

# Chapter 13

## Exponents and Powers

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### Introduction to Exponents and Powers

**Exponents and powers help in writing the large numbers in a shorter form** which we can read, understand and compare easily. A number placed in a superscript position to the right of another number or variable indicate repeated multiplication.

The short notation  $10^4$  stands for the product  $10 \times 10 \times 10 \times 10$ . Here '10' is called the base and '4' the exponent. The number  $10^4$  is read as 10 raised to the power of 4 or simply as the fourth power of 10.  $10^4$  is called the exponential form of 10,000.

**$a^b$**

**$a$  raised to the power  $b$**

For example:

$a^2$  indicates  $a \times a$

$a^3$  indicates  $a \times a \times a$

In expression  $2^5$ , 2 is called the base and 5 is called the exponent or power.

$$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

i) Express 256 as a power 2.

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8 = 256$$

|   |     |
|---|-----|
| 2 | 256 |
| 2 | 128 |
| 2 | 64  |
| 2 | 32  |
| 2 | 16  |
| 2 | 8   |
| 2 | 4   |
| 2 | 2   |
|   | 1   |

ii) Which one is greater  $6^2$  or  $2^6$ ?

$$6^2 = 6 \times 6 = 36$$

$$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$$

So,  $2^6$  is greater than  $6^2$ .

## Laws of exponents

### Laws of Exponents

#### (a) First Law

If  $a$  is any non-zero integer (base) and  $m, n$  are integers (powers),

$$\text{then } a^m a^n = a^m \times a^n = a^{m+n}$$

for example,

$$(i) (2)^5 \times (2)^{10} = (2)^{5+10} = (2)^{15}$$

(ii) Multiply  $3^2$  and  $3^4$ .

$$\Rightarrow 3^2 \times 3^4$$

$$\Rightarrow 3^{2+4}$$

$$\Rightarrow 3^6$$

$$\Rightarrow 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$\Rightarrow 729$$

(iii) Multiply  $-3^2$  and  $-3^3$ .

$$\Rightarrow (-3)^2 \times (-3)^3$$

$$\Rightarrow (-3)^{2+3}$$

$$\Rightarrow (-3)^5$$

$$\Rightarrow (-3) \times (-3) \times (-3) \times (-3) \times (-3)$$

$$\Rightarrow -243$$

#### (b) Second Law

If  $a$  is any non-zero integer (base) and  $m, n$  are whole numbers (powers).

If  $m > n$ , then  $a^m \div a^n = a^{m-n}$ .

If  $m < n$ , then  $a^m \div a^n = \frac{1}{(a)^{n-m}}$ .

For example,

(i)  $3^6 \div 3^5 = 3^{6-5} = 3^1$

(ii) Divide  $3^6$  by  $3^2$ .

$$\Rightarrow 3^6 \div 3^2$$

$$\Rightarrow 3^{6-2}$$

$$\Rightarrow 3^4$$

$$\Rightarrow 3 \times 3 \times 3 \times 3$$

$$\Rightarrow 81$$

(c) Third Law

If  $a$  and  $b$  are two non-zero integers (base) and  $m$  is a whole number (power), then  $(ab)^m = (a^m \times b^m)$

For example,

(i)  $(5^2)^3 = 5^{2 \times 3} = 5^6$

(ii) Calculate  $(3^3)^2$ .

$$\Rightarrow 3^3 \times 3^3$$

$$\Rightarrow 3^3 + 3$$

$$\Rightarrow 3^6$$

$$\Rightarrow 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$\Rightarrow 729$$

iii) Which one is greater  $(5^2) \times 3$  or  $(5^2)^3$ ?

$(5^2) \times 3$  means 52 is multiplied by 3

i.e.,  $5 \times 5 \times 3 = 75$

but  $(5^2)^3$  means  $5^2$  is multiplied by itself 3 times

$$5^2 \times 5^2 \times 5^2$$

$$5^{2+2+2}$$

$$5^6 = 15625$$

So,  $(5^2)^3$  is greater than  $(5^2) \times 3$ .

(d) Fourth Law

If  $a$  and  $b$  are two non-zero integers (base) and  $m$  is a whole number (power), then  $a^m \times b^m = (ab)^m$ .

**For example,**

$$(i) (3 \times 5)^4 = 3^4 \times 5^4$$

$$= (3 \times 3 \times 3 \times 3) \times (5 \times 5 \times 5 \times 5)$$

$$= 81 \times 625$$

$$= 50625$$

(ii) Multiply  $3^3$  and  $4^3$ .

$$\Rightarrow 3^3 \times 4^3$$

$$\Rightarrow (3 \times 4)^3$$

$$\Rightarrow 12^3$$

$$\Rightarrow 12 \times 12 \times 12$$

$$\Rightarrow 1728$$

(e) Fifth Law

If a and b are two non-zero integer (base) and m is a whole number (power),

$$\text{then } a^m \div b^m = (ab)^m = \frac{a^m}{b^m} = (a \div b)^m$$

For example,

$$(i) \left(\frac{2}{3}\right)^4$$

$$= 2^4 \div 3^4$$

$$= \frac{16}{81}$$

(ii) Divide  $3^3$  by  $4^3$ .

$$\Rightarrow 3^3 \div 4^3$$

$$\Rightarrow (3)^3 \div (4)^3$$

$$\Rightarrow (34)^3$$

(f) Sixth Law

If a is an integer (base) then  $a^0 = 1$

For example,

(i) Divide  $3^4$  by  $3^4$ .

$$\Rightarrow 3^4 \div 3^4$$

$$\Rightarrow \frac{3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3} = 1$$

$$\Rightarrow \frac{3^4}{3^4} = 1$$

$$\Rightarrow 3^{4-4} = 1$$

$$\Rightarrow 3^0 = 1$$

**So, we can say that any number (except 0) raised to the power (or exponent) 0 is 1.**

Simplify and write the answer in the exponential form

i)  $(3^8 \div 3^5)^5 \div 3^5$

$$(3^{8-5})^5 \div 3^5 \quad (\text{Using the law } a^m \div a^n = a^{m-n})$$

$$= (3^3)^5 \div 3^5 \quad (\text{Using the law } (a^m)^n = a^{mn})$$

$$= (3)^{15} \div 3^5$$

$$= 3^{(15-5)} \quad (\text{Using the law } a^m \div a^n = a^{m-n})$$

$$= 3^{10}$$

## Expressing Numbers in Standard Form

Decimal Number system

Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10.

For example:

1) 5678925 can be written as,

$$56789 = 50000 + 6000 + 700 + 80 + 9$$

$$= 5 \times 10^4 + 6 \times 10^3 + 7 \times 10^2 + 8 \times 10^1 + 9 \times 10^0$$

2) 14678925 can be written as,

$$146789 = 100000 + 40000 + 6000 + 700 + 80 + 9$$

$$= 1 \times 10^5 + 4 \times 10^4 + 6 \times 10^3 + 7 \times 10^2 + 8 \times 10^1 + 9 \times 10^0$$

Here, we can see the exponents in first example starts from 4 and go on decreasing till 0.

Similarly, we can see the exponents in first example starts from 5 and go on decreasing till 0.

### Expressing numbers in Standard Form

Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10. Such a form of a number is called its standard form.

A number is said to be in the Standard Form, if it is expressed as the product of a number between 1 and 10 and the integral power of 10.

For example,

$$153,600,000,000 = 1.536 \times 10^{11}$$

Decimal is moved 11 places towards left, therefore the power is positive.

### Large numbers in Standard Form

Express the following numbers in standard form.

i) 14320000000

ii) 2050000000

$$\text{i) } 1432000000 = 1432 \times 10000000$$

$$= 1.432 \times 1000 \times 10000000$$

$$= 1.432 \times 10^3 \times 10^7$$

$$= 1.432 \times 10^{(3+7)}$$

$$= 1.432 \times 10^{10}$$



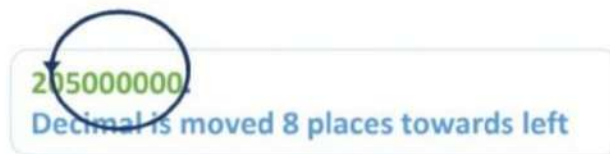
$$\text{ii) } 205000000 = 205 \times 1000000$$

$$= 2.05 \times 100 \times 1000000$$

$$= 2.05 \times 10^2 \times 10^6$$

$$= 2.05 \times 10^{(2+6)}$$

$$= 2.05 \times 10^8$$



Express 15360000000 in standard form.

$$15360000000 = 1536 \times 10000000$$

$$= 1.536 \times 1000 \times 10000000$$

$$= 1.536 \times 10^3 \times 10^7$$

$$= 1.536 \times 10^{(3+7)}$$

$$= 1.536 \times 10^{10}$$

