meter' means 'measure'. Therefore, the perimeter is defined as

“The distance covered along the boundary forming a closed figure when we go around the figure once”.

Example:

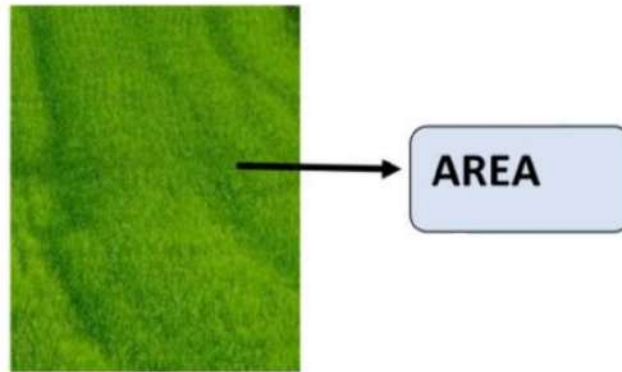


In the above figure, we see a circular ground. When we go around the ground once, we get the perimeter of the ground.

In the case of a circular figure, we do not call it perimeter we call it a circumference of the ground.

Area: The amount of plane or region or surface enclosed by the closed figure is called the area.

In the following figure, the green portion represents the area of the figure.



Now, suppose you want to fit tiles in your bedroom, how would you decide how many tiles will fit in the entire bedroom?

In such a case, we need to calculate the area of the hall.

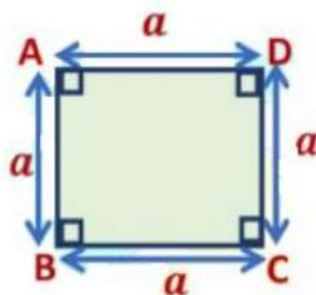
So, in this chapter, we will learn about how to find the area and perimeter of plane figures.

Squares and Rectangles

Square

A square is a regular quadrilateral;

- It has four equal sides and four equal angles and each interior angle is a right angle (90°).



In the above figure, ABCD is a square, in which the length of each side is "a" and each interior angle is 90° .

In this case perimeter of the square is four times its side.

The perimeter of a square = $4 \times$ length of the side of the square

As we know the length of each side of the square is "a".

The perimeter of a square = $4a$

In this case area of the square = Side $\times$ Side

As we know the length of each side of the square is "a".

Area of a square = $(\text{Side})^2$

Example: Find the perimeter and area of a square whose side is 5 cm

We know that,

The perimeter of a square = $4 \times$ length of its sides

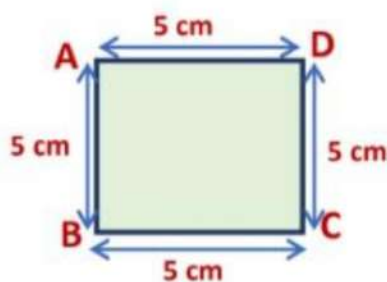
The perimeter of the square = 4×5

The perimeter of the square = 20 cm

Area of a square = $(\text{Side})^2$

Area of a square = $(5)^2$

Area of a square = 25 cm^2



Example: Find the area of a square whose side is 4 cm.

We know that,

Area of a square = Side $\times$ Side

Area of a square = $4 \text{ cm} \times 4 \text{ cm}$

$\therefore$ Square is a closed figure in which all the sides are equal

Area of a square = 16 cm^2

Example: Find the area of a square park whose perimeter is 400 m.

Suppose,

The length of each side of a park = "a"

Perimeter = 400 m

We know that,

The perimeter of a square = $4 \times$ length of its sides

$400 \text{ m} = 4 \times a$

$$4a = 400$$

$$a = \frac{400}{4}$$

$$a = 100 \text{ m}$$

$$\text{Area of a square} = (\text{side})^2 = a^2$$

$$\therefore \text{Area of a square} = (100 \times 100) \text{ m}^2$$

$$\text{Area of a square} = 10000 \text{ m}^2$$

Example: A window of length 3 m and breadth 1 m is on the wall of dimension 10 m $\times$ 10 m. Find the cost of painting the wall if the rate of painting is ₹3 per sq. m.

We have,

Length of the window = 3 m

The breadth of the window = 1 m

Side of the wall = 10 m

Area of the wall = Side $\times$ Side

$$= 10 \text{ m} \times 10 \text{ m}$$

$$= 100 \text{ m}^2$$

Area of the window = Length $\times$ Breadth

$$= 3 \text{ m} \times 1 \text{ m} = 3 \text{ m}^2$$

Thus, the required area of the wall for painting

$$= \text{Area of the wall} - \text{Area of the window}$$

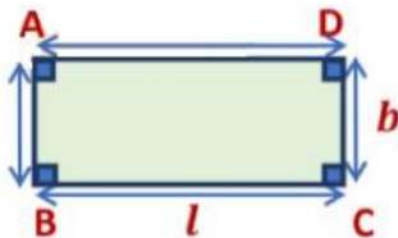
$$= (100 - 3) \text{ m}^2 = 97 \text{ m}^2$$

Rate of painting per square meter = ₹3

$$\text{Hence, the cost of painting the wall} = 97 \times ₹3 = ₹291$$

Rectangle

The rectangle is a quadrilateral with opposite sides of equal and each interior angle is a right angle.



In the above figure, ABCD is a rectangle, it has length (l) and breadth (b) and each interior angle is 90° .

In this case,

Perimeter of a rectangle = $2 \times (\text{length} + \text{breadth})$

Area of a rectangle = $\text{length} \times \text{breadth}$

Example: Find the perimeter and area of a rectangle of sides is 4 cm and 2 cm.

We know that,

Perimeter of a rectangle = $2 \times (\text{length} + \text{breadth})$

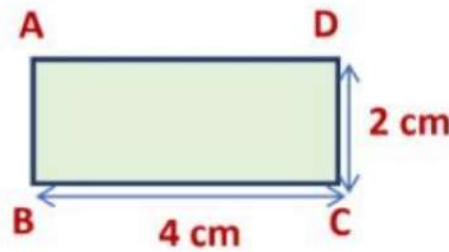
Perimeter of the rectangle = $2 \times (4 + 2)$

Perimeter of the rectangle = $2(6) = 12 \text{ cm}$

Area of a rectangle = $\text{length} \times \text{breadth}$

Area of the rectangle = $4 \text{ cm} \times 2 \text{ cm}$

Area of the rectangle = 8 cm^2



Example: Find the breadth of a rectangular park, if its area is 660 sq. m and length is 11 m. Also, find its perimeter.

We have,

$l = \text{length of a rectangular park} = 11 \text{ m}$

Area of the park = 660 sq. meter

Let the breadth of the plot be b meters.

Now,

We know that,

Area of a rectangle = $\text{length} \times \text{breadth}$

$$\text{Breadth}(b) = \frac{\text{Area of a rectangle}}{\text{Length}}$$

$$\text{Breadth}(b) = \frac{660}{11}$$

$$\text{Breadth}(b) = 60 \text{ m}$$

Perimeter of a rectangle = $2 \times (\text{length} + \text{breadth})$

Perimeter of the rectangle = $2 \times (11 + 60)$

The perimeter of the rectangle = $2 \times (71)$

The perimeter of the rectangle = 142 m

Example: A rectangular plot with perimeter 64 m and breadth 20m. What will be the area of a rectangular plot?

Let l and b be the length and breadth of the rectangular plot.

We have,

Perimeter = 64 m and breadth = 20 m

Perimeter of a rectangular plot = $2(l + b)$

$$64 = 2(l + 20)$$

$$\frac{64}{2} = (l + 20)$$

$$32 = l + 20$$

$$32 - 20 = l + 20 - 20$$

$$12 = l$$

$$l = 12 \text{ cm}$$

Now,

Area of a rectangular plot = length $\times$ breadth

Area of the rectangular plot = 12×20

Area of the rectangular plot = 240 m²

Increase of perimeter may not lead to increase in area.

We see this by using the following examples

Example: (i) Suppose we have a square having side 10 cm, then

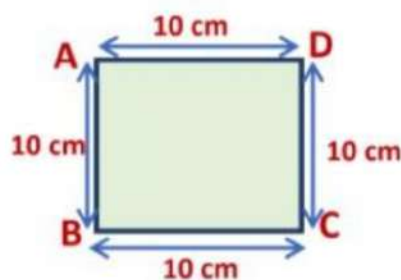
The perimeter of a square = $4 \times \text{side}$

Perimeter = 4×10

The perimeter of the square = 40 cm

Area of a square = $(\text{side})^2$

Area of the square = $(10)^2 = 100 \text{ cm}^2$



Example: (ii) Suppose we have a rectangle having length 25 cm and breadth 4 cm respectively then

Perimeter of a rectangle = $2 (\text{length} + \text{breadth})$

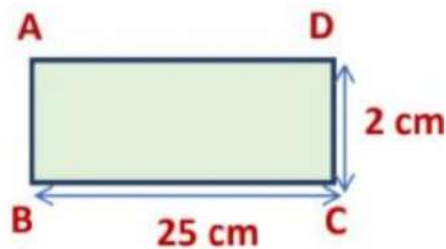
Perimeter = $2 (25 + 4)$

Perimeter = $2 (29)$

The perimeter of the rectangle = 58cm

Area of a rectangle = $\text{length} \times \text{breadth}$

Area of the rectangle = $25 \times 4 = 100\text{cm}^2$



Example: (iii) Suppose we have another rectangle having length 50

cm and breadth 2 cm respectively then

Perimeter of a rectangle = $2 (\text{length} + \text{breadth})$

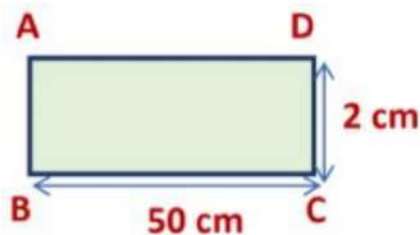
Perimeter = $2 (50 + 2)$

Perimeter = $2 (52)$

Perimeter of the rectangle = 104cm

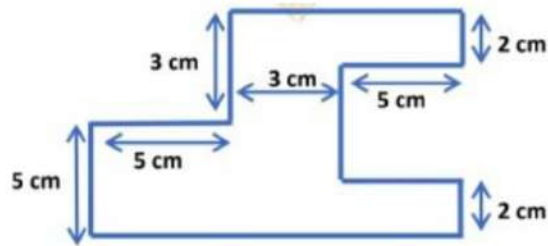
Area of a rectangle = $\text{length} \times \text{breadth}$

Area of the rectangle = $50 \times 2 = 100\text{cm}^2$

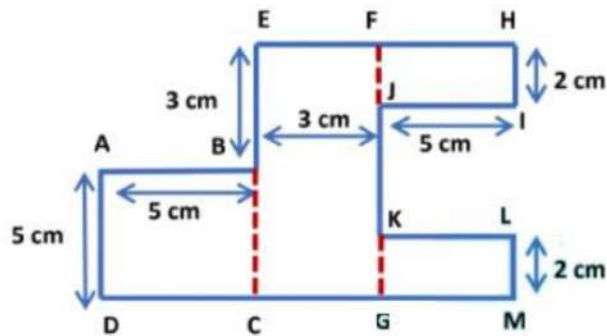


In the above examples, we see the perimeter increases but the area remains the same. Hence, from this, it is clear that the increase of perimeter need not lead to an increase in area.

Example: Find the area of the given figure.



Now, we split the above figure into one square and three rectangles.



We know,

Area of a square = (Side)²

Area of the square ABCD = $(5)^2 = 25 \text{ cm}^2$

Area of a rectangle = length $\times$ breadth

Area of the rectangle CEFG = $EF \times EC = 3 \times 8 = 24 \text{ cm}^2$

$\therefore EC = EB + BC = 3 + 5 = 8$

Area of the rectangle FHIJ = $JI \times HI = 5 \times 2 = 10 \text{ cm}^2$

Area of the rectangle GKLM = $GM \times LM = 5 \times 2 = 10 \text{ cm}^2$

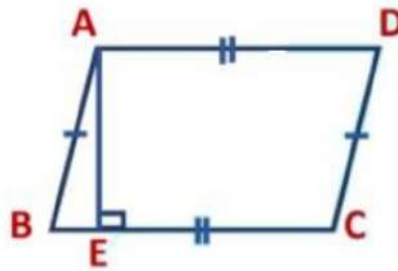
$\therefore$ Required area = Area of the square ABCD + Area of the rectangle (CEFG + FHIJ + GKLM)

$= (25 + 24 + 10 + 10) \text{ cm}^2 = 69 \text{ cm}^2$

Hence, the area of the given figure is 69 cm^2

Area of Parallelogram

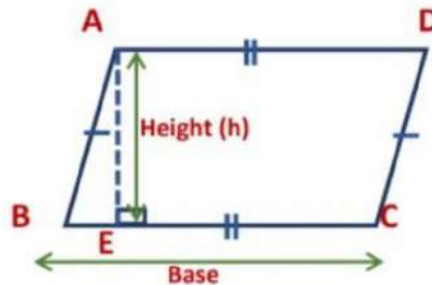
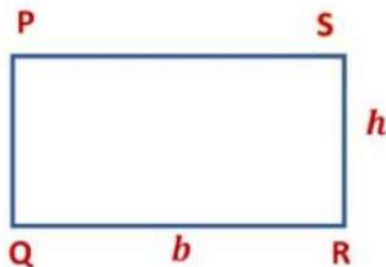
A parallelogram is a quadrilateral with opposite sides parallel (therefore opposite angles are equal.)



In this case, opposite sides and opposite angles are equal.
 $\therefore AB = DC, AD = BC$ and $\angle A = \angle C, \angle B = \angle D$

To find the area of the parallelogram

- (i) Any side of the parallelogram can be chosen as the base of the parallelogram.
- (ii) The perpendicular dropped on that side from the opposite vertex is known as height (altitude).
- (iii) In the parallelogram ABCD, AE is perpendicular to BC. Here BC is the base and AE is the height of the parallelogram.

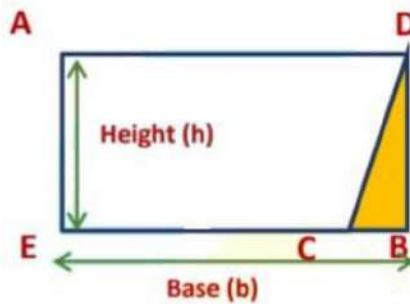


In the rectangle PQRS, length QR is the same as the base of the rectangle, and breadth SR is the same as the height of the rectangle.

Area of the rectangle PQRS = length $\times$ Breadth

Area of the rectangle PQRS = $b \times h$

Now, in the second figure, cut the triangle ABE and move to the other side of the parallelogram and look what do you get? You get a rectangle with base b and height h .



So, we can say

$$\text{Area of parallelogram ABCD} = b \times h$$

Example: Find the area of a parallelogram with base 8 cm and altitude 5 cm.

We have,

Base = 8 cm and altitude = 5 cm

Thus,

Area of a parallelogram = Base $\times$ Height

$$= 8 \text{ cm} \times 5 \text{ cm}$$

$$= 40 \text{ cm}^2$$

Example: Find the area of a parallelogram ABCD whose base BC is 10 cm and altitude AE is 3 cm. If the length of altitude AF is 5 cm to the base CD, then find CD.

Area of the parallelogram ABCD = Base $\times$ Height

$$= BC \times AE$$

$$= 10 \text{ cm} \times 3 \text{ cm}$$

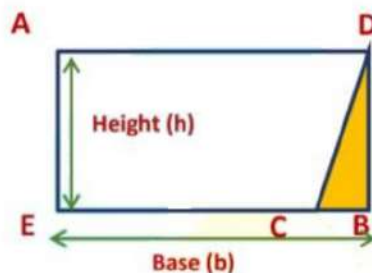
$$= 30 \text{ cm}^2$$

Also, area of the parallelogram ABCD = AF $\times$ CD

$$30 \text{ cm}^2 = 5 \text{ cm} \times CD$$

$$CD = 30 \div 5$$

$$CD = 6 \text{ cm}$$



Example: Find the height h if the area of a parallelogram is 64 cm^2 and the base is 8 cm.

$$\text{Height} = h$$

Base = 8 cm

Area of the parallelogram ABCD = Base $\times$ Height

$$64 \text{ cm}^2 = 8 \times h$$

$$\frac{64 \text{ cm}}{8 \text{ cm}} = h$$

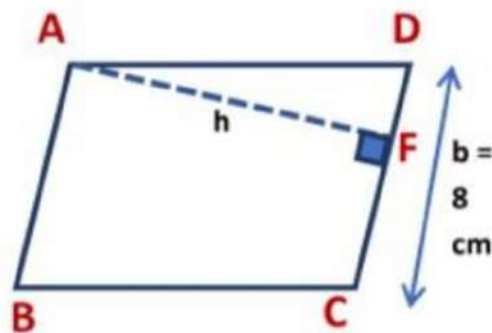
$$8 = h$$

$$H = \frac{64}{8}$$

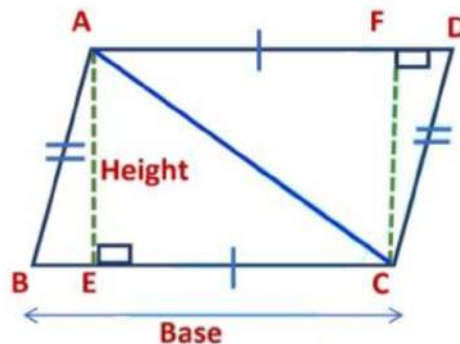
$$H = 8$$

Height (h) = 8 cm

So, the height of the parallelogram is 8 cm.



Area of Triangle



In a parallelogram ABCD, we see diagonal AC divides the parallelogram into two triangles i.e., $\triangle ABC$ and $\triangle ACD$. AE and CF are the altitudes of the parallelogram.

In figure,

$AD = BC$ and $AB = CD$

... $\because$ opposite sides of the parallelogram are equal.

$AC = AC$... common side

$\therefore \triangle ABC \cong \triangle ACD$... SSS congruency

Hence,

Area of $\triangle ABC$ = Area of $\triangle ACD$

It means the area of the parallelogram is equal to two times the area of one of these triangles.

$\therefore$ Area of parallelogram = 2 (Area of $\triangle ABC$)

$\Rightarrow \frac{1}{2}$ (Area of parallelogram) = Area of $\triangle ABC$

$\Rightarrow \frac{1}{2} \times \text{Base} \times \text{Height} = \text{Area of } \triangle ABC$

$\therefore$ we know, area of parallelogram = Base $\times$ Height

Area of $\triangle ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

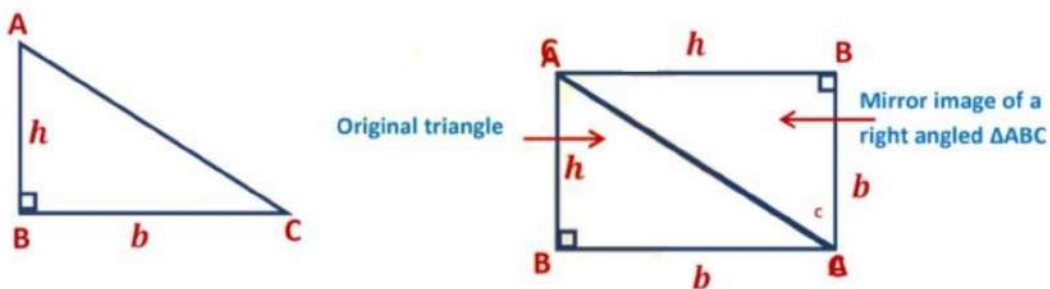
Area of $\triangle ABC = \frac{1}{2} (BC \times AE)$

Or

$\therefore$ Area of $\triangle ADC = \frac{1}{2} (AD \times CF)$

We know there are different types of triangles, so to find the area of different types of triangles can we use the same formula? Let's see

(i) Take a right-angled triangle ABC having base b and height. Now if you take the mirror image of this triangle ABC and place them in front of each other we get a rectangle.



Now, we know that,

Area of a rectangle = $b \times h$

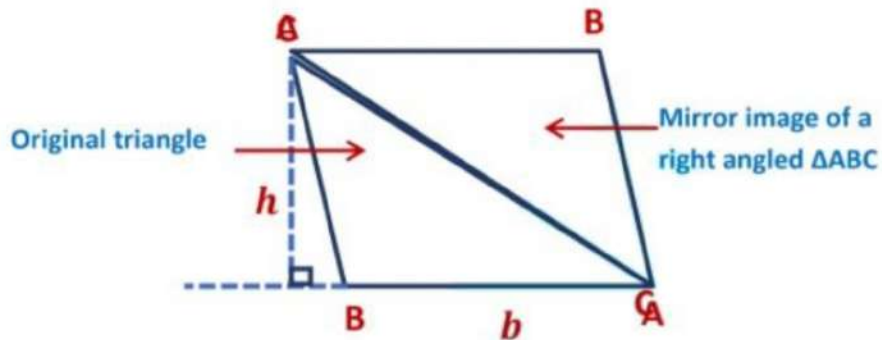
Here,

Area of rectangle = 2 $\times$ Area of triangle

$$b \times h = 2 \times \text{Area of triangle}$$

$$\text{Area of triangle} = \frac{1}{2} \times b \times h$$

(ii) Now, let's take the obtuse-angled triangle ABC having base b and height h . Now if you take the mirror image of this triangle ABC and place them in front of each other we get a parallelogram.



Now, we know,

$$\text{Area of parallelogram} = b \times h$$

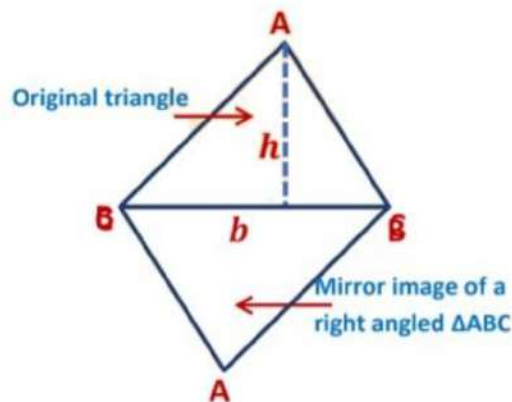
Here,

$$\text{Area of parallelogram} = 2 \times \text{Area of triangle}$$

$$b \times h = 2 \times \text{Area of triangle}$$

$$\text{Area of triangle} = \frac{1}{2} \times b \times h$$

(iii) Now, let's take the acute-angled triangle having base b and height. Now if you take the mirror image of this triangle and place them in front of each other we get a parallelogram.



Now, we know,

Area of parallelogram = $b \times h$

Here,

Area of parallelogram = $2 \times$ Area of triangle

$b \times h = 2 \times$ Area of triangle

Area of triangle = $\frac{1}{2} \times b \times h$

So, we conclude that to find the area of any triangle we use the same formula.

Example: The area of a triangle is 32cm^2 . If the height is 4 cm, what is its base?

We have,

Area of triangle = $\frac{1}{2} \times b \times h$

Base = $\frac{2 \times \text{Area}}{\text{Height}}$

Here,

Height = 8 cm and area = 32cm^2

Base of the triangle = $\frac{2 \times 32}{8}$

Base of the triangle = $4 = 16\text{ cm}$

Example: $\triangle ABC$ is right angle at A. AD is perpendicular to BC. If AB = 3 cm, BC = 5 cm and AC = 4 cm, find the area of $\triangle ABC$. Also, find the length of AD.

Area of $\triangle ABC$,

When base is AB = 3 cm and Height AC = 4 cm

Area of a $\triangle ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

Area of the $\triangle ABC = \frac{1}{2} \times AB \times AC$

Area of the $\triangle ABC = \frac{1}{2} \times 3 \times 4$

Area of the $\triangle ABC = 6\text{ cm}^2$

Now, find AD = ?

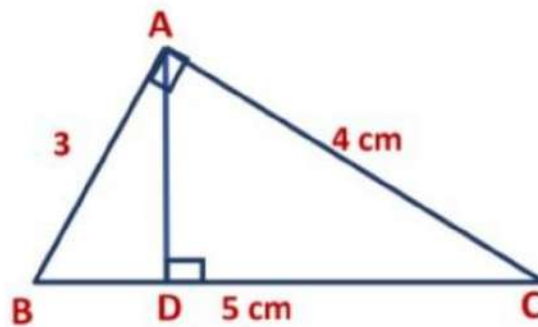
When area of the $\triangle ABC = 6 \text{ cm}^2$ and base $BC = 5 \text{ cm}$

Area of a $\triangle ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$6 = \frac{1}{2} \times 5 \times AD$$

$$12 = 5 \times AD$$

$$AD = \frac{12}{5} = 2.4 \text{ cm}$$



Example: A rectangular park ABCD having dimension 30 m by 15 m. ADE is a triangle such that $EF \perp AD$ and $EF = 8 \text{ m}$. Calculate the area of the shaded region.

We know,

Area of a rectangle = $b \times h$

Area of the rectangle ABCD = $AB \times BC$

$$= 30 \text{ m} \times 15 \text{ m}$$

$$= 450 \text{ m}^2$$

Area of the triangle = $\frac{1}{2} \times b \times h$

Area of the triangle = $\frac{1}{2} \times AD \times FE$

Area of the triangle ADE = $\frac{1}{2} \times BC \times FE$

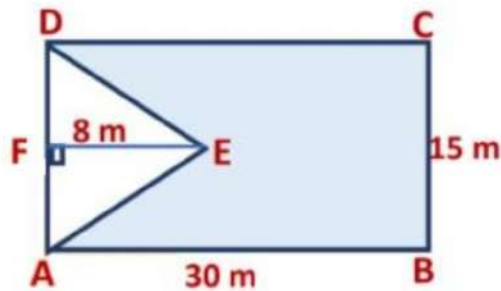
Area of the triangle ADE = $\frac{1}{2} \times AD \times FE \dots \because AD = BC$

Area of the triangle ADE = $\frac{1}{2} \times 15 \times 8$

Area of the triangle = $15 \text{ m} \times 4 \text{ m}$

Area of the triangle = 60 m^2

Now, Area of the shaded region = Area of the rectangle - Area of the triangle
= $(450 - 60) \text{ m}^2 = 390 \text{ m}^2$



Triangles as a Part of Rectangle and Squares

Triangles as Parts of Rectangles

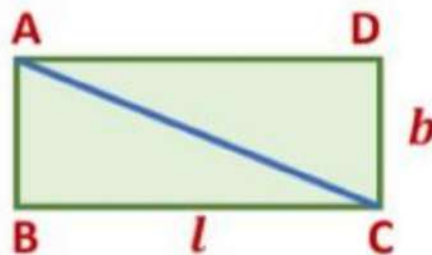
When we draw a diagonal in a rectangle we see the rectangle is divided into two equal triangles. So, there are two triangles located inside the rectangle. Also, the sum of the areas of two triangles is the same as the area of the rectangle.

Consider the rectangle ABCD,

Area of the rectangle ABCD = $l \times b$

= $AB \times BC = AD \times DC$

AC is the diagonal divides the rectangle into two equal triangles i.e., ABC and ADC.



$$\text{Area of the } \triangle ABC = \frac{1}{2} \times AB \times BC$$

$$\text{Area of the } \triangle ADC = \frac{1}{2} \times AD \times DC$$

$$\text{Area of rectangle } ABCD = \text{Area of } \triangle ABC + \text{Area of } \triangle ADC$$

$$= \frac{1}{2} \times (AB \times BC) + \frac{1}{2} \times (AD \times DC)$$

But in rectangle opposite sides are equal.

$$\therefore AD = BC, AB = CD$$

$$= \frac{1}{2} \times (AB \times BC) + \frac{1}{2} \times (BC \times AB)$$

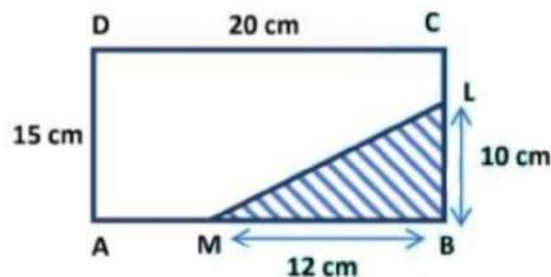
$$= \frac{2}{2} (AB \times BC)$$

$$= (AB \times BC)$$

This is the area of the rectangle ABCD.

Example: A rectangular piece of measures 20 cm by 15 cm. A triangular piece, with base 12 cm and height 10 cm is cut off from the canvas. Find the area of the remaining piece.

Let ABCD is a rectangular piece of canvas. $\triangle LBM$ is the triangular piece to be cut off.



We know,

Area of a rectangle = length $\times$ breadth

$$\text{Area of rectangle } ABCD = 20 \times 15 = 300\text{cm}^2$$

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Area of triangle LMB} = \frac{1}{2} \times \text{MB} \times \text{LB}$$

$$\text{Area of triangle LMB} = \frac{1}{2} \times 12 \times 10$$

$$\text{Area of triangle LMB} = 60 \text{ cm}^2$$

$$\begin{aligned} \text{Now, required area} &= \text{Area of rectangle ABCD} - \text{Area of triangle LMB} \\ &= 300 \text{ cm}^2 - 60 \text{ cm}^2 = 240 \text{ cm}^2 \end{aligned}$$

Hence, the area of the remaining piece is 240 cm^2

Triangles as Parts of squares

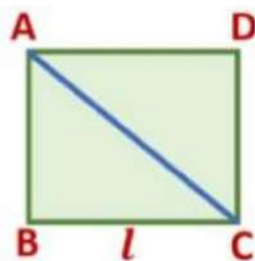
As we see when we draw a diagonal in a rectangle the rectangle is divided into two equal triangles. In a very similar way triangles are located inside the square also.

So, in the given figure,

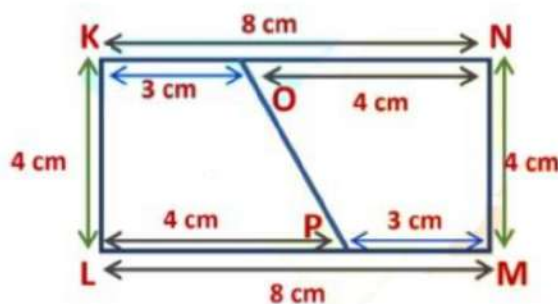
$$\triangle ABC \cong \triangle ADC$$

$$\text{Area of } \triangle ABC \cong \text{Area of } \triangle ADC$$

$$= \frac{1}{2} \times (\text{area of square ABCD})$$



Generalizing for Other Congruent Parts of Rectangles



In rectangle KLMN line segment OP divide the rectangle KLMN into two equal parts into two equal parts (KLPO and OPMN), Where $KO = PM = 3$ cm and $LP = ON = 4$ cm. Are they congruent?

$KO = PM = 3$ cm, $LP = ON = 4$ cm and $KL = MN = 4$ cm

SO, $KLPO \cong OPMN$ [$\because$ OP is common]

The area of each congruent part = $\frac{1}{2}$ (Area of rectangle)

The area of quadrilateral KLPO = $\frac{1}{2}$ ($KL \times LM$)

$$= \frac{1}{2} (4 \text{ cm} \times 8 \text{ cm})$$

$$= \frac{1}{2} (32 \text{ cm}) = 16 \text{ cm}^2$$

The area of quadrilateral MPON = $\frac{1}{2}$ ($KN \times MN$)

$$= \frac{1}{2} (8 \text{ cm} \times 4 \text{ cm})$$

$$= \frac{1}{2} (32 \text{ cm}) = 16 \text{ cm}^2$$

Area of rectangle KLMN = Area of quadrilateral KLPO + Area of quadrilateral MPON
 $MPON = 16 \text{ cm}^2 + 16 \text{ cm}^2 = 32 \text{ cm}^2$

Here, the two parts are congruent to each other. So, the area of one part is equal to the area of the other part.

Example: ABCD is a rectangle with perimeter 100 cm and breadth 15 cm. What will be the area of ΔACD ?

Let l and b be the length and breadth of the rectangle ABCD.

Perimeter of the rectangle ABCD = $2(l + b)$

$$100 \text{ cm} = 2(l + 15)$$

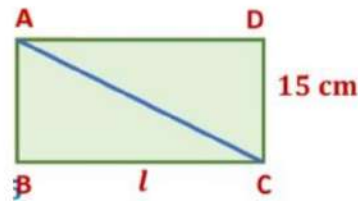
$$\frac{100}{2} = (l + 15)$$

$$50 = l + 15$$

$$50 - 15 = l + 15 - 15$$

$$35 = l$$

$$l = 35 \text{ cm}$$



Now,

$$\text{Area of } \triangle ADC = \frac{1}{2} (\text{Area of rectangle})$$

$$= \frac{1}{2} \times (l \times b)$$

$$= \frac{1}{2} \times (35 \times 15)$$

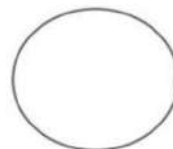
$$= \frac{1}{2} \times (525)$$

$$= 262.5 \text{ m}^2$$

Circle

Circle is a very unique figure. We see circles all around us in different things. Some examples of a circle are a dining plate, a coin, watch, tyres, etc...which are in the shape of a circle.

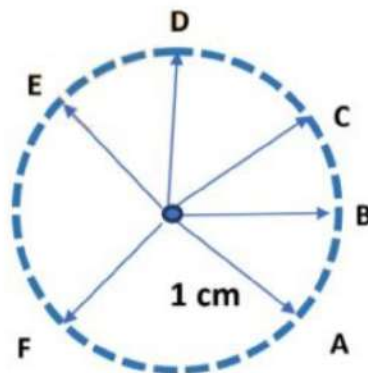
Can you tell what exactly is a circle?



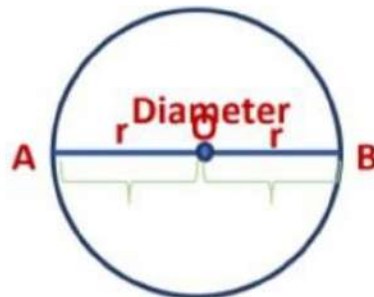
“Circle is a simple closed curve in which all the points are at equal distance from a fixed point inside it”.

In the following figure, we have taken a point O as a fixed point and we draw another point A, 1 cm away from the fixed point O. Similarly we take other points, which are also 1 cm away from the fixed point O.

Now, if we join all the points together they form a circle.
[Represented by a dotted line]



Parts of a Circle



- The center of a circle is the point that is equidistant from all points on the circle. In the above figure, O is the center of the circle.
- The fixed distance between the center of the circle to any point on the circle is called the radius. In the above figure, OA and OB are the radii of the circle.
- The distance from one point on a circle through the center to another point on the circle is called the diameter.
- In the above figure, AB is the diameter.

Diameter is twice the radius.
 $\therefore \text{Diameter} = 2 \times \text{Radius}$

- The distance around the circular region is known as the circumference. The circumference of a circle is the same as the perimeter.

We know the relation between diameter and radius, $d = 2r$.

To understand the circumference of a circle, firstly we have to understand the number called Pi.

What is Pi?

Pi is the ratio of two different distances on a circle, which is constant for every circle.

Distance around a circle

Distance across a circle = $\text{Pi}(\pi)$

Circumference

Diameter = $\text{Pi}(\pi) \dots \dots (i)$

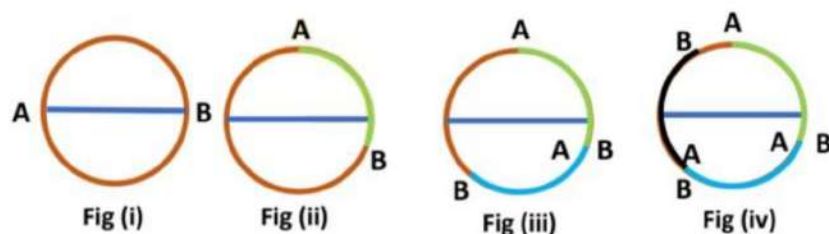
Pi (π) is an ancient Greek symbol.

What is this Pi(π) means?

Suppose in a circle we draw a diameter AB and if we wrap it around the boundary of the circle as shown in fig. (ii)

We again take the same diameter and wrap it around the boundary of the circle one more time as shown in fig. (iii)

Similarly, we wrap the same diameter around the circle for the third time as shown in fig. (iv), then what do you observe in the following figures?



We observe when we wrap the diameter three times around the boundary of the circle then also a very small boundary left over.

So, from this, we can say that,

$$3 \times \text{diameter} \approx \text{Circumference}$$

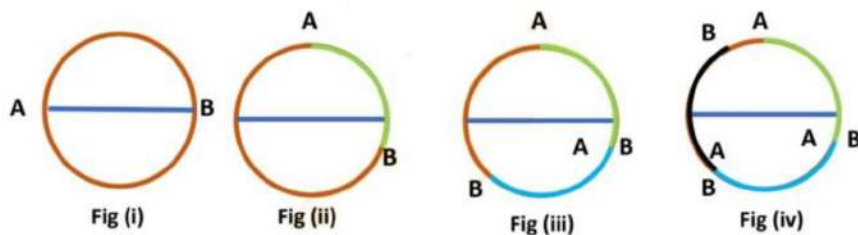
$$3 \approx \frac{\text{Circumference}}{\text{Diameter}}$$

Here, the same diameter is wrapped around the circle three times so, we can write it as

$$\frac{3}{1} \approx \frac{\text{Circumference}}{\text{Diameter}}$$

So, the ratio of circumference to the diameter is approximately 3:1.

Now, let's take another circle, in which if we wrap the diameter around its circumference three times then we again see a small boundary left over.



So, in this case, also we can say that,

$$3 \times \text{diameter} \approx \text{Circumference}$$

$$3 \approx \frac{\text{Circumference}}{\text{Diameter}}$$

Here, the same diameter is wrapped around the circle three times so, we can write it as

$$\frac{3}{1} \approx \frac{\text{Circumference}}{\text{Diameter}}$$

So, in this case, also the ratio of circumference to the diameter is approximately 3:1.

In the above cases, we see our diameter is 1 but if we measure our circumferences by using a better tape we get the values like 3.1 or 3.14. If we measure the circumference by using even better tape we will get the number 3.1459 and in this number, only the digits get added and they never repeat.

This is true for every circle. Hence, the ratio of circumference to diameter is always taken as 3.1459.

$$\therefore \frac{\text{Circumference}}{\text{Diameter}} = 3.1459 \text{ (approximately)}$$

This ratio is denoted by π (Pi).

Thus, we have

$$\pi = 3.1459 \text{ (approximately)} = \frac{22}{7} \text{ (approximately)}$$

$$\text{Now, } \frac{\text{Circumference}}{\text{Diameter}} = \pi$$

We represent,

Circumference - C, diameter - d, radius - r

$$\frac{C}{d} = \pi$$

$$C = \pi d$$

$$C = \pi \times 2r \dots \because [d = 2r]$$

$$\text{Circumference} = 2\pi r$$

Example: Find the circumference of the circle of radius 14 cm.

We have,

$$\text{Radius} = 14 \text{ cm}$$

$$\text{Circumference of circle} = 2\pi r$$

$$= 2 \times 3.14 \times 14$$

$$= 87.92 \text{ cm.}$$

So, the circumference of the circle is 87.92 cm.

Example: The diameter of a wheel of a bullock cart is 1 m. Find the distance traveled by the bullock cart during the period, the wheel makes 1000 revolutions.

It may be noted that in one revolution, the bullock cart covers a distance equal to the circumference of the wheel.

The diameter of the wheel = 1 m

We know that,

Circumference = $\pi \times d$

$\Rightarrow$ Circumference of the wheel = 3.14×1

= 3.14 m

Thus, the bullock cart covers 3.14 m in one revolution.

Now,

The distance covered by the bullock cart in 1000 revolutions

= $3.14 \text{ m} \times 1000$

= 3140 m

The distance travelled by the bullock cart in 1000 revolutions is 3140m.

Example: Find the circumference of the semicircle if its radius is 4cm.

We have, radius = 4 cm

Circumference of circle = $2\pi r$

$\therefore$ Circumference of semicircle = $\frac{1}{2} \times 2\pi r$

= $\frac{1}{2} \times 2 \times \frac{22}{7} \times 4$

= $\frac{88}{7}$

= 12.57cm

So, the circumference of the semicircle is 12.57cm

Area of Circle

The area of a circle is the region enclosed by the circle. The area of the circle is equal to pi (π) multiplied by its radius squared.

Rena wants to polish a tabletop of radius 1 m. What will be the cost of polishing a circular table-top at the rate of ₹10 per square meter?

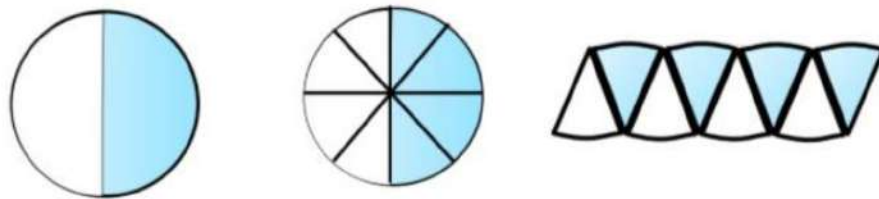
To find the cost of polishing a circular table-top, Rena needs to find the area of the circular tabletop.

Let us see how to find the area of the circle by using graph paper. Rena draws a circle of radius 1 cm on a graph paper.

By counting a number of squares enclosed by a circle, Rena finds the area of the circle.

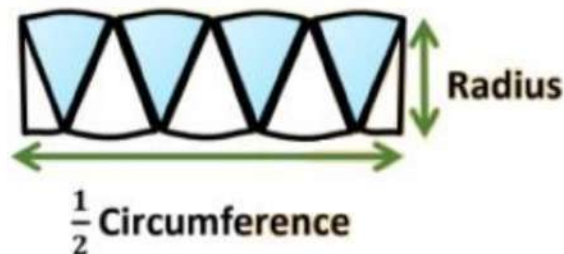
But we know, the edge of the circle is not straight hence, Rena gets a rough estimate of the area of the circle by this method.

There is another way of finding the area of a circle as follows.



In a circle, we shade one half of the circle. Now we fold the circle into eighths and cut along the folds. What do we get? We get four shaded triangles and four unshaded triangles. If we arrange that eight triangles in a straight line as shown in the figure we get a parallelogram.

Now, if we cut the first unshaded triangle into two half and put one half in last after the shaded triangle we get a rectangle as shown below.



What is the breadth of this rectangle? The breadth of this rectangle is nothing but the radius of the circle, 'r'. What is the length of this rectangle? The length is nothing but half of the circumference of the circle.

Hence,

Area of the circle = Area of rectangle

We know that, area of rectangle = $l \times b$

Area of the circle = [half of the circumference] $\times$ Radius

Area of the circle = $\left[\frac{1}{2} \times 2\pi r \right] \times r$

$$\therefore \text{Area of the circle} = \pi r^2$$

Example: Find the area of a circle whose radius is 6 cm

We have,

Radius of a circle = 6 cm

We know,

Area of the circle = πr^2

$$\text{Area of the circle} = \frac{22}{7} \times (6)^2$$

$$= \frac{22}{7} \times 36$$

$$= \frac{792}{7} = 113.13 \text{ cm}^2$$

Hence, the area of a circle is 113.13 cm²

Example: The area of a circle is 154 cm². Find the radius of the circle.

Let the radius of the circle be r cm.

We have,

Area of the circle (A) = 154 cm²

$$\pi r^2 = 154 \text{ cm}^2$$

$$\frac{22}{7} \times (r)^2 = 154 \text{ cm}^2$$

$$(r)^2 = \frac{154 \times 7}{22}$$

$$(r)^2 = \frac{1078}{22}$$

$$(r)^2 = 49$$

$$r = (7)^2$$

$$r = 7 \text{ cm}$$

Hence, the radius of the circle is 7 cm.

Example: An aluminium wire when bent in the form of a square encloses an area of 121 cm^2 . If the same wire is bent in the form of a circle, find the area of the circle.

We have:

$$\text{Area of the square} = 121 \text{ cm}^2$$

We know that,

$$\text{Area of the square} = (\text{Side})^2 = (11)^2 \text{ cm}^2$$

$$\text{Side} = 11 \text{ cm.}$$

So,

$$\text{The perimeter of the square} = 4 \times (\text{side})$$

$$= (4 \times 11) \text{ cm}$$

$$= 44 \text{ cm}$$

Now,

Let r be the radius of the circle.

Circumference of the circle = Perimeter of the square

$$2\pi r = 44$$

$$2 \times \frac{22}{7} \times r = 44$$

$$r = \frac{44 \times 7}{2 \times 22}$$

$$r = 7 \text{ cm.}$$

$$\text{Area of a circle (A)} = \pi r^2$$

$$\text{Area of the circle} = \frac{22}{7} \times 7 \times 7$$

$$= 154 \text{ cm}^2$$

Conversion of Units

Conversion of units is the conversion between different units of measurement for the same quantity.

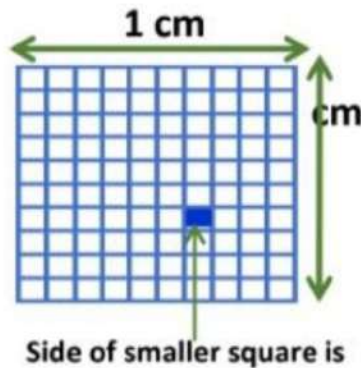
We know

$$1 \text{ kilometer} = 1000 \text{ meter}$$

$$1 \text{ meter} = 100 \text{ cm}$$

1 centimeter = 10 millimeters

1 meter² = ? centimeter²



To find this let us consider an example, take a graph paper and draw a square of side 1 cm then divide the square into 100 smaller squares, each side 1 mm. So, the area of the smaller square of side 1 mm
 $= 1 \text{ mm} \times 1 \text{ mm} = 1 \text{ mm}^2 \dots(i)$

Now, the area of the square bigger square (having side 1cm) is equal to 100 times the area of the smaller square.

So, area of the bigger square = $100 \times$ area of the smaller square

$1 \text{ cm}^2 = 100 \times 1 \text{ mm}^2 \dots \text{from (i)}$

Or

$$1 \text{ cm}^2 = 100 \text{ mm}^2$$

Similarly,

$$1 \text{ meter}^2 = 1 \text{ m} \times 1 \text{ m}$$

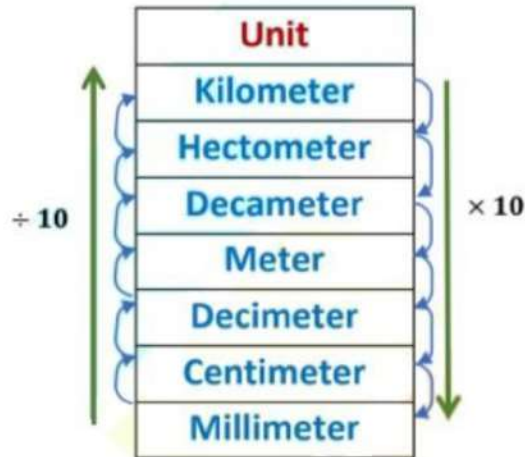
$$1 \text{ meter}^2 = 100 \text{ cm} \times 100 \text{ cm} \dots [1 \text{ m} = 100 \text{ cm}]$$

$$1 \text{ meter}^2 = 10000 \text{ cm}^2$$

Look at the following table in which Kilo, Hecto, Deca, Centi, and Milli are in a particular sequence.

Here we are talking about the distance hence it will become Kilometer, Hectometer, Decameter, Meter, Decimeter, Centimeter, and Millimeter.

In the following figure, if you go down the conversion factor is multiplies by 10 and if you go up the conversion factor is divide by 10.



Example: If you have to convert 1 kilometre to meters then look at the figure, there are three steps from km (kilometre) to m (metre).

So, multiply 1 km by $10 \times 10 \times 10$

1 kilometer = $10 \times 10 \times 10$ meter

1 kilometer = 1000 meter

Example: If you have to convert 1 millimetre to centimetres then look at the figure from millimetre you have to move to centimetre which means you have to move one step up. So, we divide this 1 millimetre by 10.

$\therefore$ 1 millimeter = centimeter

1 millimeter = 0.1 cm

When we convert a unit of area to a smaller unit, the resulting number of units will be bigger.

Example: Suppose Ram asks Raghav his home is how far from the office? Raghav said my home is 10 km far from the office. If Raghav wants he can say this distance in meter or millimetre also like, 10,000 meters or 10,000,000 millimetre

What can we see from this as we convert a higher unit of area to a lower unit, the resulting number of units will be bigger?

Example: Convert 7 km to m

We know that,

$$1 \text{ km} = 1000 \text{ m}$$

There are three steps from km (kilometre) to m (meter). So, multiply by 1000 m.

$$7 \text{ km} = 7 \times 1000$$

$$8 \text{ km} = 7000 \text{ m}$$

Example: Convert 10 m 50 cm into cm.

We know that,

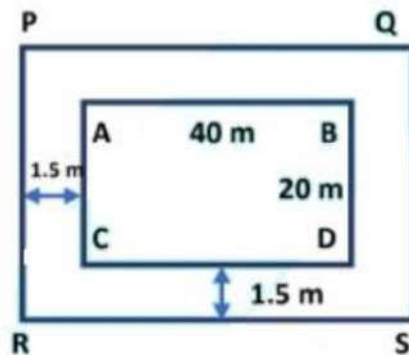
$$1 \text{ m} = 100 \text{ cm}$$

There are two steps from m (meter) to cm (centimeter). So, Multiply by 100

$$\begin{aligned} 10 \text{ m } 50 \text{ cm} &= 10 \times 100 \text{ cm} + 50 \text{ cm} \\ &= 1000 \text{ cm} + 50 \text{ cm} \\ &= 1050 \text{ cm} \end{aligned}$$

Applications of Perimeter and Area

Example: A rectangular park is 40 m long and 20 m wide. A path 1.5 m wide is constructed outside the park. Find the area of the path.
(REFERENCE: NCERT)



Let ABCD represent rectangular park and the shaded region represents path constructed outside the park which is 1.5 m wide.

To find the area of the path, we need to find Area of rectangle PQRS and Area of rectangle ABCD

We have,

$$PQ = (40 + 1.5 + 1.5) \text{ m} = 43 \text{ m}$$

$$PS = (20 + 1.5 + 1.5) \text{ m} = 23 \text{ m}$$

Now,

$$\text{Area of the rectangle ABCD} = l \times b =$$

$$= 40 \times 20 \text{ m}^2$$

$$= 800 \text{ m}^2$$

$$\text{Now, Area of the rectangle PQRS} = l \times b$$

$$= 43 \times 23 \text{ m}^2$$

$$= 989 \text{ m}^2$$

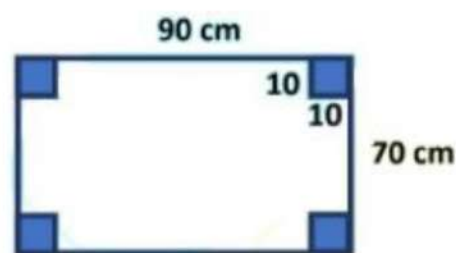
$$\text{Area of the path} = \text{Area of the rectangle PQRS} - \text{Area of the rectangle ABCD}$$

$$= (989 - 800) \text{ m}^2$$

$$= 189 \text{ m}^2$$

Example: From a rectangular sheet of copper, of size 90 cm by 70 cm, are cut four squares of side 10 cm from each corner. Find the area of the remaining sheet.

(REFERENCE: NCERT)



Length of the rectangular sheet = 90 cm

The breadth of the rectangular sheet = 70 cm

Area of the rectangular sheet of copper

$$= 90 \text{ cm} \times 70 \text{ cm}$$

$$= 6300 \text{ cm}^2$$

Side of the square at the corner of the sheet = 10 cm

Area of one square at the corner of the sheet = $(10 \text{ cm})^2 = 100 \text{ cm}^2$

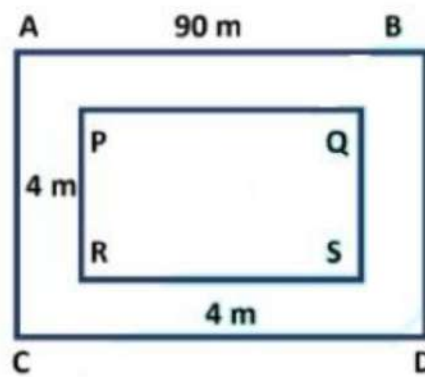
Area of 4 squares at the corner of the sheet = $4 \times 100 \text{ cm}^2 = 400 \text{ cm}^2$

Hence,

Area of the remaining sheet of copper = Area of the rectangular sheet – Area of the 4 squares

Area of the remaining sheet of copper = $(6300 - 400) \text{ cm}^2 = 5900 \text{ cm}^2$

Example: A path 4 m wide runs along inside a square park of side 90 m. Find the area of the path. Also, find the cost of cementing it at the rate of ₹200 per 10 m^2 . (REFERENCE: NCERT)



Let ABCD be the square park of side 100 m. The shaded region represents the path 4 m wide.

$$PQ = 90 - (4 + 4) = 82 \text{ m}$$

$$\text{Area of square ABCD} = (\text{side})^2 = (90)^2 \text{ m}^2 = 8100 \text{ m}^2$$

$$\text{Area of square PQRS} = (\text{side})^2 = (82)^2 \text{ m}^2 = 6724 \text{ m}^2$$

$$\text{Therefore, area of the path} = (8100 - 6724) \text{ m}^2 = 1376 \text{ m}^2$$

$$\text{Cost of cementing } 10 \text{ m}^2 = ₹ 200$$

Therefore,

$$\text{Cost of cementing } 1 \text{ m}^2 = \frac{200}{10} = 20$$

$$\begin{aligned} \text{So, the cost of cementing } 1376 \text{ m}^2 &= ₹20 \times 1376 \\ &= ₹ 27520 \end{aligned}$$