

18. Binomial Theorem

Very Short Answer

1. Question

Write the number of terms in the expansion of $(2 + \sqrt{3}x)^{10} + (2 - \sqrt{3}x)^{10}$.

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^k a^{n-k}$$

$$(2 + \sqrt{3}x)^{10} = \sum_{k=0}^{10} \binom{10}{k} 2^k (\sqrt{3}x)^{10-k}$$

$$= \binom{10}{0} 2^0 (\sqrt{3}x)^{10} + \binom{10}{1} 2^1 (\sqrt{3}x)^{10-1} + \dots + \binom{10}{9} 2^9 (\sqrt{3}x)^{10-9} + \binom{10}{10} 2^{10} (\sqrt{3}x)^{10-10} \quad (1)$$

$$(2 - \sqrt{3}x)^{10} = \sum_{k=0}^{10} \binom{10}{k} 2^k (-\sqrt{3}x)^{10-k}$$

$$= \binom{10}{0} 2^0 (-\sqrt{3}x)^{10} + \binom{10}{1} 2^1 (-\sqrt{3}x)^{10-1} + \dots + \binom{10}{9} 2^9 (-\sqrt{3}x)^{10-9} + \binom{10}{10} 2^{10} (-\sqrt{3}x)^{10-10} \quad (2)$$

Add both equations;

$$\begin{aligned} & (2 + \sqrt{3}x)^{10} + (2 - \sqrt{3}x)^{10} \\ &= \binom{10}{0} 2^0 (\sqrt{3}x)^{10} + \binom{10}{1} 2^1 (\sqrt{3}x)^{10-1} + \dots + \binom{10}{9} 2^9 (\sqrt{3}x)^{10-9} \\ & \quad + \binom{10}{10} 2^{10} (\sqrt{3}x)^{10-10} + \\ & \binom{10}{0} 2^0 (-\sqrt{3}x)^{10} + \binom{10}{1} 2^1 (-\sqrt{3}x)^{10-1} + \dots + \binom{10}{9} 2^9 (-\sqrt{3}x)^{10-9} \\ & \quad + \binom{10}{10} 2^{10} (-\sqrt{3}x)^{10-10} \end{aligned}$$

The even terms; i.e. $k=1,3,5,7$ & 9 cancel each other

So, we are left with only terms with $k=0,2,4,6,8$ & 10

So total number of terms = 6

2. Question

Write the sum of the coefficients in the expansion $(1 - 3x + x^2)^{111}$.

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(1-3x+x^2)^{111}$$

For sum of coefficients; put $x=1$

We have;

$$(1-3+1)^{111}=(-1)^{111}$$

$$= -1$$

3. Question

Write the number of terms in the expansion of $(1-3x+3x^2-x^3)^8$.

Answer

Given:

$$(1-3x+3x^2-x^3)^8$$

$$\text{Highest power is } (x^3)^8 = x^{24}$$

$$\text{And lowest power is } x^0$$

So the expansion contains all the terms ranging from 0 to 24

Therefore, total number of terms = 25

4. Question

Write the middle term in the expansion of $\left(\frac{2x^2}{3} + \frac{3}{2x^2}\right)^{10}$.

Answer

Given:

$$(x+a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\left(\frac{2x^2}{3} + \frac{3}{2x^2}\right)^{10} = \sum_{k=0}^{10} \binom{10}{k} \left(\frac{2x^2}{3}\right)^{10-k} \left(\frac{3}{2x^2}\right)^k$$

$$\text{Total number of terms} = n+1 = 11$$

So middle term = 6th term, i.e. $k=5$

$$= \binom{10}{5} \left(\frac{2x^2}{3}\right)^{10-5} \left(\frac{3}{2x^2}\right)^5$$

$$= \frac{10!}{5! \times 5!}$$

$$= 252$$

5. Question

Which term is independent of x , in the expansion of $\left(x - \frac{1}{3x^2}\right)^9$?

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\left(x - \frac{1}{3x^2}\right)^9 = \sum_{k=0}^9 \binom{9}{k} x^{9-k} \left(\frac{-1}{3x^2}\right)^k$$

$$= \sum_{k=0}^9 \binom{9}{k} x^{9-k} \left(\frac{-1}{3}\right)^k x^{-2k}$$

$$\Rightarrow x^{(9-k-2k)} = x^0$$

$$\Rightarrow 9-3k=0$$

$$\Rightarrow k=3$$

So 4th term is independent of x.

6. Question

If a and b denote respectively the coefficients of x^m and x^n in the expansion of $(1+x)^{m+n}$, then write the relation between a and b.

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(1 + x)^{m+n} = \sum_{k=0}^{m+n} \binom{m+n}{k} 1^{m+n-k} x^k$$

Coefficient of x^m ; $k=m$

$$a = \binom{m+n}{m} 1^{m+n-m}$$

$$a = \frac{(m+n)!}{m! \times n!} \dots (1)$$

Coefficient of x^n ; $k=n$

$$b = \binom{m+n}{n} 1^{m+n-n}$$

$$b = \frac{(m+n)!}{n! \times m!} \dots (2)$$

Divide both equations;

We get;

$$a=b$$

7. Question

If a and b are coefficients of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then write the relation between a and b.

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(1 + x)^{2n} = \sum_{k=0}^{2n} \binom{2n}{k} 1^{2n-k} x^k$$

Coefficient of x^n ; $k=n$

$$a = \binom{2n}{n} 1^{2n-n} \dots (1)$$

$$(1 + x)^{2n-1} = \sum_{k=0}^{2n-1} \binom{2n-1}{k} 1^{2n-1-k} x^k$$

Coefficient of x^n ; $k=n$

$$b = \binom{2n-1}{n} 1^{2n-1-n} \dots (2)$$

Divide both equations;

$$\frac{a}{b} = \frac{2n!}{n! \times n!} \times \frac{n! \times (n-1)!}{(2n-1)!}$$

$$\frac{a}{b} = \frac{2n(2n-1)!}{n! \times (n-1)!} \times \frac{n! \times (n-1)!}{(2n-1)!}$$

$$\frac{a}{b} = 2n$$

$$a = 2b$$

8. Question

Write the middle term in the expansion of $\left(x + \frac{1}{x}\right)^{10}$.

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

Total terms = $n+1 = 11$

So middle term = 6th term ; i.e. $k=5$

$$\left(x + \frac{1}{x}\right)^{10} = \sum_{k=0}^{10} \binom{10}{k} x^{10-k} \left(\frac{1}{x}\right)^k$$

For $k=5$;

$$= \binom{10}{5} x^{10-5} \left(\frac{1}{x}\right)^5$$

$$= {}^{10}C_5$$

9. Question

If a and b denote the sum of the coefficients in the expansions of $(1 - 3x + 10x^2)^n$ and $(1 + x^2)^n$ respectively, then write the relation between a and b .

Answer

Given:

$$(1-3x+10x^2)^n$$

Sum of coefficients = a

$$a = (1 - 3 + 10)^n$$

$$=(2^3)^n$$

$$=(2^n)^3$$

$$(1+x^2)^n$$

Sum of coefficients = b

$$b = (1+1)^n$$

$$=2^n$$

Put value of b in a; we get:

$$a=b^3$$

10. Question

Write the coefficient of the middle term in the expansion of $(1 + x)^{2n}$

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(1 + x)^{2n} = \sum_{k=0}^{2n} \binom{2n}{k} 1^{2n-k} x^k$$

Total terms = $2n+1$

Middle term = $(2n+1)/2$

i.e. $(n+1)$ th term

so $k=n$

$$= \binom{2n}{n} 1^{2n-n} x^n$$

$$= {}^{2n}C_n$$

11. Question

Write the number of terms in the expansion of $\{(2x + y^3)^4\}^7$

Answer

Given:

$$\{(2x + y^3)^4\}^7 = (2x + y^3)^{28}$$

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(2x + y^3)^{28} = \sum_{k=0}^{28} \binom{28}{k} 2x^{28-k} (y^3)^k$$

So total number of terms = n+1

$$= 28+1$$

$$= 29$$

12. Question

Find the sum of coefficients of two middle terms in the binomial expansion of $(1 + x)^{2n-1}$

Answer

Given:

$$\text{Total terms after expansion} = 2n-1+1=2n$$

$$\text{Middle term} = 2n/2 = n\text{th term}$$

So two required middle terms are : nth & (n+1)th term

k = (n-1) & n for both terms respectively.

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(1 + x)^{2n-1} = \sum_{k=0}^{2n-1} \binom{2n-1}{k} 1^{2n-1-k} x^k$$

Coefficient of nth term;

$$= {}^{2n-1}C_{n-1}$$

Coefficient of (n+1)th term ;

$$= {}^{2n-1}C_n$$

$$\text{Sum of coefficients} = {}^{2n-1}C_{n-1} + {}^{2n-1}C_n$$

$$= {}^{2n-1+1}C_n$$

$$= {}^{2n}C_n$$

13. Question

Find the ratio of the coefficients of x^p and x^q in the expansion of $(1 + x)^{p+q}$.

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(1 + x)^{p+q} = \sum_{k=0}^{p+q} \binom{p+q}{k} 1^{p+q-k} x^k$$

For x^p ; k=p

$$\text{Coefficient} = {}^{p+q}C_p (1)$$

For x^q ; k=q

$$\text{Coefficient} = {}^{p+q}C_q \quad (2)$$

Divide both equations;

$$\frac{p}{q} = \frac{(p+q)!}{p! \times q!} \times \frac{p! \times q!}{(p+q)!}$$

$$\frac{p}{q} = 1$$

14. Question

Write last two digits of the number 3^{400} .

Answer

Given:

$$3^{400} = (3^2)^{200}$$

$$= 9^{200}$$

$$= (10 - 1)^{200}$$

By binomial expansion, $(x + a)^n = \sum_{k=0}^n {}^nC_k x^{n-k} a^k$

$$(1 - 10)^{200} = \sum_{k=0}^{200} {}^{200}C_k 1^{200-k} (-10)^k$$

$$= {}^{200}C_0 (-10)^0 + {}^{200}C_1 (-10)^1 + {}^{200}C_2 (-10)^2 + \dots + {}^{200}C_{200} (-10)^{200}$$

$$= 1 - 2000 + 10^2 \{I\}$$

$$= 1 + 100(I - 20)$$

So, the last two digits would be 01.

15. Question

Find the number of terms in the expansion of $(a + b + c)^n$.

Answer

Given:

$$T_n = \frac{n!}{p! \times q! \times r!} a^p b^q c^r;$$

Where $p + q + r = n$

Since number of ways in which we can divide n different things into r different things is : ${}^{n+r-1}C_{r-1}$

Here, $n=n$ & $r=3$

$$\text{So, } {}^{n+3-1}C_{3-1} = {}^{n+2}C_2$$

$$= \frac{(n+2)!}{2! \times n!}$$

$$= \frac{(n+2)(n+1)n!}{2! \times n!}$$

$$= \frac{(n+1)(n+2)}{2}$$

so, the number of terms = $\frac{(n+1)(n+2)}{2}$

16. Question

If a and b are the coefficients of x^n in the expansions $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, find $\frac{a}{b}$.

Answer

Given:

$$(x+a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(1+x)^{2n} = \sum_{k=0}^{2n} \binom{2n}{k} 1^{2n-k} x^k$$

Coefficient of x^n ; $k=n$

$$a = \binom{2n}{n} 1^{2n-n} (1)$$

$$(1+x)^{2n-1} = \sum_{k=0}^{2n-1} \binom{2n-1}{k} 1^{2n-1-k} x^k$$

Coefficient of x^n ; $k=n$

$$b = \binom{2n-1}{n} 1^{2n-1-n} (2)$$

Divide both equations;

$$\frac{a}{b} = \frac{2n!}{n! \times n!} \times \frac{n! \times (n-1)!}{(2n-1)!}$$

$$\frac{a}{b} = \frac{2n(2n-1)!}{n! \times (n-1)!} \times \frac{n! \times (n-1)!}{(2n-1)!}$$

$$\frac{a}{b} = 2n$$

$$a=2b$$

17. Question

Write the total number of terms in the expansion of $(x+a)^{100} + (x-a)^{100}$.

Answer

Given:

$$(x+a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(x+a)^{100} = \sum_{k=0}^{100} \binom{100}{k} x^{100-k} a^k$$

$$= \binom{100}{0} x^{100} a^0 + \binom{100}{1} x^{100-1} a^1 + \dots + \binom{100}{99} x^1 a^{99} + \binom{100}{100} x^0 a^{100}$$

$$\begin{aligned}
(x-a)^n &= \sum_{k=0}^{100} \binom{100}{k} x^{100-k} (-a)^k \\
&= \binom{100}{0} x^{100-0} (-a)^0 + \binom{100}{1} x^{100-1} (-a)^1 + \dots + \binom{100}{99} x^1 (-a)^{99} \\
&\quad + \binom{100}{100} x^0 (-a)^{100} \\
(x+a)^{100} &+ (x-a)^{100} \\
&= \binom{100}{0} x^{100} a^0 + \binom{100}{1} x^{100-1} a^1 + \dots + \binom{100}{99} x^1 a^{99} + \binom{100}{100} x^0 a^{100} + \\
&\quad \binom{100}{0} x^{100-0} (-a)^0 + \binom{100}{1} x^{100-1} (-a)^1 + \dots + \binom{100}{99} x^1 (-a)^{99} \\
&\quad + \binom{100}{100} x^0 (-a)^{100} \\
&= 2 \left\{ \binom{100}{0} x^{100} a^0 + \binom{100}{2} x^{100-2} a^2 + \dots + \binom{100}{100} x^0 a^{100} \right\}
\end{aligned}$$

So odd powers of x cancel each other, we are left with even powers of x or say odd terms of expansion.

So total number of terms are $T_1, T_3, \dots, T_{99}, T_{101}$

$$= \frac{1+101}{2}$$

$$= 51$$

18. Question

If $(1-x+x^2)^n = a_0 + a_1x + a_2x^2 + \dots + a_{2n}x^{2n}$, find the value of $a_0 + a_2 + a_4 + \dots + a_{2n}$.

Answer

$$(1-x+x^2)^n = a_0 + a_1x + a_2x^2 + \dots + a_{2n}x^{2n}$$

At $x = 1$

$$(1-1+1^2)^n = a_0 + a_1(1) + a_2(1)^2 + \dots + a_{2n}(1)^{2n}$$

$$a_0 + a_1 + a_2 + \dots + a_{2n} = 1 \dots (1)$$

At $x = -1$

$$(1-(-1)+(-1)^2)^n = a_0 + a_1(-1) + a_2(-1)^2 + \dots + a_{2n}(-1)^{2n}$$

$$a_0 - a_1 + a_2 - \dots + a_{2n} = 3^n \dots (2)$$

On adding eq.1 and eq.2

$$(a_0 + a_1 + a_2 + \dots + a_{2n}) + (a_0 - a_1 + a_2 - \dots + a_{2n}) = 1 + 3^n$$

$$2(a_0 + a_2 + a_4 + \dots + a_{2n}) = 1 + 3^n$$

$$a_0 + a_2 + a_4 + \dots + a_{2n} = \frac{1+3^n}{2}$$

MCQ

1. Question

Mark the correct alternative in the following :

If in the expansion of $(1+x)^{20}$, the coefficient of r th and $(r+4)$ th terms are equal, then r is equal to

- A. 7
- B. 8
- C. 9
- D. 10

Answer

Given:

$$(x+a)^n = \sum_{k=0}^n \binom{n}{k} x^k a^{n-k}$$

In r th term; $k=r-1$

& in $(r+4)$ th term ; $k=r+3$

So, the terms are;

$$\binom{20}{r-1} 1^{21-r} x^{r-1} \text{ \& } \binom{20}{r+3} 1^{17-r} x^{r+3}$$

Coefficients of both terms are equal:

$$\binom{20}{r-1} = \binom{20}{r+3}$$

$$\frac{20!}{(r-1)!(21-r)!} = \frac{20!}{(r+3)!(17-r)!}$$

$$\frac{1}{(r-1)!(21-r)(20-r)(19-r)(18-r)(17-r)!} = \frac{1}{(r+3)(r+2)(r+1)r(r-1)!(17-r)!}$$

$$\frac{1}{(21-r)(20-r)(19-r)(18-r)} = \frac{1}{(r+3)(r+2)(r+1)r}$$

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$$(r+3)(r+2)(r+1)r = (21-r)(20-r)(19-r)(18-r)$$

So, $r = (21-r)$;

$(r+1) = (20-r)$;

$(r+2) = (19-r)$;

$(r+3) = (18-r)$

We get;

$$r=9$$

2. Question

Mark the correct alternative in the following :

The term without x in the expansion of $\left(2x - \frac{1}{2x^2}\right)^{12}$ is

- A. 495

- B. -495
C. -7920
D. 7920

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^k a^{n-k}$$

$$\left(2x - \frac{1}{2x^2}\right)^{12} = \sum_{k=0}^{12} \binom{12}{k} (2x)^{12-k} \left(\frac{-1}{2x^2}\right)^k$$

$$= \sum_{k=0}^{12} \binom{12}{k} 2^{12-k} (x)^{12-k} \left(\frac{-1}{2}\right)^k x^{-2k}$$

The term without x is where :

$$x^{12-k-2k} = x^0$$

$$12-3k=0$$

$$k=4$$

for k=4; the term is :

$$= \binom{12}{4} (2x)^{12-4} \left(\frac{-1}{2x^2}\right)^4$$

$$= \frac{12!}{4! \times 8!} \times 2^8 \times x^8 \times \left(\frac{-1}{2}\right)^4 \times x^{-8}$$

$$= 7920$$

3. Question

Mark the correct alternative in the following :

If rth term in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$ is without x, then r is equal to.

- A. 8
B. 7
C. 9
D. 10

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^k a^{n-k}$$

$$\left(2x^2 - \frac{1}{x}\right)^{12} = \sum_{k=0}^{12} \binom{12}{k} (2x^2)^{12-k} \left(\frac{-1}{x}\right)^k$$

$$= \sum_{k=0}^{12} \binom{12}{k} 2^k (-1)^{12-k} x^{2(12-k)} x^{-k}$$

For term without x:

$$x^{2(12-k)-k} = x^0$$

$$24-2k-k=0$$

$$24-3k=0$$

$$k=8$$

for k= 8;

$$\text{term} = 8+1=9^{\text{th}} \text{ term}$$

4. Question

Mark the correct alternative in the following :

If in the expansion of $(a+b)^n$ and $(a+b)^{n+3}$, the ratio of the coefficients of second and third terms, and third and fourth terms respectively are equal, then n is

A. 3

B. 4

C. 5

D. 6

Answer

Given:

$$(x+a)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} x^k$$

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

$$T_2 = \binom{n}{1} a^{n-1} b^1 ; T_3 = \binom{n}{2} a^{n-2} b^2$$

$$\frac{T_1}{T_3} = \frac{\binom{n}{1} a^{n-1} b^1}{\binom{n}{2} a^{n-2} b^2}$$

$$= \frac{n! \times 2! \times (n-2)! \times a^n a^{-1} b}{n! \times 1! \times (n-1)! \times a^n a^{-2} b^2}$$

$$= \frac{2 \times (n-2)! \times a}{(n-1)(n-2)! \times b}$$

$$= \frac{2a}{(n-1)b} (1)$$

$$(a+b)^{n+3} = \sum_{k=0}^{n+3} \binom{n+3}{k} a^{n+3-k} b^k$$

$$T_3 = \binom{n+3}{2} a^{n+3-2} b^2 ; T_4 = \binom{n+3}{3} a^{n+3-3} b^3$$

$$\begin{aligned}\frac{T_3}{T_4} &= \frac{\binom{n+3}{2} a^{n+1} b^2}{\binom{n+3}{3} a^n b^3} \\&= \frac{n! \times 3! \times n! \times a^{n+1} b^2}{n! \times 2! \times (n+1)! \times a^n b^3} \\&= \frac{3! \times n! \times a}{2! \times (n+1)n! \times b} \\&= \frac{3a}{(n+1)b} \quad (2)\end{aligned}$$

Equating both equations:

$$\frac{2a}{(n-1)b} = \frac{3a}{(n+1)b}$$

$$2(n+1) = 3(n-1)$$

$$2n+2 = 3n-3$$

$$n=5$$

5. Question

Mark the correct alternative in the following :

If A and B are the sums of odd and even terms respectively in the expansion of $(x+a)^n$, then

$(x+a)^{2n} - (x-a)^{2n}$ is equal to

A. 4 (A+B)

B. 4 (A - B)

C. AB

D. 4 AB

Answer

Given:

$$\begin{aligned}(x+a)^n &= \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k \\&= \binom{n}{0} x^n a^0 + \binom{n}{1} x^{n-1} a^1 + \dots + \binom{n}{n-1} x^1 a^{n-1} + \binom{n}{n} x^0 a^n\end{aligned}$$

$$\begin{aligned}(x-a)^n &= \sum_{k=0}^n \binom{n}{k} x^{n-k} (-a)^k \\&= \binom{n}{0} x^n (-a)^0 + \binom{n}{1} x^{n-1} (-a)^1 + \dots + \binom{n}{n-1} x^1 (-a)^{n-1} + \binom{n}{n} x^0 (-a)^n\end{aligned}$$

$$\text{So, } (x+a)^n + (x-a)^n = \binom{n}{0} x^n a^0 + \binom{n}{1} x^{n-1} a^1 + \dots + \binom{n}{n-1} x^1 a^{n-1} + \binom{n}{n} x^0 a^n +$$

$$\binom{n}{0} x^n (-a)^0 + \binom{n}{1} x^{n-1} (-a)^1 + \dots + \binom{n}{n-1} x^1 (-a)^{n-1} + \binom{n}{n} x^0 (-a)^n$$

$$= 2 \left\{ \binom{n}{0} x^n a^0 + \binom{n}{2} x^{n-2} a^2 + \dots + \binom{n}{n} x^0 a^n \right\}$$

$$= 2A$$

$$\text{So, } (x+a)^n - (x-a)^n = \binom{n}{0}x^na^0 + \binom{n}{1}x^{n-1}a^1 + \dots + \binom{n}{n-1}x^1a^{n-1} + \binom{n}{n}x^0a^n -$$

$$\binom{n}{0}x^{n-0}(-a)^0 + \binom{n}{1}x^{n-1}(-a)^1 + \dots + \binom{n}{n-1}x^1(-a)^{n-1} + \binom{n}{n}x^0(-a)^n$$

$$= 2 \left\{ \binom{n}{1}x^{n-1}a^1 + \binom{n}{3}x^{n-3}a^3 + \dots + \binom{n}{n-1}x^1a^{n-1} \right\}$$

$$= 2B$$

$$(x+a)^{2n} - (x-a)^{2n} = [(x+a)^n]^2 - [(x-a)^n]^2$$

$$= \{(x+a)^n + (x-a)^n\} \times \{(x+a)^n - (x-a)^n\}$$

$$= 2A \times 2B$$

$$= 4AB$$

6. Question

Mark the correct alternative in the following :

The number of irrational terms in the expansion of $(4^{1/5} + 7^{1/10})^{45}$ is

- A. 40
- B. 5
- C. 41
- D. None of these

Answer

Given:

$$(x+a)^n = \sum_{k=0}^n \binom{n}{k} x^k a^{n-k}$$

$$(4^{1/5} + 7^{1/10})^{45} = \sum_{k=0}^{45} \binom{45}{k} (4^{1/5})^k (7^{1/10})^{45-k}$$

Total number of terms in expansion = n+1

$$= 45+1$$

$$= 46$$

irrational terms = total terms - rational terms

For rational terms; the power of each term should be integer.

Therefore, k must be divisible by 5 and (45-k) by 10.

i.e. the terms having power as multiples of 5.

i.e. 0, 5, 10, 15, 20, 25, 30, 35, 40 & 45

for k = 5, 15, 25, 35 & 45;

(45-k) do not give an integral power, so these powers have to be rejected.

Now, we have k = 0, 10, 20, 30 & 40 which give us rational terms.

$$\text{Hence, irrational terms} = 46 - 5 = 41$$

7. Question

Mark the correct alternative in the following :

The coefficient of x^{-17} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is

- A. 1365
- B. -1365
- C. 3003
- D. -3003

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\left(x^4 - \frac{1}{x^3}\right)^{15} = \sum_{k=0}^{15} \binom{15}{k} (x^4)^{15-k} \left(\frac{-1}{x^3}\right)^k$$

$$= \sum_{k=0}^{15} \binom{15}{k} x^{4(15-k)} (-1)^k x^{-3k}$$

$$x^{4(15-k)-3k} = x^{-17}$$

$$60-4k-3k = -17$$

$$-7k = -77$$

$$k = 11$$

$$= \binom{15}{11} x^{4(15-11)} (-1)^{11} x^{-3 \times 11}$$

$$= \binom{15}{11} x^{16} (-1)^{11} x^{-33}$$

$$= -\frac{15!}{11! \times 4!} x^{-17}$$

$$\text{Coefficient} = -1365$$

8. Question

Mark the correct alternative in the following :

In the expansion of $\left(x^2 - \frac{1}{3x}\right)^9$, the term without x is equal to

- A. $\frac{28}{81}$
- B. $\frac{-28}{243}$
- C. $\frac{28}{243}$

D. None of these

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\left(x^2 - \frac{1}{3x}\right)^9 = \sum_{k=0}^9 \binom{9}{k} (x^2)^{9-k} \left(\frac{-1}{3x}\right)^k$$

$$= \sum_{k=0}^9 \binom{9}{k} x^{2(9-k)} \left(\frac{-1}{3}\right)^k x^{-k}$$

$$\Rightarrow x^{2(9-k)-k} = x^0$$

$$\Rightarrow 18-2k-k = 0$$

$$\Rightarrow 18-3k = 0$$

$$\Rightarrow k = 6$$

$$= \frac{9!}{6! \times 3!} \left(\frac{-1}{3}\right)^6$$

$$= \frac{28}{243}$$

9. Question

Mark the correct alternative in the following :

If in the expansion of $(1+x)^{15}$, the coefficients of $(2r+3)^{\text{th}}$ and $(r-1)^{\text{th}}$ terms are equal, then the value of r is

A. 5

B. 6

C. 4

D. 3

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(1+x)^{15} = \sum_{k=0}^{15} \binom{15}{k} 1^{15-k} x^k$$

For $(2r+3)^{\text{th}}$ term; $k=(2r+2)$

$$\binom{15}{2r+2} 1^{15-2r-2} x^{2r+2}$$

For $(r-1)^{\text{th}}$ term; $k=r-2$

$$\binom{15}{r-2} 1^{15-r+2} x^{r-2}$$

Coefficients of both terms are equal;

$$\binom{15}{2r+2} = \binom{15}{r-2}$$

$$\Rightarrow \frac{15!}{(2r+2)!(13-2r)!} = \frac{15!}{(r-2)!(17-r)!}$$

$$\Rightarrow \frac{1}{(2r+2)(2r+1)(2r)!(13-2r)!} = \frac{1}{(r-2)(r-1)r!(17-r)!}$$

$$\Rightarrow \frac{1}{2(2r+2)(2r+1)(13-2r)!} = \frac{1}{(r-2)(r-1)(17-r)!}$$

10. Question

Mark the correct alternative in the following :

The middle term in the expansion of $\left(\frac{2x^2}{3} + \frac{3}{2x^2}\right)^{10}$ is

- A. 251
- B. 252
- C. 250
- D. None of these

Answer

Given:

$$n = 10$$

Total number of terms on expansion = $n+1 = 11$

So middle term is 6th term; i.e. $k=5$

$$= \frac{10!}{5! \times 5!} \left(\frac{2x^2}{3}\right)^5 \left(\frac{3}{2x^2}\right)^5$$

$$= 252$$

11. Question

Mark the correct alternative in the following :

If in the expansion of $\left(x^4 - \frac{1}{3}\right)^{15}$, x^{-17} occurs in r^{th} term, then

- A. $r = 10$
- B. $r = 11$
- C. $r = 12$
- D. $r = 13$

Answer

Given:

$$(x+a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\left(x^4 - \frac{1}{x^3}\right)^{15} = \sum_{k=0}^{15} {}^{(15)}_k (x^4)^{15-k} \left(\frac{-1}{x^3}\right)^k$$

$$= \sum_{k=0}^{15} {}^{(15)}_k x^{4(15-k)} (-1)^k x^{-3k}$$

$$\Rightarrow x^{4(15-k)-3k} = x^{-17}$$

$$\Rightarrow 60-4k-3k = -17$$

$$\Rightarrow -7k = -77$$

$$\Rightarrow k = 11$$

So, the term is 12th term.

12. Question

Mark the correct alternative in the following :

In the expansion of $\left(x - \frac{1}{3x^2}\right)^9$, the term independent of x is

A. T₃

B. T₄

C. T₅

D. None of these

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n {}^{(n)}_k x^{n-k} a^k$$

$$\left(x - \frac{1}{3x^2}\right)^9 = \sum_{k=0}^9 {}^{(9)}_k x^{9-k} \left(\frac{-1}{3x^2}\right)^k$$

$$= \sum_{k=0}^9 {}^{(9)}_k x^{9-k} \left(\frac{-1}{3}\right)^k x^{-2k}$$

$$\Rightarrow x^{9-k-2k} = x^0$$

$$\Rightarrow 9-3k = 0$$

$$\Rightarrow k = 3$$

So, the term is 4th term.

13. Question

Mark the correct alternative in the following :

If in the expansion of $(1 + y)^n$, the coefficients of 5th, 6th and 7th terms are in A.P., then n is equal to

A. 7, 11

B. 7, 14

C. 8, 16

D. None of these

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(1 + y)^n = \sum_{k=0}^n \binom{n}{k} 1^{n-k} y^k$$

$$T_5 = \binom{n}{4} 1^{n-4} y^4; T_6 = \binom{n}{5} 1^{n-5} y^5 \text{ \& } T_7 = \binom{n}{6} 1^{n-6} y^6$$

Since T_5 , T_6 & T_7 are in AP

$$\text{Then; } 2(T_6) = T_5 + T_7$$

$$\text{i.e. } \binom{n}{4} 1^{n-4} y^4 + \binom{n}{6} 1^{n-6} y^6 = 2 \times \binom{n}{5} 1^{n-5} y^5$$

$$\frac{n!}{4!(n-4)!} + \frac{n!}{6!(n-6)!} = 2 \times \frac{n!}{5!(n-5)!}$$

$$\frac{1}{4!(n-4)(n-5)(n-6)!} + \frac{1}{6!(n-6)!} = 2 \times \frac{1}{5!(n-5)(n-6)!}$$

$$\frac{1}{(n-4)(n-5)} + \frac{1}{30} - \frac{2}{5(n-5)} = 0$$

$$\frac{30 + (n-4)(n-5) - 12(n-4)}{30(n-4)(n-5)} = 0$$

$$\Rightarrow 30 + (n-4)(n-5) - 12(n-4) = 0$$

$$\Rightarrow 30 + n^2 - 9n + 20 - 12n + 48 = 0$$

$$\Rightarrow n^2 - 21n + 98 = 0$$

$$\Rightarrow (n-7)(n-14) = 0$$

$$\Rightarrow n = 7, 14$$

14. Question

Mark the correct alternative in the following :

In the expansion of $\left(\frac{1}{2}x^{1/3} + x^{-1/5}\right)^8$, the term independent of x is

A. T_5

B. T_6

C. T_7

D. T_8

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\left(\frac{1}{2}x^{\frac{1}{3}} + x^{-\frac{1}{5}}\right)^8 = \sum_{k=0}^8 \binom{8}{k} \left(\frac{1}{2}x^{\frac{1}{3}}\right)^{8-k} (x^{-1/5})^k$$

$$= \sum_{k=0}^8 \binom{8}{k} \left(\frac{1}{2}\right)^{8-k} x^{\frac{(8-k)}{3} - \frac{k}{5}}$$

$$\Rightarrow x^{\frac{(8-k)}{3} - \frac{k}{5}} = x^0$$

$$\Rightarrow \frac{(8-k)}{3} - \frac{k}{5} = 0$$

$$\Rightarrow \frac{5(8-k) - 3k}{15} = 0$$

$$\Rightarrow 40 - 5k - 3k = 0$$

$$\Rightarrow 40 - 8k = 0$$

$$\Rightarrow k = 5$$

So, the term is 6th term.

15. Question

Mark the correct alternative in the following :

If the sum of odd numbered terms and the sum of even numbered terms in the expansion of $(x + a)^n$ are A and B respectively, then the value of $(x^2 - a^2)^n$ is.

A. $A^2 - B^2$

B. $A^2 + B^2$

C. 4 AB

D. None of these

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$= \binom{n}{0} x^n a^0 + \binom{n}{1} x^{n-1} a^1 + \dots + \binom{n}{n-1} x^1 a^{n-1} + \binom{n}{n} x^0 a^n$$

$$= A + B$$

$$(x - a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} (-a)^k$$

$$= \binom{n}{0} x^n (-a)^0 + \binom{n}{1} x^{n-1} (-a)^1 + \dots + \binom{n}{n-1} x^1 (-a)^{n-1} + \binom{n}{n} x^0 (-a)^n$$

$$= A - B$$

$$(x^2 - a^2)^n = [(x+a)(x-a)]^n$$

$$= (x+a)^n (x-a)^n$$

$$= (A+B)(A-B)$$

$$= A^2 - B^2$$

16. Question

Mark the correct alternative in the following :

If the coefficient of x in $\left(x^2 + \frac{\lambda}{x}\right)^5$ is 270, then $\lambda =$

A. 3

B. 4

C. 5

D. None of these

Answer

Given:

$$(x+a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\left(x^2 + \frac{\lambda}{x}\right)^5 = \sum_{k=0}^5 \binom{5}{k} (x^2)^{5-k} \left(\frac{\lambda}{x}\right)^k$$

$$= \sum_{k=0}^5 \binom{5}{k} x^{2(5-k)} \lambda^k x^{-k}$$

$$\Rightarrow x^{2(5-k)-k} = x^1$$

$$\Rightarrow 10 - 2k - k = 1$$

$$\Rightarrow 9 - 3k = 0$$

$$\Rightarrow k = 3$$

for $k=3$;

$$\Rightarrow \binom{5}{3} x^{2(5-3)} \lambda^3 x^{-3}$$

$$\Rightarrow \frac{5!}{3! \times 2!} \lambda^3 = 270$$

$$\Rightarrow \lambda^3 = 27$$

$$\Rightarrow \lambda = 3$$

17. Question

Mark the correct alternative in the following :

The coefficient of x^4 in $\left(\frac{x}{2} - \frac{3}{2}\right)^{10}$ is.

A. $\frac{405}{256}$

B. $\frac{504}{259}$

C. $\frac{450}{263}$

D. None of these

Answer

Given:

$$\Rightarrow (x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\Rightarrow \left(\frac{x}{2} - \frac{3}{2}\right)^{10} = \sum_{k=0}^{10} \binom{10}{k} \left(\frac{x}{2}\right)^{10-k} \left(\frac{-3}{2}\right)^k$$

$$\Rightarrow x^{10-k} = x^4$$

$$\Rightarrow 10-k=4$$

$$\Rightarrow k=6$$

for $k=6$;

$$= \binom{10}{6} \left(\frac{x}{2}\right)^{10-6} \left(\frac{-3}{2}\right)^6$$

$$= \frac{10!}{6! \times 4!} \times \frac{(-3)^6}{2^{10}} x^4$$

So, the coefficient of $x^4 = 105 \times \frac{729}{512}$

18. Question

Mark the correct alternative in the following :

The total number of terms in the expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is

A. 202

B. 51

C. 50

D. None of these

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(x + a)^{2n} = \sum_{k=0}^{2n} \binom{2n}{k} x^{2n-k} a^k$$

$$= \binom{2n}{0} x^{2n} a^0 + \binom{2n}{1} x^{2n-1} a^1 + \dots + \binom{2n}{99} x^1 a^{99} + \binom{2n}{2n} x^0 a^{2n}$$

$$(x - a)^n = \sum_{k=0}^{2n} \binom{2n}{k} x^{2n-k} (-a)^k$$

$$= \binom{2n}{0} x^{2n-0} (-a)^0 + \binom{2n}{1} x^{2n-1} (-a)^1 + \dots + \binom{2n}{99} x^1 (-a)^{99} + \binom{2n}{2n} x^0 (-a)^{2n}$$

$$(x + a)^{100} + (x - a)^{100}$$

$$= \binom{2n}{0} x^{2n} a^0 + \binom{2n}{1} x^{2n-1} a^1 + \dots + \binom{2n}{99} x^1 a^{99} + \binom{2n}{2n} x^0 a^{2n} +$$

$$\binom{2n}{0} x^{2n-0} (-a)^0 + \binom{2n}{1} x^{2n-1} (-a)^1 + \dots + \binom{2n}{99} x^1 (-a)^{99} + \binom{2n}{2n} x^0 (-a)^{2n}$$

$$= 2 \left\{ \binom{2n}{0} x^{2n} a^0 + \binom{2n}{2} x^{2n-2} a^2 + \dots + \binom{2n}{2n} x^0 a^{2n} \right\}$$

So odd powers of x cancel each other, we are left with even powers of x or say odd terms of expansion.

So total number of terms are $T_1, T_3, \dots, T_{99}, T_{101}$

$$= \frac{1 + 101}{2}$$

$$= 51$$

19. Question

Mark the correct alternative in the following :

If T_2 / T_3 in the expansion of $(a + b)^n$ and T_3 / T_4 in the expansion of $(a + b)^{n+3}$ are equal, then $n =$

A. 3

B. 4

C. 5

D. 6

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} x^k$$

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

$$T_2 = \binom{n}{1} a^{n-1} b^1 ; T_3 = \binom{n}{2} a^{n-2} b^2$$

$$\frac{T_2}{T_3} = \frac{\binom{n}{1} a^{n-1} b^1}{\binom{n}{2} a^{n-2} b^2}$$

$$= \frac{n! \times 2! \times (n-2)! \times a^{n-1} b}{n! \times 1! \times (n-1)! \times a^{n-2} b^2}$$

$$= \frac{2 \times (n-2)! \times a}{(n-1)(n-2)! \times b}$$

$$= \frac{2a}{(n-1)b} \quad (1)$$

$$(a+b)^{n+3} = \sum_{k=0}^{n+3} \binom{n+3}{k} a^{n+3-k} b^k$$

$$T_3 = \binom{n+3}{2} a^{n+3-2} b^2; T_4 = \binom{n+3}{3} a^{n+3-3} b^3$$

$$\frac{T_3}{T_4} = \frac{\binom{n+3}{2} a^{n+1} b^2}{\binom{n+3}{3} a^n b^3}$$

$$= \frac{n! \times 3! \times n! \times a^n a^1 b^2}{n! \times 2! \times (n+1)! \times a^n b^3}$$

$$= \frac{3! \times n! \times a}{2! \times (n+1)n! \times b}$$

$$= \frac{3a}{(n+1)b} \quad (2)$$

Equating both equations:

$$\frac{2a}{(n-1)b} = \frac{3a}{(n+1)b}$$

$$\Rightarrow 2(n+1) = 3(n-1)$$

$$\Rightarrow 2n+2 = 3n-3$$

$$\Rightarrow n=5$$

20. Question

Mark the correct alternative in the following :

The coefficient of $\frac{1}{x}$ in the expansion of $(1+x)^n \left(1 + \frac{1}{x}\right)^n$ is.

A. $\frac{n!}{\{(n-1)!(n+1)!\}}$

B. $\frac{(2n)!}{[(n-1)!(n+1)!]}$

C. $\frac{(2n)!}{(2n-1)!(2n+1)!}$

D. None of these

Answer

Given:

$$(x+a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\begin{aligned}
 (1+x)^n \left(1 + \frac{1}{x}\right)^n &= \frac{(1+x)^n (1+x)^n}{x^n} \\
 &= \frac{(1+x)^{2n}}{x^n} \\
 &= \frac{1}{x^n} \sum_{k=0}^{2n} \binom{2n}{k} 1^{2n-k} x^k
 \end{aligned}$$

For x^{-1} ;

$$\Rightarrow \frac{x^k}{x^n} = x^{-1}$$

$$\Rightarrow x^{k-n} = x^{-1}$$

$$\Rightarrow k-n=-1$$

$$\Rightarrow k=n-1$$

$$\text{So, coefficient} = \frac{2n!}{(n-1)!(n+1)!}$$

21. Question

Mark the correct alternative in the following :

If the sum of the binomial coefficients of the expansion $\left(2x + \frac{1}{x}\right)^n$ is equal to 256, then the term independent of x is

- A. 1120
- B. 1020
- C. 512
- D. None of these

Answer

Given:

$$\text{Sum of binomial coefficients} = 2^n$$

$$= 256$$

$$\Rightarrow 2^n = 2^8$$

$$\Rightarrow n = 8$$

$$\text{so total terms} = n+1$$

$$= 9$$

$$\text{Middle term} = 5^{\text{th}} \text{ term; i.e. } k=4$$

$$\text{So, term independent of } x = \binom{8}{4} (2x)^{8-4} \left(\frac{1}{x}\right)^4$$

$$= \frac{8!}{4! \times 4!} 2^4$$

$$= 1120$$

22. Question

Mark the correct alternative in the following :

If the fifth term of the expansion $\left(a^{2/3} + a^{-1}\right)^n$ does not contain 'a'. Then n is equal to

- A. 2
- B. 5
- C. 10
- D. None of these

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\left(a^{2/3} + a^{-1}\right)^n = \sum_{k=0}^n \binom{n}{k} \left(a^{2/3}\right)^{n-k} (a^{-1})^k$$

$$= \sum_{k=0}^n \binom{n}{k} a^{\frac{2(n-k)}{3} - k}$$

Term 5 ; i.e. k=4:

$$a^{\frac{2(n-k)}{3} - k} = a^0$$

$$\frac{2(n-4) - 3(4)}{3} = 0$$

$$\Rightarrow 2n - 8 - 12 = 0$$

$$\Rightarrow n = 10$$

23. Question

Mark the correct alternative in the following :

The coefficient of x^{-3} in the expansion of $\left(x - \frac{m}{x}\right)^{11}$ is

- A. $-924m^7$
- B. $-792m^5$
- C. $-792m^6$
- D. $-330m^7$

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\left(x - \frac{m}{x}\right)^{11} = \sum_{k=0}^{11} \binom{11}{k} x^{11-k} \left(\frac{-m}{x}\right)^k$$

$$= \sum_{k=0}^{11} \binom{11}{k} x^{11-k} (-m)^k x^{-k}$$

$$\Rightarrow x^{11-k-k} = x^{-3}$$

$$\Rightarrow 11-2k=-3$$

$$\Rightarrow 14-2k=0$$

$$\Rightarrow k=7$$

for $k=7$; coefficient is:

$$= \frac{11!}{7! \times 4!} (-m)^7$$

$$= -330m^7$$

24. Question

Mark the correct alternative in the following :

The coefficient of the term independent of x in the expansion of $\left(ax + \frac{b}{x}\right)^{14}$ is

A. $14!a^7b^7$

B. $\frac{14!}{7!}a^7b^7$

C. $\frac{14!}{(7!)^2}a^7b^7$

D. $\frac{14!}{(7!)^3}a^7b^7$

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\left(ax + \frac{b}{x}\right)^{14} = \sum_{k=0}^{14} \binom{14}{k} (ax)^{14-k} \left(\frac{b}{x}\right)^k$$

$$= \sum_{k=0}^{14} \binom{14}{k} a^{14-k} b^k x^{14-k-k}$$

$$x^{14-k-k} = x^0$$

$$14-2k=0$$

$$k = 7$$

So, the coefficient is:

$$= \frac{14!}{7! \times 7!} a^7 b^7$$

25. Question

Mark the correct alternative in the following :

The coefficient of x^5 in the expansion of $(1+x)^{21} + (1+x)^{22} + \dots + (1+x)^{30}$ is.

A. ${}^{51}C_5$

B. 9C_5

C. ${}^{31}C_6 - {}^{21}C_6$

D. ${}^{30}C_5 + {}^{20}C_5$

Answer

Given:

$$(x+a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(1+x)^{21} + (1+x)^{22} + \dots + (1+x)^{30}$$

Coefficient of x^5 in any expansion = $\binom{n}{5} 1^{n-5} x^5$; i.e. nC_5

So, coefficient of x^5 in above expansion = ${}^{21}C_5 + {}^{22}C_5 + {}^{23}C_5 + \dots + {}^{30}C_5$

26. Question

Mark the correct alternative in the following :

The coefficient of $x^8 y^{10}$ in the expansion $(x+y)^{18}$ is.

A. ${}^{18}C_8$

B. ${}^{18}P_{10}$

C. 2^{18}

D. None of these

Answer

Given:

$$(x+y)^{18} = \sum_{k=0}^{18} \binom{18}{k} x^{18-k} y^k$$

For $x^8 y^{10}$; $k=10$

So coefficient is ${}^{18}C_{10}$

Also ${}^{18}C_{10} = {}^{18}C_8$

So coefficient = $^{18}C_8$

27. Question

Mark the correct alternative in the following :

If the coefficients of the $(n+1)^{\text{th}}$ term and the $(n+3)^{\text{th}}$ term in the expansion of $(1+x)^{20}$ are equal, then the value of n is

- A. 10
- B. 8
- C. 9
- D. None of these

Answer

Given:

$$(x+a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(1+x)^{20} = \sum_{k=0}^{20} \binom{20}{k} 1^{20-k} x^k$$

For nth term ; $k=n-1$

So for $(n+1)^{\text{th}}$ term ; $k= n$

& for $(n+3)^{\text{th}}$ term ; $k = n+2$

Coefficients for the above terms are equal;

$$\frac{20!}{n!(20-n)!} = \frac{20!}{(n+2)!(18-n)!}$$

$$\frac{1}{n! \times (20-n)(19-n)(18-n)!} = \frac{1}{(n+2)(n+1)n! \times (18-n)!}$$

$$(20-n)(19-n) = (n+2)(n+1)$$

$$380-39n+n^2 = n^2+3n+2$$

$$42n-378=0$$

$$n=9$$

28. Question

Mark the correct alternative in the following :

If the coefficients of 2^{nd} , 3^{rd} and 4^{th} terms in the expansion of $(1+x)^n$, $n \in \mathbb{N}$ are in A.P., then n =

- A. 7
- B. 14
- C. 2
- D. None of these

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(1 + x)^n = \sum_{k=0}^n \binom{n}{k} 1^{n-k} x^k$$

$$T_2 = \binom{n}{1} 1^{n-1} x^1; T_3 = \binom{n}{2} 1^{n-2} x^2 \text{ \& } T_4 = \binom{n}{3} 1^{n-3} x^3$$

Since T_2 , T_3 & T_4 are in AP

$$\text{Then; } 2(T_3) = T_2 + T_4$$

$$\text{i.e. } \binom{n}{1} 1^{n-1} x^1 + \binom{n}{3} 1^{n-3} x^3 = 2 \times \binom{n}{2} 1^{n-2} x^2$$

$$\frac{n!}{1!(n-1)!} + \frac{n!}{3!(n-3)!} = 2 \times \frac{n!}{2!(n-2)!}$$

$$\frac{1}{1!(n-1)(n-2)(n-3)!} + \frac{1}{3!(n-3)!} = 2 \times \frac{1}{2!(n-2)(n-3)!}$$

$$\frac{1}{(n-1)(n-2)} + \frac{1}{6} = \frac{1}{(n-2)}$$

$$\frac{1}{(n-1)(n-2)} + \frac{1}{6} - \frac{1}{(n-2)} = 0$$

$$\frac{6}{6(n-1)(n-2)} + \frac{(n-1)(n-2)}{6(n-1)(n-2)} - \frac{6(n-1)}{6(n-1)(n-2)} = 0$$

$$(n-1)(n-2) - 6(n-1) + 6 = 0$$

$$n^2 - 3n + 2 - 6n + 6 + 6 = 0$$

$$n^2 - 9n + 14 = 0$$

$$(n-2)(n-7) = 0$$

$$n = 2, 7$$

$n=2$ rejected for term 3rd

So $n=7$

29. Question

Mark the correct alternative in the following :

The middle term in the expansion of $\left(\frac{2x}{3} - \frac{3}{2x^2}\right)^{2n}$ is.

A. ${}^{2n}C_n$

B. $(-1)^n {}^{2n}C_n X^{-n}$

C. ${}^{2n}C_n X^{-n}$

D. None of these

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\left(\frac{2x}{3} - \frac{3}{2x^2}\right)^{2n} = \sum_{k=0}^{2n} \binom{2n}{k} \left(\frac{2x}{3}\right)^{2n-k} \left(\frac{-3}{2x^2}\right)^k$$

For middle term,

$$T_{n+1} = \binom{2n}{n} \left(\frac{2x}{3}\right)^{2n-k} \left(\frac{-3}{2x^2}\right)^k$$

$$= \binom{2n}{n} \left(\frac{2}{3}\right)^n \left(\frac{3}{2}\right)^n (-1)^n x^{n-n} x^{-2n}$$

$$= \binom{2n}{n} (-1)^n x^{-n}$$

$$= (-1)^n {}^{2n}C_n x^{-n}$$

30. Question

Mark the correct alternative in the following :

If r^{th} term is the middle term in the expansion of $\left(x^2 - \frac{1}{2x}\right)^{20}$, then $(r+3)^{\text{th}}$ term is

A. ${}^{20}C_{14} \left(\frac{x}{2^{14}}\right)$

B. ${}^{20}C_{12} x^2 2^{-12}$

C. $-{}^{20}C_7 x \cdot 2^{-13}$

D. None of these

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\left(x^2 - \frac{1}{2x}\right)^{20} = \sum_{k=0}^{20} \binom{20}{k} (x^2)^{20-k} \left(\frac{-1}{2x}\right)^k$$

$$\text{Total terms} = n+1 = 21$$

$$\text{Mid term} = 21/2 = 11^{\text{th}} \text{ term}$$

For $k=10$, it is r^{th} term.

$$\text{So } (r+3)^{\text{th}} \text{ term} = 11^{\text{th}} \text{ term}$$

$$k=13$$

$$T_{14} = \binom{20}{13} (x^2)^{20-13} \left(\frac{-1}{2x}\right)^{13}$$

$$= \binom{20}{13} (x^2)^7 \left(\frac{-1}{2}\right)^{13} x^{-13}$$

$$= \binom{20}{13} \left(\frac{-1}{2}\right)^{13} x^{-13} x^{14}$$

$$= {}^{20}C_{13} x \cdot 2^{-13}$$

$$= {}^{20}C_7 x \cdot 2^{-13}$$

31. Question

Mark the correct alternative in the following :

The number of terms with integral coefficients in the expansion of $(17^{1/3} + 35^{1/2}x)^{600}$ is

- A. 2n
- B. 50
- C. 150
- D. 101

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\left(17^{1/3} + 35^{1/2}x\right)^{600} = \sum_{k=0}^{600} \binom{600}{k} (17^{1/3})^{600-k} (35^{1/2}x)^k$$

For integral coefficients; (600-k) should be divisible by 3 and k should be divisible by 2.

It indicates that k should be multiple of 6.

So, the values of k would be = 6, 12, 18, ..., 594, 600

32. Question

Mark the correct alternative in the following :

Constant term in the expansion of $\left(x - \frac{1}{x}\right)^{10}$ is

- A. 152
- B. -152
- C. -252
- D. 252

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$\left(x - \frac{1}{x}\right)^{10} = \sum_{k=0}^{10} \binom{10}{k} x^{10-k} \left(\frac{-1}{x}\right)^k$$

$$= \sum_{k=0}^{10} \binom{10}{k} x^{10-k} (-1)^k x^{-k}$$

For constant term,

$$x^{10-k-k} = x^0$$

$$10-2k = 0$$

$$k = 5$$

$$\text{Term} = \binom{10}{5} x^{10-5} (-1)^5 x^{-5}$$

$$= -252$$

33. Question

Mark the correct alternative in the following :

If the coefficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ are the same, then the value of a is.

A. $-\frac{7}{9}$

B. $-\frac{9}{7}$

C. $\frac{7}{9}$

D. $\frac{9}{7}$

Answer

Given:

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} a^k$$

$$(3 + ax)^9 = \sum_{k=0}^9 \binom{9}{k} 3^{9-k} (ax)^k$$

Coefficient of x^2 ; $k=2$

$$= \binom{9}{2} 3^{9-2} a^2$$

$$= \binom{9}{2} 3^7 a^2 (1)$$

Coefficient of x^3 ; $k=3$

$$= \binom{9}{3} 3^{9-3} a^3$$

$$= \binom{9}{3} 3^6 a^3 (2)$$

Equate both equations;

$$\binom{9}{2} 3^7 a^2 = \binom{9}{3} 3^6 a^3$$

$$\frac{9!}{2! \times 7!} \times 3 = \frac{9!}{3! \times 6!} a$$

$$\frac{1}{7} \times 3 = \frac{1}{3} a$$

$$\frac{9}{7} = a$$