

Fundamental Concepts (Including Fundamental Operations)

POINTS TO REMEMBER

1. **Constants and Variables** : The numbers which has fixed value is called constant and same at English alphabet which can be assigned any value according to the requirement is called variables.
2. **Term** : A term is a number, (constant), a variable or a combination of numbers and variables.
3. **Algebraic Expression** : An algebraic expression is a collection of one or more terms, which are separated from each other by addition (+) or subtraction (-) signs.
4. **Types of algebraic expressions** :
 - (i) Monomial : It has only one term
 - (ii) Binomial : It has two terms
 - (iii) Trinomial : It has three terms
 - (iv) Multinomial : It has more than three terms
 - (v) Polynomial : It has two or more than two terms.

Note : An expression of the type $x^{\frac{2}{5}}$ does not form a monomial unless JC is not equal to zero.
5. **Product**: When two or more quantities are multiplied together, the result is called their product.
6. **Factors** : Each of the quantities (numbers or variables) multiplied together to form a term is called a factor of the given term.
7. **Co-efficient**: In a monomial, any factor or group of factors of a term is called the co-efficient of the remaining part of the monomial.
8. **Degree of a monomial**: The degree of a monomial is the exponent of its variable or the sum of the exponents of its variables.
9. **Degree of a polynomial**: The degree of a polynomial is the degree of its highest degree term.
10. **Like and unlike terms** : Terms having the same literal co-efficients or alphabetic letters are called like terms ; whereas the terms with different literal co-efficients are called unlike terms.
11. **Addition and subtraction** : Addition and subtraction of only like terms is possible by adding or subtracting the numerical co-efficients.
12. **Multiplication and division** :
 - (A) **Multiplication** :
 - (i) Multiplications of monomials.
 - (a) Multiply the numerical co-efficient together
 - (ii) Multiply the literal co-efficients separately together.
 - (iii) Combine the like terms.
 - (B) **Division** :
 - (i) Dividing a polynomial by a monomial Divide each term of the polynomial by monomial and simplify each fractions.

(ii) While dividing one polynomial by another polynomial ; arrange the terms of both the dividend and the divisor both in descending or in ascending order of their powers and then divide.

SOME IMPORTANT POINTS

TYPES OF BRACKETS:

The name of different types of brackets and the order in which they are removed is shown below:

- (a) _____ ; Bar (Vinculum) bracket
- (b) () ; Circular bracket .
- (c) { } ; Curly bracket and then
- (d) [] ; square bracket

EXERCISE 11 (A)

Question 1.

Separate constant terms and variable terms from the following :

$$(i) \ 8, x, 6xy, 6 + x, -5xy^2, 15az^2, \frac{32z}{xy}, \frac{y^2}{3x}$$

Solution:

Constant is only 8 others are variables

Question 2.

Constant is only 8 others are variables

(i) $2x \div 15$

(ii) $ax + 9$

(iii) $3x^2 \times 5x$

(iv) $5 + 2a - 3b$

(v) $2y - \frac{7}{3}z \div x$

(vi) $3p \times q \div z$

(vii) $12z \div 5x + 4$

(viii) $12 - 5z - 4$

(ix) $a^3 - 3ab^2 \times c$

Answer:

$$(i) 2x \div 15 = \frac{2x}{15}$$

It is a monomial as it has one term.

(ii) $ax + 9$: It is binomial

($\because$ It has two terms)

(iii) $3x^2 \times 5x = 15x^3$: It is monomial

($\because$ It has one term)

(iv) $5 + 2a - 3b$: It is trinomial

($\because$ It has three terms)

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$$(v) 2y - \frac{7}{3}z \div x = 2y - \frac{7z}{3x} : \text{It is binomial}$$

($\because$ It has two terms)

$$(vi) 3p \times q \div z = \frac{3pq}{z} : \text{It is monomial}$$

($\because$ It has one term)

$$(vii) 12z \div 5x + 4 = \frac{12z}{5x} + 4 : \text{It is binomial}$$

($\because$ It has two terms)

(viii) $12 - 5z - 4 = 8 - 5z$: It is binomial

($\because$ It has two terms)

$$(ix) a^3 - 3ab^2 \times c = a^3 - 3ab^2c : \text{It is binomial}$$

($\because$ It has two terms)

Question 3.

Write the coefficient of:

(i) xy in $-3axy$

(ii) z^2 in p^2yz^2

(iii) mn in $-mn$

(iv) 15 in $-15p^2$

Solution:

(i) Co-efficient of xy in $-3axy = -3a$

(ii) Co-efficient of z^2 in $p^2yz^2 = p^2y$

- (iii) Co-efficient of mn in $-mn = -1$
- (iv) Co-efficient of 15 in $-15p^2$ is $-p^2$

Question 4.

For each of the following monomials, write its degree :

- (i) $7y$
- (ii) $-x^2y$
- (iii) xy^2z
- (iv) $-9y^2z^3$
- (v) $3m^3n^4$
- (vi) $-2p^2q^3r^4$

Solution:

- (i) Degree of $7y = 1$
- (ii) Degree of $-x^2y = 2+1=3$
- (iii) Degree of $xy^2z = 1 + 2 + 1 = 4$
- (iv) Degree of $-9y^2z^3 = 2 + 3 = 5$
- (v) Degree of $3m^3n^4 = 3 + 4 = 7$
- (vi) Degree of $-2p^2q^3r^4 = 2 + 3 + 4 = 9$

Question 5.

Write the degree of each of the following polynomials :

- (i) $3y^3 - x^2y^2 + 4x$
- (ii) $p^3q^2 - 6p^2q^5 + p^4q^4$
- (iii) $-8mn^6 + 5m^3n$
- (iv) $7 - 3x^2y + y^2$
- (v) $3x - 15$
- (vi) $2y^2z + 9yz^3$

Solution:

- (i) The degree of $3y^3 - x^2y^2 + 4x$ is 4 as x^2y^2 is the term which has highest degree.
- (ii) The degree of $p^3q^2 - 6p^2q^5 + p^4q^4$ is 8 as p^4q^4 is the term which has highest degree.
- (iii) The degree of $-8mn^6 + 5m^3n$ is 7 as $-8mn^6$ is the term which has the highest degree.
- (iv) The degree of $7 - 3x^2y + y^2$ is 3 as $-3x^2y$ is the term which has the highest degree.
- (v) The degree of $3x - 15$ is 1 as $3x$ is the term which is highest degree.
- (vi) The degree of $2y^2z + 9yz^3$ is 4 as $9yz^3$ has the highest degree.

Question 6.

Group the like term together :

- (i) $9x^2$, xy , $-3x^2$, x^2 and $-2xy$
- (ii) ab , $-a^2b$, $-3ab$, $5a^2b$ and $-8a^2b$
- (iii) $7p$, $8pq$, $-5pq - 2p$ and $3p$

Solution:

- (i) $9x^2$, $-3x^2$ and x^2 are like terms
 xy and $-2xy$ are like terms
(ii) ab , $-3ab$, are like terms,
 $-a^2b$, $5a^2b$, $-8a^2b$ are like terms
(iii) $7p$, $-2p$ and $3p$ are like terms,
 $8pq$, $-5pq$ are like terms.

Question 7.

Write numerical co-efficient of each of the followings :

- (i) y
(ii) $-y$
(iii) $2x^2y$
(iv) $-8xy^3$
(v) $3py^2$
(vi) $-9a^2b^3$

Solution:

- (i) Co-efficient of $y = 1$
(ii) Co-efficient of $-y = -1$
(iii) Co-efficient of $2x^2y$ is $= 2$
(iv) Co-efficient of $-8xy^3$ is $= -8$
(v) Co-efficient of lpy^2 is $= 3$
(vi) Co-efficient of $-9a^2b^3$ is $= -9$

Question 8.

In $-5x^3y^2z^4$; write the coefficient of:

- (i) z^2
(ii) y^2
(iii) yz^2
(iv) x^3y
(v) $-xy^2$
(vi) $-5xy^2z$

Also, write the degree of the given algebraic expression.

Solution:

$-5x^3y^2z^4$

- (i) Co-efficient of z^2 is $-5x^3y^2z^2$
(ii) Co-efficient of y^2 is $-5x^3z^4$
(iii) Co-efficient of yz^2 is $-5x^3yz^2$
(iv) Co-efficient of x^3y is $-5yz^4$
(v) Co-efficient of $-xy^2$ is $5x^2z^4$
(vi) Co-efficient of $-5xy^2z$ is x^2z^3

Degree of the given expression is $3 + 2 + 4 = 9$

EXERCISE 11 (B)

Question 1.

Fill in the blanks :

(i) $8x + 5x = \dots\dots\dots$

(ii) $8x - 5x = \dots\dots\dots$

(iii) $6xy^2 + 9xy^2 = \dots\dots\dots$

(iv) $6xy^2 - 9xy^2 = \dots\dots\dots$

(v) The sum of $8a$, $6a$ and $5b = \dots\dots\dots$

(vi) The addition of 5 , $7xy$, 6 and $3xy = \dots\dots\dots$

(vii) $4a + 3b - 7a + 4b = \dots\dots\dots$

(viii) $-15x + 13x + 8 = \dots\dots\dots$

(ix) $6x^2y + 13xy^2 - 4x^2y + 2xy^2 = \dots\dots\dots$

(x) $16x^2 - 9x^2 =$ and $25xy^2 - 17xy^2 = \dots\dots\dots$

Solution :

(i) $8x + 5x = 13x$

(ii) $8x - 5x = 3x$

(iii) $6xy^2 + 9xy^2 = 15xy^2$

(iv) $6xy^2 - 9xy^2 = -3xy^2$

(v) The sum of $8a$, $6a$ and $5b$
 $= 8a + 6a + 5b = 14a + 5b$

(vi) The addition of 5 , $7xy$, 6 and $3xy$
 $= 5 + 6 + 7xy + 3xy = 11 + 10xy$

(vii) $4a + 3b - 7a + 4b$
 $= 4a - 7a + 3b + 4b = -3a + 7b = 7b - 3a$

(viii) $-15x + 13x + 8$
 $= -2x + 8 = 8 - 2x$

(ix) $6x^2y + 13xy^2 - 4x^2y + 2xy^2$
 $= 6x^2y - 4x^2y + 13xy^2 + 2xy^2 = 2x^2y + 15xy^2$

(x) $16x^2 - 9x^2 = 7x^2$ and
 $25xy^2 - 17xy^2 = 8xy^2$

Question 2.

Add :

(i)- $9x$, $3x$ and $4x$

(ii) $23y^2$, $8y^2$ and $-12y^2$

(iii) $18pq - 15pq$ and $3pq$

Solution:

$$(i) - 9x + 3x + 4x$$

$$= - 9x + 7x = - 2x$$

$$(ii) 23y^2 + 8y^2 - 12y^2$$

$$= 31y^2 - 12y^2 = 19y^2$$

$$(iii) 18pq - 15pq + 3pq$$

$$= 18pq + 3pq - 15pq = 21pq - 15pq = 6pq$$

Question 3.

Simplify :

(i) $3m + 12m - 5m$

(ii) $7n^2 - 9n^2 + 3n^2$

(iii) $25zy - 8zy - 6zy$

(iv) $-5ax^2 + 7ax^2 - 12ax^2$

(v) $-16am + 4mx + 4am - 15mx + 5am$

Solution:

$$(i) 3m + 12m - 5m = 15m - 5m = 10m$$

$$(ii) 7n^2 - 9n^2 + 3n^2$$

$$= 7n^2 + 3n^2 - 9n^2 = 10n^2 - 9n^2 = n^2$$

$$(iii) 25zy - 8zy - 6zy$$

$$= 25zy - 14zy = 11zy$$

$$(iv) -5ax^2 + 7ax^2 - 12ax^2$$

$$= -5ax^2 - 12ax^2 + 7ax^2$$

$$= -17ax^2 + 7ax^2 = -10ax^2$$

$$(v) -16am + 4mx + 4am - 15mx + 5am$$

$$= -16am + 4am + 5am + 4mx - 15mx$$

$$= -16am + 9am + 4mx - 15mx = -7am - 11mx$$

Question 4.

Add :

(i) $a + b$ and $2a + 3b$

(ii) $2x + y$ and $3x - 4y$

(iii) $-3a + 2b$ and $3a + b$

(iv) $4 + x$, $5 - 2x$ and $6x$

Solution:

(i) $a + b$ and $2a + 3b$ (ii) $2x + y$ and $3x - 4y$

(iii) $-3a + 2b$ and $3a + b$ (iv) $4 + x$, $5 - 2x$ and $6x$

(i) $a + b + 2a + 3b$

$$= a + 2a + b + 3b = 3a + 4b$$

(ii) $2x + y + 3x - 4y$

$$= 2x + 3x + y - 4y = 5x - 3y$$

(iii) $-3a + 2b + 3a + b$

$$= -3a + 3a + 2b + b = 0 + 3b = 3b$$

(iv) $4 + x + 5 - 2x + 6x$

$$= x - 2x + 6x + 4 + 5$$

$$= 7x - 2x + 9 = 5x + 9$$

Question 5.

Find the sum of:

(i) $3x + 8y + 7z$, $6y + 4z - 2x$ and $3y - 4x + 6z$

(ii) $3a + 5b + 2c$, $2a + 3b - c$ and $a + b + c$.

(iii) $4x^2 + 8xy - 2y^2$ and $8xy - 5y^2 + x^2$

(iv) $9x^2 - 6x + 7$, $5 - 4x$ and $6 - 3x^2$

(v) $5x^2 - 2xy + 3y^2$ and $-2x^2 + 5xy + 9y^2$

and $3x^2 - xy - 4y^2$

(vi) $a^2 + b^2 + 2ab$, $2b^2 + c^2 + 2bc$

and $4c^2 - a^2 + 2ac$

(vii) $9ax - 6bx + 8$, $4ax + 8bx - 7$

and $-6ax - 4bx - 3$

(viii) $abc + 2ba + 3ac$, $4ca - 4ab + 2bca$

and $2ab - 3abc - 6ac$

(ix) $4a^2 + 5b^2 - 6ab$, $3ab$, $6a^2 - 2b^2$

and $4b^2 - 5ab$

(x) $x^2 + x - 2$, $2x - 3x^2 + 5$ and $2x^2 - 5x + 7$

(xi) $4x^3 + 2x^2 - x + 1$, $2x^3 - 5x^2 - 3x + 6$, $x^2 + 8$ and $5x^3 - 7x$

Solution:

$$\begin{aligned} (i) \quad & 3x + 8y + 7z + 6y + 4z - 2x \\ & \quad \quad \quad + 3y - 4x + 6z \\ & = 3x - 2x - 4x + 8y + 6y + 3y + 7z + 4z + 6z \\ & = 3x - 6x + 17y + 17z \\ & = -3x + 17y + 17z \end{aligned}$$

$$\begin{aligned} (ii) \quad & 3a + 5b + 2c + 2a + 3b - c + a + b + c \\ & = 3a + 2a + a + 5b + 3b + b + 2c - c + c \\ & = 6a + 9b + 3c - c = 6a + 9b + 2c \end{aligned}$$

$$\begin{aligned} (iii) \quad & 4x^2 + 8xy - 2y^2 \text{ and } 8xy - 5y^2 + x^2 \\ & 4x^2 + 8xy - 2y^2 + 8xy - 5y^2 + x^2 \\ & = 4x^2 + x^2 + 8xy + 8xy - 2y^2 - 5y^2 \\ & = 5x^2 + 16xy - 7y^2 \end{aligned}$$

$$\begin{aligned} (iv) \quad & 9x^2 - 6x + 7, 5 - 4x \text{ and } 6 - 3x^2 \\ & 9x^2 - 6x + 7 + 5 - 4x + 6 - 3x^2 \\ & = 9x^2 - 3x^2 - 6x - 4x + 7 + 5 + 6 \\ & = 6x^2 - 10x + 18 \end{aligned}$$

$$\begin{aligned} (v) \quad & 5x^2 - 2xy + 3y^2, -2x^2 + 5xy + 9y^2 \\ & \text{and } 3x^2 - xy - 4y^2 \\ & 5x^2 - 2xy + 3y^2 - 2x^2 + 5xy + 9y^2 \\ & \quad \quad \quad + 3x^2 - xy - 4y^2 \end{aligned}$$

$$\begin{aligned}
&= 5x^2 - 2x^2 + 3x^2 - 2xy + 5xy - xy \\
&\quad + 3y^2 + 9y^2 - 4y^2 \\
&= 8x^2 - 2x^2 + 5xy - 3xy + 12y^2 - 4y^2 \\
&= 6x^2 + 2xy + 8y^2
\end{aligned}$$

$$\begin{aligned}
\text{(vi)} \quad &a^2 + b^2 + 2ab, 2b^2 + c^2 + 2bc \\
&\text{and } 4c^2 - a^2 + 2ac \\
&a^2 + b^2 + 2ab + 2b^2 + c^2 + 2bc \\
&\quad + 4c^2 - a^2 + 2ac \\
&= a^2 - a^2 + b^2 + 2b^2 + c^2 + 4c^2 + 2ab \\
&\quad + 2bc + 2ac \\
&= 3b^2 + 5c^2 + 2ab + 2bc + 2ac
\end{aligned}$$

$$\begin{aligned}
\text{(vii)} \quad &9ax - 6bx + 8, 4ax + 8bx - 7 \\
&\text{and } -6ax - 4bx - 3 \\
&9ax - 6bx + 8 + 4ax + 8bx - 7 \\
&\quad - 6ax - 4bx - 3 \\
&= 9ax + 4ax - 6ax - 6bx + 8bx - 4bx \\
&\quad + 8 - 7 - 3 \\
&= 13ax - 6ax + 8bx - 10bx + 8 - 10 \\
&= 7ax - 2bx - 2
\end{aligned}$$

$$\begin{aligned}
\text{(viii)} \quad &abc + 2ba + 3ac, 4ca - 4ab + 2bca \\
&\text{and } 2ab - 3abc - 6ac \\
&abc + 2ab + 3ca + 4ca - 4ab + 2abc \\
&\quad + 2ab - 3abc - 6ca \\
&= abc + 2abc - 3abc + 2ab - 4ab + 2ab \\
&\quad + 3ca + 4ca - 6ca \\
&= 3abc - 3abc + 4ab - 4ab + 7ca - 6ca \\
&= 0 + 0 + ca = ca
\end{aligned}$$

$$\begin{aligned}
 & \text{(ix) } 4a^2 + 5b^2 - 6ab, 3ab, 6a^2 - 2b^2 \\
 & \text{and } 4b^2 - 5ab \\
 & 4a^2 + 5b^2 - 6ab + 3ab + 6a^2 - 2b^2 \\
 & \qquad \qquad \qquad + 4b^2 - 5ab \\
 & = 4a^2 + 6a^2 + 5b^2 - 2b^2 + 4b^2 - 6ab \\
 & \qquad \qquad \qquad + 3ab - 5ab \\
 & = 10a^2 + 9b^2 - 2b^2 - 11ab + 3ab \\
 & = 10a^2 + 7b^2 - 8ab
 \end{aligned}$$

$$\begin{aligned}
 & \text{(x) } x^2 + x - 2, 2x - 3x^2 + 5 \text{ and } 2x^2 - 5x + 7 \\
 & = x^2 + x - 2 + 2x - 3x^2 + 5 + 2x^2 - 5x + 7 \\
 & = x^2 - 3x^2 + 2x^2 + x + 2x - 5x - 2 + 5 + 7 \\
 & = 3x^2 - 3x^2 + 3x - 5x - 2 + 12 \\
 & = 0 - 2x + 10 \\
 & = -2x + 10
 \end{aligned}$$

$$\begin{aligned}
 & \text{(xi) } 4x^3 + 2x^2 - x + 1, 2x^3 - 5x^2 - 3x + 6, x^2 + 8 \\
 & \text{and } 5x^3 - 7x \\
 & 4x^3 + 2x^2 - x + 1 + 2x^3 - 5x^2 - 3x + 6 \\
 & \qquad \qquad \qquad + x^2 + 8 + 5x^3 - 7x \\
 & = 4x^3 + 2x^3 + 5x^3 + 2x^2 - 5x^2 + x^2 - x \\
 & \qquad \qquad \qquad - 3x - 7x + 1 + 6 + 8 \\
 & = 11x^3 + 3x^2 - 5x^2 - 11x + 15 \\
 & = 11x^3 - 2x^2 - 11x + 15
 \end{aligned}$$

Question 6.

Find the sum of:

- (i) x and $3y$
- (ii) $-2a$ and $+5$
- (iii) $-4x^2$ and $+7x$
- (iv) $+4a$ and $-7b$
- (v) $x^3 + 3x^2y$ and $2y^2$
- (vi) 11 and $-by$

Solution:

$$\begin{aligned}
 & \text{(i) } x + 3y \qquad \qquad \text{(ii) } -2a + 5 \\
 & \text{(iii) } -4x^2 + 7x \qquad \text{(iv) } 4a - 7b \\
 & \text{(v) } x^3 + 3x^2y + 2y^2 \quad \text{(vi) } 11 - by
 \end{aligned}$$

Question 7.

The sides of a triangle are $2x + 3y$, $x + 5y$ and $7x - 2y$, find its perimeter.

Solution:

Sides of a triangle are $2x + 3y$, $x + 5y$, $7x - 2y$

$\therefore$ Perimeter = sum of three sides of the triangle

$$= 2x + 3y + x + 5y + 7x - 2y$$

$$= 2x + x + 7x + 3y + 5y - 2y$$

$$= 10x + 8y - 2x = 10x + 6y$$

Question 8.

The two adjacent sides of a rectangle are $6a + 9b$ and $8a - 4b$. Find its, perimeter.

Solution

Sides of a rectangle are $6a + 9b$

and $8a - 4b$

Let, length = $6a + 9b$

and breadth = $8a - 4b$

$\therefore$ Perimeter = $2(\text{length} + \text{breadth})$

$$= 2(6a + 9b + 8a - 4b)$$

$$= 2(14a + 5b) = 28a + 10b$$

Question 9.

Subtract the second expression from the first:

(i) $2a + b$, $a + b$ (ii) $-2b + 2c$, $b + 3c$

(iii) $5a + b$, $-6b + 2a$ (iv) $a^3 - 1 + a$, $3a - 2a^2$

(v) $p + 2$, 1

(vi) $x + 2y + z$, $-x - y - 3z$

(vii) $3a^2 - 8ab - 2b^2$, $3a^2 - 4ab + 6b^2$

(viii) $4pq - 6p^2 - 2q^2$, $9p^2$

(ix) $10abc$, $2a^2 + 2abc - 4b^2$

(x) $a^2 + ab + c^2$, $a^2 - d^2$

Solution:

$$\begin{aligned}(i) \quad (2a + b) - (a + b) \\&= 2a + b - a - b = 2a - a + b - b \\&= a + 0 = a\end{aligned}$$

$$\begin{aligned}(ii) \quad (-2b + 2c) - (b + 3c) \\&= -2b + 2c - b - 3c \\&= -2b - b + 2c - 3c \\&= -3b - c\end{aligned}$$

$$\begin{aligned}(iii) \quad (5a + b) - (-6b + 2a) \\&= 5a + b + 6b - 2a \\&= 5a - 2a + b + 6b \\&= 3a + 7b\end{aligned}$$

$$\begin{aligned}(iv) \quad (a^3 - 1 + a) - (3a - 2a^2) \\&= a^3 - 1 + a - 3a + 2a^2 \\&= a^3 + 2a^2 - 2a - 1\end{aligned}$$

$$(v) \quad (p + 2) - 1 = p + 2 - 1 = p + 1$$

$$\begin{aligned}(vi) \quad (x + 2y + z) - (-x - y - 3z) \\&= x + 2y + z + x + y + 3z \\&= x + x + 2y + y + z + 3z \\&= 2x + 3y + 4z\end{aligned}$$

$$\begin{aligned}(vii) \quad (3a^2 - 8ab - 2b^2) - (3a^2 - 4ab + 6b^2) \\&= 3a^2 - 8ab - 2b^2 - 3a^2 + 4ab - 6b^2 \\&= 3a^2 - 3a^2 - 2b^2 - 6b^2 - 8ab + 4ab \\&= 0 - 8b^2 - 4ab \\&= -4ab - 8b^2\end{aligned}$$

$$\begin{aligned}(viii) \quad (4pq - 6p^2 - 2q^2) - (9p^2) \\&= 4pq - 6p^2 - 2q^2 - 9p^2 \\&= 4pq - 15p^2 - 2q^2\end{aligned}$$

$$\begin{aligned}
 \text{(ix)} \quad & 10abc - (2a^2 + 2abc - 4b^2) \\
 &= 10abc - 2a^2 - 2abc + 4b^2 \\
 &= 10abc - 2abc - 2a^2 + 4b^2 \\
 &= 8abc - 2a^2 + 4b^2
 \end{aligned}$$

$$\begin{aligned}
 \text{(x)} \quad & (a^2 + ab + c^2) - (a^2 - d^2) \\
 &= a^2 + ab + c^2 - a^2 + d^2 \\
 &= a^2 - a^2 + ab + c^2 + d^2 \\
 &= ab + c^2 + d^2
 \end{aligned}$$

Question 10.

Subtract:

(i) $4x$ from $8 - x$

(ii) $-8c$ from $c + 3d$

(iii) $-5a - 2b$ from $b + 6c$

(iv) $4p + p^2$ from $3p^2 - 8p$

(v) $5a - 3b + 2c$ from $4a - b - 2c$

(vi) $-xy + yz - zx$ from $xy - yz + xz$

(vii) $2x^2 - 7xy - y^2$ from $3x^2 - 5xy + 3y^2$

(viii) $a^2 - 3ab - 6b^2$ from $2b^2 - a^2 + 2ab$

(ix) $4x^2 - 5x^2y + y^2$ from $-3y^2 + 5xy^2$
 $-7x^2 - 9x^2y$

(x) $6m^3 + 4m^2 + 7m - 3$ from $3m^3 + 4$

Solution:

(i) $4x$ from $8 - x$

$$(8 - x) - 4x = 8 - x - 4x = 8 - 5x$$

(ii) $-8c$ from $c + 3d$

$$(c + 3d) - (-8c)$$

$$c + 3d + 8c = 9c + 3d$$

(iii) $-5a - 2b$ from $b + 6c$

$$(b + 6c) - (-5a - 2b)$$

$$= b + 6c + 5a + 2b = 5a + 3b + 6c$$

(iv) $4p + p^2$ from $3p^2 - 8p$

$$(3p^2 - 8p) - (4p + p^2)$$

$$= 3p^2 - 8p - 4p - p^2 = 2p^2 - 12p$$

(v) $5a - 3b + 2c$ from $4a - b - 2c$

$$(4a - b - 2c) - (5a - 3b + 2c)$$

$$= 4a - b - 2c - 5a + 3b - 2c$$

$$= 4a - 5a - b + 3b - 2c - 2c$$

$$= -a + 2b - 4c$$

(vi) $-xy + yz - zx$ from $xy - yz + xz$

$$\begin{aligned}(xy - yz + xz) - (-xy + yz - xz) \\&= xy - yz + xz + xy - yz + xz \\&= xy + xy - yz - yz + xz + xz \\&= 2(xy - yz + xz)\end{aligned}$$

(vii) $2x^2 - 7xy - y^2$ from $3x^2 - 5xy + 3y^2$

$$\begin{aligned}(3x^2 - 5xy + 3y^2) - (2x^2 - 7xy - y^2) \\&= 3x^2 - 5xy + 3y^2 - 2x^2 + 7xy + y^2 \\&= 3x^2 - 2x^2 - 5xy + 7xy + 3y^2 + y^2 \\&= x^2 + 2xy + 4y^2\end{aligned}$$

(viii) $a^2 - 3ab - 6b^2$ from $2b^2 - a^2 + 2ab$

$$\begin{aligned}(2b^2 - a^2 + 2ab) - (a^2 - 3ab - 6b^2) \\&= 2b^2 - a^2 + 2ab - a^2 + 3ab + 6b^2 \\&= -a^2 - a^2 + 2b^2 + 6b^2 + 2ab + 3ab \\&= -2a^2 + 8b^2 + 5ab \\&= 8b^2 + 5ab - 2a^2\end{aligned}$$

(ix) $4x^2 - 5x^2y + y^2$ from $-3y^2 + 5xy^2 - 7x^2 - 9x^2y$

$$\begin{aligned}(-3y^2 + 5xy^2 - 7x^2 - 9x^2y) \\&\quad - (4x^2 - 5x^2y + y^2) \\&= -3y^2 + 5xy^2 - 7x^2 - 9x^2y - 4x^2 \\&\quad + 5x^2y - y^2 \\&= -3y^2 - y^2 + 5xy^2 - 7x^2 - 4x^2 \\&\quad - 9x^2y + 5x^2y \\&= -4y^2 + 5xy^2 - 11x^2 - 4x^2y\end{aligned}$$

(x) $6m^3 + 4m^2 + 7m - 3$ from $3m^3 + 4$

$$\begin{aligned}(3m^3 + 4) - (6m^3 + 4m^2 + 7m - 3) \\&= 3m^3 + 4 - 6m^3 - 4m^2 - 7m + 3 \\&= 3m^3 - 6m^3 - 4m^2 - 7m + 4 + 3 \\&= -3m^3 - 4m^2 - 7m + 7\end{aligned}$$

Question 11.

Subtract $-5a^2 - 3a + 1$ from the sum of $4a^2 + 3 - 8a$ and $9a - 7$.

Solution:

$$\begin{aligned}
 &\text{Sum of } 4a^2 + 3 - 8a \text{ and } 9a - 7 \\
 &= 4a^2 + 3 - 8a + 9a - 7 = 4a^2 + a - 4 \\
 &\therefore (4a^2 + a - 4) - (-5a^2 - 3a + 1) \\
 &= 4a^2 + a - 4 + 5a^2 + 3a - 1 \\
 &= 4a^2 + 5a^2 + a + 3a - 4 - 1 \\
 &= 9a^2 + 4a - 5
 \end{aligned}$$

Question 12.

By how much does $8x^3 - 6x^2 + 9x - 10$ exceed $4x^3 + 2x^2 + 7x - 3$?

Solution:

$$\begin{aligned}
 &8x^3 - 6x^2 + 9x - 10 \text{ exceeds } 4x^3 + 2x^2 \\
 &\qquad\qquad\qquad + 7x - 3 \\
 &= (8x^3 - 6x^2 + 9x - 10) \\
 &\qquad\qquad\qquad - (4x^3 + 2x^2 + 7x - 3) \\
 &= 8x^3 - 6x^2 + 9x - 10 - 4x^3 - 2x^2 - 7x + 3 \\
 &= 8x^3 - 4x^3 - 6x^2 - 2x^2 + 9x - 7x - 10 + 3 \\
 &= 4x^3 - 8x^2 + 2x - 7
 \end{aligned}$$

Question 13.

What must be added to $2a^3 + 5a - a^2 - 6$ to get $a^2 - a - a^3 + 1$?

Solution:

$$\begin{aligned}
 &\text{We get, the required result by subtracting} \\
 &2a^3 - a^2 + 5a - 6 \text{ from } -a^3 + a^2 - a + 1 \\
 &= (-a^3 + a^2 - a + 1) - (2a^3 - a^2 + 5a - 6) \\
 &= -a^3 + a^2 - a + 1 - 2a^3 + a^2 - 5a + 6 \\
 &= -a^3 - 2a^3 + a^2 + a^2 - a - 5a + 1 + 6 \\
 &= -3a^3 + 2a^2 - 6a + 7
 \end{aligned}$$

Question 14.

What must be subtracted from $a^2 + b^2 + 2ab$ to get $-4ab + 2b^2$?

Solution:

We get, the required result by subtracting

$-4ab + 2b^2$ from $a^2 + b^2 + 2ab$.

$$= a^2 + b^2 + 2ab - (-4ab + 2b^2)$$

$$= a^2 + b^2 + 2ab + 4ab - 2b^2$$

$$= a^2 + b^2 - 2b^2 + 2ab + 4ab$$

$$= a^2 - b^2 + 6ab$$

Question 15.

Find the excess of $4m^2 + 4n^2 + 4p^2$ over $m^2 + 3n^2 - 5p^2$

Solution:

The required result will be by subtracting

$m^2 + 3n^2 - 5p^2$ from $4m^2 + 4n^2 + 4p^2$

$$= 4m^2 + 4n^2 + 4p^2 - (m^2 + 3n^2 - 5p^2)$$

$$= 4m^2 + 4n^2 + 4p^2 - m^2 - 3n^2 + 5p^2$$

$$= 4m^2 - m^2 + 4n^2 - 3n^2 + 4p^2 + 5p^2$$

$$= 3m^2 + n^2 + 9p^2$$

Question 16.

By how much is $3x^3 - 2x^2y + xy^2 - y^3$ less than $4x^3 - 3x^2y - 7xy^2 + 2y^3$

Solution:

We can get the required result by subtracting

$3x^3 - 2x^2y + xy^2 - y^3$ from $4x^3 - 3x^2y - 7xy^2 + 2y^3$

$$= (4x^3 - 3x^2y - 7xy^2 + 2y^3)$$

$$- (3x^3 - 2x^2y + xy^2 - y^3)$$

$$= 4x^3 - 3x^2y - 7xy^2 + 2y^3 - 3x^3 + 2x^2y$$

$$- xy^2 + y^3$$

$$= 4x^3 - 3x^3 - 3x^2y + 2x^2y - 7xy^2 - xy^2$$

$$+ 2y^3 + y^3$$

$$= x^3 - x^2y - 8xy^2 + 3y^3$$

Question 17.

Subtract the sum of $3a^2 - 2a + 5$ and $a^2 - 5a - 7$ from the sum of $5a^2 - 9a + 3$ and $2a - a^2 - 1$

Solution:

$$\begin{aligned} &\text{Sum of } 3a^2 - 2a + 5 \text{ and } a^2 - 5a - 7 \\ &= 3a^2 - 2a + 5 + a^2 - 5a - 7 \\ &= 3a^2 + a^2 - 2a - 5a + 5 - 7 \\ &= 4a^2 - 7a - 2 \end{aligned}$$

$$\begin{aligned} &\text{and sum of } 5a^2 - 9a + 3 \text{ and } 2a - a^2 - 1 \\ &= 5a^2 - 9a + 3 + 2a - a^2 - 1 \\ &= 5a^2 - a^2 - 9a + 2a + 3 - 1 \\ &= 4a^2 - 7a + 2 \end{aligned}$$

$$\begin{aligned} &\text{Now } (4a^2 - 7a + 2) - (4a^2 - 7a - 2) \\ &= 4a^2 - 7a + 2 - 4a^2 + 7a + 2 \\ &= 4a^2 - 4a^2 - 7a + 7a + 2 + 2 \\ &= 0 + 0 + 4 = 4. \end{aligned}$$

Question 18.

The perimeter of a rectangle is $28x^3 + 16x^2 + 8x + 4$. One of its sides is $8x^2 + 4x$. Find the other side

Solution:

$$\begin{aligned} &\text{Perimeter of a rectangle } (2l + 2b) \\ &= 28x^3 + 16x^2 + 8x + 4 \\ &\text{Let one side } (l) = 8x^2 + 4x \\ &\therefore 2l = 2(8x^2 + 4x) = 16x^2 + 8x \\ &\therefore 2b = (28x^3 + 16x^2 + 8x + 4) \\ &\quad \quad \quad - (16x^2 + 8x) \\ &= 28x^3 + 16x^2 + 8x + 4 - 16x^2 - 8x \\ &= 28x^3 + 4 \end{aligned}$$

$$\begin{aligned} \therefore \text{Other side } (b) &= \frac{28x^3 + 4}{2} \\ &= 14x^3 + 2 \end{aligned}$$

Question 19.

The perimeter of a triangle is $14a^2 + 20a + 13$. Two of its sides are $3a^2 + 5a + 1$ and $a^2 + 10a - 6$. Find its third side.

Solution:

$$\text{Perimeter of a triangle} = 14a^2 + 20a + 13$$

Sum of two sides

$$= 3a^2 + 5a + 1 + a^2 + 10a - 6$$

$$= 3a^2 + a^2 + 5a + 10a + 1 - 6$$

$$= 4a^2 + 15a - 5$$

$$\therefore \text{Third side} = (14a^2 + 20a + 13)$$

$$- (4a^2 + 15a - 5)$$

$$= 14a^2 + 20a + 13 - 4a^2 - 15a + 5$$

$$= 14a^2 - 4a^2 + 20a - 15a + 13 + 5$$

$$= 10a^2 + 5a + 18$$

Question 20.

$$x = 4a^2 + b^2 - 6ab$$

$$y = 3b^2 - 2a^2 + 8ab$$

$$z = 6a^2 + 8b^2 - 6ab$$

$$\begin{aligned} (i) \ x + y + z &= 4a^2 + b^2 - 6ab + 3b^2 - 2a^2 \\ &\quad + 8ab + 6a^2 + 8b^2 - 6ab \\ &= 4a^2 - 2a^2 + 6a^2 + b^2 + 3b^2 + 8b^2 - 6ab \\ &\quad + 8ab - 6ab \end{aligned}$$

$$= 10a^2 - 2a^2 + 12b^2 - 12ab + 8ab$$

$$= 8a^2 + 12b^2 - 4ab$$

$$\begin{aligned} (ii) \ x - y - z &= (4a^2 + b^2 - 6ab) \\ &\quad - (3b^2 - 2a^2 + 8ab) - (6a^2 + 8b^2 - 6ab) \\ &= 4a^2 + b^2 - 6ab - 3b^2 + 2a^2 - 8ab \\ &\quad - 6a^2 - 8b^2 + 6ab \\ &= 4a^2 + 2a^2 - 6a^2 + b^2 - 3b^2 - 8b^2 \\ &\quad - 6ab - 8ab + 6ab \\ &= 6a^2 - 6a^2 + b^2 - 11b^2 - 14ab + 6ab \\ &= -10b^2 - 8ab \end{aligned}$$

Solution:

$$\text{If } x = 4a^2 + b^2 - 6ab, y = 3b^2 - 2a^2 + 8ab$$

and $z = 6a^2 + 8b^2 - 6ab$ find :

$$(i) \ x + y + z$$

$$(ii) \ x - y - z$$

Question 21.

If $m = 9x^2 - 4xy + 5y^2$ and $n = -3x^2 + 2xy - y^2$ find :

(i) $2m - n$

(ii) $m + 2n$

(iii) $m - 3n$.

Solution:

$$m = 9x^2 - 4xy + 5y^2$$

$$n = -3x^2 + 2xy - y^2$$

$$\begin{aligned} \text{(i) } 2m - n &= 2(9x^2 - 4xy + 5y^2) \\ &\quad - (-3x^2 + 2xy - y^2) \\ &= 18x^2 - 8xy + 10y^2 + 3x^2 - 2xy + y^2 \\ &= 18x^2 + 3x^2 - 8xy - 2xy + 10y^2 + y^2 \\ &= 21x^2 - 10xy + 11y^2 \end{aligned}$$

$$\begin{aligned} \text{(ii) } m + 2n &= (9x^2 - 4xy + 5y^2) \\ &\quad + 2(-3x^2 + 2xy - y^2) \\ &= 9x^2 - 4xy + 5y^2 - 6x^2 + 4xy - 2y^2 \\ &= 9x^2 - 6x^2 - 4xy + 4xy + 5y^2 - 2y^2 \\ &= 3x^2 + 3y^2 \end{aligned}$$

$$\text{(iii) } m = 9x^2 - 4xy + 5y^2$$

$$n = -3x^2 + 2xy - y^2$$

Now,

$$\begin{aligned} m - 3n &= 9x^2 - 4xy + 5y^2 - 3(-3x^2 + 2xy - y^2) \\ &= 9x^2 - 4xy + 5y^2 + 9x^2 - 6xy + 3y^2 \\ &= 18x^2 - 10xy + 8y^2 \end{aligned}$$

Question 22.

Simplify:

$$(i) 3x + 5(2x + 6) - 7x$$

$$(ii) 3(4y - 10) + 2(y - 1)$$

$$(iii) -(7 + 6x) - 7(x + 2)$$

$$(iv) x - (x - y) - y - (y - x)$$

$$(v) 4x + 7y - [5y - 8] - 2x$$

$$(vi) -2m + 5 + 4(m - 3)$$

$$(vii) 2x - y + 5 - (x - y)$$

$$(viii) 2(x - y) - (x - 8)$$

$$(ix) 4(3x - 8) - 3(5x + 3) - 2(6x - 8)$$

$$(x) 5(x - 4) - 3(x - 4) + 7(x - 4)$$

Solution:

$$(i) 3x + 5(2x + 6) - 7x$$

$$\Rightarrow 3x + 10x + 30 - 7x$$

$$\Rightarrow 3x + 10x - 7x + 30$$

$$\Rightarrow 13x - 7x + 30$$

$$\Rightarrow 6x + 30$$

$$(ii) 3(4y - 10) + 2(y - 1)$$

$$\Rightarrow 12y - 30 + 2y - 2$$

$$\Rightarrow 12y + 2y - 30 - 2$$

$$\Rightarrow 14y - 32$$

$$(iii) -(7 + 6x) - 7(x + 2)$$

$$\Rightarrow -7 - 6x - 7x - 14$$

$$\Rightarrow -7x - 6x - 7 - 14$$

$$\Rightarrow -13x - 21$$

$$(iv) x - (x - y) - y - (y - x)$$

$$\Rightarrow x - x + y - y - y + x$$

$$\Rightarrow 2x - x - 2y + y$$

$$\Rightarrow x - y$$

$$(v) 4x + 7y - [5y - 8] - 2x$$

$$\Rightarrow 4x + 7y - 5y + 8 - 2x$$

$$\Rightarrow 4x - 2x + 7y - 5y + 8$$

$$\Rightarrow 2x + 2y + 8$$

$$(vi) -2m + 5 + 4(m - 3)$$

$$\Rightarrow -2m + 5 + 4m - 12$$

$$\Rightarrow -2m + 4m + 5 - 12$$

$$\Rightarrow 2m - 7$$

$$(vii) 2x - y + 5 - (x - y)$$

$$\Rightarrow 2x - y + 5 - x + y$$

$$\Rightarrow 2x - x + 5$$

$$\Rightarrow x + 5$$

$$(viii) 2(x - y) - (x - 8)$$

$$\Rightarrow 2x - 2y - x + 8$$

$$\Rightarrow 2x - x - 2y + 8$$

$$\Rightarrow x - 2y + 8$$

$$(ix) 4(3x - 8) - 3(5x + 3) - 2(6x - 8)$$

$$\Rightarrow 12x - 32 - 15x - 9 - 12x + 16$$

$$\Rightarrow 12x - 15x - 12x - 32 - 9 + 16$$

$$\Rightarrow 12x - 27x - 41 + 16$$

$$\Rightarrow -15x - 25$$

$$(x) 5(x - 4) - 3(x - 4) + 7(x - 4)$$

$$\Rightarrow 5x - 20 - 3x + 12 + 7x - 28$$

$$\Rightarrow 5x + 7x - 3x - 20 - 28 + 12$$

$$\Rightarrow 12x - 3x - 48 + 12$$

$$\Rightarrow 9x - 36$$

EXERCISE 11 (C)

Question 1.

Multiply:

(i) $3x$, $5x^2y$ and $2y$

(ii) 5 , $3a$ and $2ab^2$

(iii) $5x + 2y$ and $3xy$

(iv) $6a - 5b$ and $-2a$

(v) $4a + 5b$ and $4a - 5b$

(vi) $9xy + 2y^2$ and $2x - 3y$

(vii) $-3m^2n + 5mn - 4mn^2$ and $6m^2n$

(viii) $6xy^2 - 7x^2y^2 + 10x^3$ and $-3x^2y^3$

Solution:

(i) Product of $3x$, $5x^2y$ and $2y$

$$\begin{aligned} &= 3x \times 5x^2y \times 2y \\ &= 3 \times 5 \times 2 \times x \times x^2 \times y \times y \\ &= 30x^3y^2 \end{aligned}$$

(ii) Product of 5 , $3a$ and $2ab^2$

$$\begin{aligned} &= 5 \times 3a \times 2ab^2 \\ &= 5 \times 3 \times 2 \times a \times ab^2 \\ &= 30a^2b^2 \end{aligned}$$

(iii) Product of $5x + 2y$ and $3xy$

$$\begin{aligned} &= 3xy (5x + 2y) \\ &= 3xy \times 5x + 3xy \times 2y \\ &= 15x^2y + 6xy^2 \end{aligned}$$

(iv) Product of $6a - 5b$ and $-2a$

$$\begin{aligned} &= -2a (6a - 5b) \\ &= -2a \times 6a + (-2a) (-5b) \\ &= -12a^2 + 10ab \end{aligned}$$

(v) Product of $4a + 5b$ and $4a - 5b$

$$= 16a^2 - 25b^2$$

$$\begin{array}{r} 4a + 5b \\ \times 4a - 5b \\ \hline 16a^2 + 20ab \\ - 20ab - 25b^2 \\ \hline 16a^2 \qquad - 25b^2 \end{array}$$

(vi) Product of $9xy + 2y^2$ and $2x - 3y$

$$= 18x^2y - 23xy^2 - 6y^3$$

$$\begin{array}{r} 9xy + 2y^2 \\ \times 2x - 3y \\ \hline 18x^2y + 4xy^2 \\ - 27xy^2 - 6y^3 \\ \hline 18x^2y - 23xy^2 - 6y^3 \end{array}$$

(vii) Product of $-3m^2n + 5mn - 4mn^2$ and $6m^2n$

$$\begin{aligned}
 &= 6m^2n(-3m^2n + 5mn - 4mn^2) \\
 &= 6m^2n \times (-3m^2n) + 6m^2n \times 5mn \\
 &\quad + 6m^2n \times (-4mn^2) \\
 &= -18m^4n^2 + 30m^3n^2 - 24m^3n^3
 \end{aligned}$$

(viii) Product of $6xy^2 - 7x^2y^2 + 10x^3$ and $-3x^2y^3$

$$\begin{aligned}
 &= -3x^2y^3(6xy^2 - 7x^2y^2 + 10x^3) \\
 &= -3x^2y^3 \times 6xy^2 + (-3x^2y^3)(-7x^2y^2) \\
 &\quad + (-3x^2y^3) \times 10x^3 \\
 &= -18x^3y^5 + 21x^4y^5 - 30x^5y^3
 \end{aligned}$$

Question 2.

Copy and complete the following multiplications :

$$\begin{array}{r}
 (i) \quad 3a + 2b \\
 \times \quad -3xy \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 (ii) \quad 9x + 5y \\
 \times \quad -3xy \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 (iii) \quad 3xy - 2x^2 - 6x \\
 \times \quad -5x^2y \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 (iv) \quad a + b \\
 \times \quad a + b \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 (v) \quad ax - b \\
 \times 2ax + 2b^2 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 (vi) \quad 2a - b + 3c \\
 \times \quad 2a - 4b \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 (vii) \quad 3m^2 + 5m - 2n \\
 \times \quad 5n - 3m \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 (viii) \quad 6 - 3x + 2x^2 \\
 \times 1 + 5x - x^2 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 (ix) \quad 4x^3 - 10x^2 + 6x - 8 \\
 \times \quad 3 + 2x - x^2 \\
 \hline
 \end{array}$$

Solution:

$$\begin{array}{r} (i) \quad 3a + 2b \\ \times \quad -3xy \\ \hline -9axy - 6bxy \end{array}$$

$$\begin{array}{r} (ii) \quad 9x - 5y \\ \times \quad -3xy \\ \hline -27x^2y + 15xy^2 \end{array}$$

$$\begin{array}{r} (iii) \quad 3xy - 2x^2 - 6x \\ \times \quad -5x^2y \\ \hline -15x^3y^2 + 10x^4y + 30x^3y \end{array}$$

$$\begin{array}{r} (iv) \quad a + b \\ \times \quad a + b \\ \hline a^2 + ab \\ \quad ab + b^2 \\ \hline a^2 + 2ab + b^2 \end{array}$$

$$\begin{array}{r} (v) \quad ax - b \\ \times \quad 2ax + 2b^2 \\ \hline 2a^2x^2 - 2abx + 2ab^2x - 2b^3 \end{array}$$

$$\begin{array}{r}
 \text{(vi)} \quad 2a - b + 3c \\
 \times \quad 2a - 4b \\
 \hline
 4a^2 - 2ab + 6ac \\
 - 8ab + 4b^2 - 12bc \\
 \hline
 4a^2 - 10ab + 6ac + 4b^2 - 12bc
 \end{array}$$

$$\begin{array}{r}
 \text{(vii)} \quad 3m^2 + 6m - 2n \\
 \times \quad 5n - 3m \\
 \hline
 15m^2n + 30mn - 10n^2 - 9m^3 - 18m^2 \\
 + 6mn \\
 \hline
 15m^2n + 36mn - 10n^2 - 9m^3 - 18m^2
 \end{array}$$

$$\begin{array}{r}
 \text{(viii)} \quad 6 - 3x + 2x^2 \\
 \times 1 + 5x - x^2 \\
 \hline
 6 - 3x + 2x^2 \\
 + 30x - 15x^2 + 10x^3 \\
 - 6x^2 + 3x^3 - 2x^4 \\
 \hline
 6 + 27x - 19x^2 + 13x^3 - 2x^4
 \end{array}$$

$$\begin{array}{r}
 \text{(ix)} \quad 4x^3 - 10x^2 + 6x - 8 \\
 \times \quad 3 + 2x - x^2 \\
 \hline
 12x^3 - 30x^2 + 18x - 24 \\
 8x^4 - 20x^3 + 12x^2 - 16x \\
 4x^5 + 10x^4 - 6x^3 + 8x^2 \\
 \hline
 4x^5 + 18x^4 - 14x^3 - 10x^2 + 2x - 24
 \end{array}$$

Question 3.

Evaluate :

(i) $(c + 5)(c - 3)$ (ii) $(3c - 5d)(4c - 6d)$

(iii) $\left(\frac{1}{2}a + \frac{1}{2}b\right)\left(\frac{1}{2}a - \frac{1}{2}b\right)$.

(iv) $(a^2 + 2ab + b^2)(a + b)$

(v) $(3x - 1)(4x^3 - 2x^2 + 6x - 3)$

(vi) $(4m - 2)(m^2 + 5m - 6)$

(vii) $(8 - 12x + 7x^2 - 6x^3)(5 - 2x)$

(viii) $(4x^2 - 4x + 1)(2x^3 - 3x^2 + 2)$

(ix) $(6p^2 - 8pq + 2q^2)(-5p)$

(x) $-4y(15x + 12y - 8z)(x - 2y)$

(xi) $(a^2 + b^2 + c^2 - ab - bc - ca)(a + b + c)$

Solution:

$$\begin{aligned}(i) \quad (c + 5)(c - 3) &= c(c - 3) + 5(c - 3) \\ &= c^2 - 3c + 5c - 15 \\ &= c^2 + 2c - 15\end{aligned}$$

$$\begin{aligned}(ii) \quad (3c - 5d)(4c - 6d) \\ &= 3c(4c - 6d) - 5d(4c - 6d) \\ &= 12c^2 - 18cd - 20cd + 30d^2 \\ &= 12c^2 - 38cd + 30d^2\end{aligned}$$

$$\begin{aligned}(iii) \quad \left(\frac{1}{2}a + \frac{1}{2}b\right)\left(\frac{1}{2}a - \frac{1}{2}b\right) \\ &= \frac{1}{2}a\left(\frac{1}{2}a - \frac{1}{2}b\right) + \frac{1}{2}b\left(\frac{1}{2}a - \frac{1}{2}b\right) \\ &= \frac{1}{4}a^2 - \frac{1}{4}ab + \frac{1}{4}ab - \frac{1}{4}b^2 \\ &= \frac{1}{4}a^2 - \frac{1}{4}b^2\end{aligned}$$

$$\begin{aligned}(iv) \quad (a^2 + 2ab + b^2)(a + b) \\ &= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2) \\ &= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3 \\ &= a^3 + 3a^2b + 3ab^2 + b^3\end{aligned}$$

$$\begin{aligned}(v) \quad (3x - 1)(4x^3 - 2x^2 + 6x - 3) \\ &= 3x(4x^3 - 2x^2 + 6x - 3) - 1(4x^3 - 2x^2 + 6x - 3) \\ &= 12x^4 - 6x^3 + 18x^2 - 9x - 4x^3 + 2x^2 - 6x + 3 \\ &= 12x^4 - 6x^3 - 4x^3 + 18x^2 + 2x^2 - 9x - 6x + 3 \\ &= 12x^4 - 10x^3 + 20x^2 - 15x + 3\end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad & (4m - 2)(m^2 + 5m - 6) \\
 &= 4m(m^2 + 5m - 6) - 2(m^2 + 5m - 6) \\
 &= 4m^3 + 20m^2 - 24m - 2m^2 - 10m + 12 \\
 &= 4m^3 + 20m^2 - 2m^2 - 24m - 10m + 12 \\
 &= 4m^3 + 18m^2 - 34m + 12 \text{ Ans.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii)} \quad & (8 - 12x + 7x^2 - 6x^3)(5 - 2x) \\
 &= 5(8 - 12x + 7x^2 - 6x^3) \\
 &\quad - 2x(8 - 12x + 7x^2 - 6x^3) \\
 &= 40 - 60x + 35x^2 - 30x^3 - 16x + 24x^2 \\
 &\quad - 14x^3 + 12x^4 \\
 &= 40 - 60x - 16x + 35x^2 + 24x^2 - 30x^3 \\
 &\quad - 14x^3 + 12x^4 \\
 &= 40 - 76x + 59x^2 - 44x^3 + 12x^4
 \end{aligned}$$

$$\begin{aligned}
 \text{(viii)} \quad & (4x^2 - 4x + 1)(2x^3 - 3x^2 + 2) \\
 &= 4x^2(2x^3 - 3x^2 + 2) - 4x(2x^3 - 3x^2 + 2) \\
 &\quad + 1(2x^3 - 3x^2 + 2) \\
 &= 8x^5 - 12x^4 + 8x^2 - 8x^4 + 12x^3 - 8x + 2x^3 \\
 &\quad - 3x^2 + 2 \\
 &= 8x^5 - 12x^4 - 8x^4 + 12x^3 + 2x^3 + 8x^2 \\
 &\quad - 3x^2 - 8x + 2 \\
 &= 8x^5 - 20x^4 + 14x^3 + 5x^2 - 8x + 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(ix)} \quad & (6p^2 - 8pq + 2q^2)(-5p) \\
 &= -5p \times 6p^2 - 5p \times (-8pq) - 5p(2q^2) \\
 &= -30p^3 + 40p^2q - 10pq^2
 \end{aligned}$$

$$\begin{aligned}
 \text{(x)} \quad & -4y(15 + 12y - 8z)(x - 2y) \\
 &= -4y(x - 2y)(15x + 12y - 8z) \\
 &= (-4xy + 8y^2)(15x + 12y - 8z) \\
 &= -4xy(15x + 12y - 8z) + 8y^2 \\
 &\quad (15x + 12y - 8z) \\
 &= -60x^2y - 48xy^2 + 32xyz + 120xy^2 \\
 &\quad + 96y^3 - 64y^2z \\
 &= -60x^2y - 48xy^2 + 120xy^2 - 64y^2z + 96y^3 \\
 &\quad + 32xyz \\
 &= -60x^2y + 72xy^2 - 64y^2z + 96y^3 + 32xyz
 \end{aligned}$$

$$\begin{aligned}
& (xi) (a^2 + b^2 + c^2 - ab - bc - ca) (a + b + c) \\
& = a (a^2 + b^2 + c^2 - ab - bc - ca) \\
& \quad + b (a^2 + b^2 + c^2 - ab - bc - ca) \\
& \quad + c (a^2 + b^2 + c^2 - ab - bc - ca) \\
& = a^3 + ab^2 + ac^2 - a^2b - abc - ca^2 + a^2b \\
& \quad + b^3 + bc^2 - ab^2 - b^2c \\
& \quad - abc + a^2c + b^2c + c^3 - abc - bc^2 - c^2a \\
& = a^3 + b^3 + c^3 - a^2b + a^2b - ca^2 + a^2c \\
& \quad + bc^2 - bc^2 - ab^2 + ab^2 \\
& \quad - abc - abc - abc + ac^2 - ac^2 + b^2c - b^2c \\
& = a^3 + b^3 + c^3 - 3abc
\end{aligned}$$

Question 4.

Evaluate:

- (i) $(a + b) (a - b)$
- (ii) $(a^2 + b^2) (a + b) (a - b)$; using the result of (i).
- (iii) $(a^4 + b^4) (a^2 + b^2) (a + b) (a - b)$; using the result of (ii).

Solution:

$$\begin{aligned}
& (i) (a + b) (a - b) \\
& = a (a - b) + b (a - b) \\
& = a^2 - ab + ab - b^2 = a^2 - b^2 \\
& (ii) (a^2 + b^2) (a + b) (a - b) \\
& = (a^2 + b^2) (a^2 - b^2) \quad \{\text{from (i)}\} \\
& = a^2 (a^2 - b^2) + b^2 (a^2 - b^2) \\
& = a^4 - a^2b^2 + a^2b^2 - b^4 = a^4 - b^4 \\
& (iii) (a^4 + b^4) (a^2 + b^2) (a + b) (a - b) \\
& = (a^4 + b^4) (a^4 - b^4) \quad \{\text{from (ii)}\} \\
& = a^4 (a^4 - b^4) + b^4 (a^4 - b^4) \\
& = a^8 - a^4b^4 + a^4b^4 - b^8 = a^8 - b^8
\end{aligned}$$

Question 5.

Evaluate :

(i) $(3x - 2y)(4x + 3y)$

(ii) $(3x - 2y)(4x + 3y)(8x - 5y)$

(iii) $(a + 5)(3a - 2)(5a + 1)$

(iv) $(a + 1)(a^2 - a + 1)$ and $(a - 1)(a^2 + a + 1)$;

and then : $(a + 1)(a^2 - a + 1)$

+ $(a - 1)(a^2 + a + 1)$

(v) $(5m - 2n)(5m + 2n)(25m^2 + 4n^2)$

Solution:

(i) $(3x - 2y)(4x + 3y)$

$= 3x(4x + 3y) - 2y(4x + 3y)$

$= 12x^2 + 9xy - 8xy - 6y^2$

$= 12x^2 + xy - 6y^2$

(ii) $(3x - 2y)(4x + 3y)(8x - 5y)$

$= (12x^2 + xy - 6y^2)(8x - 5y)$ { from (i) }

$= 8x(12x^2 + xy - 6y^2)$

$- 5y(12x^2 + xy - 6y^2)$

$= 96x^3 + 8x^2y - 48xy^2 - 60x^2y - 5xy^2$

+ $30y^3$

$= 96x^3 + 8x^2y - 60x^2y - 48xy^2 - 5xy^2 + 30y^3$

$= 96x^3 - 52x^2y - 53xy^2 + 30y^3$

(iii) $(a + 5)(3a - 2)(5a + 1)$

$= \{a(3a - 2) + 5(3a - 2)\}(5a + 1)$

$$\begin{aligned}
&= (3a^2 - 2a + 15a - 10)(5a + 1) \\
&= (3a^2 + 13a - 10)(5a + 1) \\
&= 5a(3a^2 + 13a - 10) + 1(3a^2 + 13a - 10) \\
&= 15a^3 + 65a^2 - 50a + 3a^2 + 13a - 10 \\
&= 15a^3 + 68a^2 - 37a - 10
\end{aligned}$$

$$(iv) (a+1)(a^2 - a + 1) \text{ and } (a-1)(a^2 + a + 1);$$

$$\begin{aligned}
&= a(a^2 - a + 1) + 1(a^2 - a + 1) \\
&= a^3 - a^2 + a + a^2 - a + 1 = a^3 + 1
\end{aligned}$$

$$(a-1)(a^2 + a + 1)$$

$$= a(a^2 + a + 1) - 1(a^2 + a + 1)$$

$$= a^3 + a^2 + a - a^2 - a - 1 = a^3 - 1$$

$$\text{Now, } (a+1)(a^2 - a + 1) + (a-1)(a^2 + a + 1)$$

$$= a^3 + 1 + a^3 - 1 = 2a^3$$

$$(v) (5m - 2n)(5m + 2n)(25m^2 + 4n^2)$$

$$= \{5m(5m + 2n) - 2n(5m + 2n)\} (25m^2 + 4n^2)$$

$$= (25m^2 + 10mn - 10mn - 4n^2)$$

$$(25m^2 + 4n^2)$$

$$= (25m^2 - 4n^2)(25m^2 + 4n^2)$$

$$= 25m^2(25m^2 + 4n^2) - 4n^2(25m^2 + 4n^2)$$

$$= 625m^4 + 100m^2n^2 - 100m^2n^2 - 16n^4$$

$$= 625m^4 - 16n^4$$

Question 6.

Multiply:

$$(i) mn^4, m^3n \text{ and } 5m^2n^3$$

$$(ii) 2mnpq, 4mnpq \text{ and } 5mnpq$$

$$(iii) pq - pm \text{ and } p^2m$$

$$(iv) x^3 - 3y^3 \text{ and } 4x^2y^2$$

$$(v) a^3 - 4ab \text{ and } 2a^2b$$

$$(vi) x^2 + 5yx - 3y^2 \text{ and } 2x^2y.$$

Solution:

- (i) mn^4 , m^3n and $5m^2n^3$
 $\Rightarrow 5m^2n^3 \times mn^4 \times m^3n$
 $\Rightarrow 5m^{(2+1+3)}n^{(3+4+1)}$
 $= 5m^6n^8$
- (ii) $2mnpq$, $4mnpq$ and $5mnpq$
 $\Rightarrow 5mnpq \times 2mnpq \times 4mnpq$
 $\Rightarrow 5 \times 2 \times 4 m^{(1+1+1)}n^{(1+1+1)}p^{(1+1+1)}q^{(1+1+1)}$
 $\Rightarrow 40m^3n^3p^3q^3$
- (iii) $pq - pm$ and $p^2m \Rightarrow p^2m \times (pq - pm)$
 $\Rightarrow p^3qm - p^3m^2$
- (iv) $x^3 - 3y^3$ and $4x^2y^2$
 $\Rightarrow 4x^2y^2 \times (x^3 - 3y^3)$
 $\Rightarrow 4x^5y^2 - 12x^2y^5$
- (v) $a^3 - 4ab$ and $2a^2b$
 $\Rightarrow 2a^2b \times (a^3 - 4ab)$
 $\Rightarrow 2a^5b - 8a^3b^2$
- (vi) $x^2 + 5yx - 3y^2$ and $2x^2y$
 $\Rightarrow 2x^2y \times (x^2 + 5yx - 3y^2)$
 $\Rightarrow 2x^4y + 10x^3y^2 - 6x^2y^3$

Question 7.

Multiply:

- (i) $(2x + 3y)(2x + 3y)$ (ii) $(2x - 3y)(2x + 3y)$
(iii) $(2x + 3y)(2x - 3y)$ (iv) $(2x - 3y)(2x - 3y)$
(v) $(-2x + 3y)(2x - 3y)$ (vi) $(xy + 2b)(xy - 2b)$
(vii) $(x - a)(x + 3b)$
(viii) $(2x + 5y + 6)(3x + y - 8)$
(ix) $(3x - 5y + 2)(5x - 4y - 3)$
(x) $(6x - 2y)(3x - y)$
(xi) $(1 + 6x^2 - 4x^3)(-1 + 3x - 3x^2)$

Solution:

$$(i) (2x + 3y) (2x + 3y)$$

$$\Rightarrow 2x (2x + 3y) + 3y (2x + 3y)$$

$$\Rightarrow 4x^2 + 6xy + 6xy + 9y^2$$

$$\Rightarrow 4x^2 + 12xy + 9y^2$$

$$(ii) (2x - 3y) (2x + 3y)$$

$$\Rightarrow 2x (2x + 3y) - 3y (2x + 3y)$$

$$\Rightarrow 2x \times 2x + 2x \times 3y - 3y \times 2x - 3y \times 3y$$

$$\Rightarrow 4x^2 + 6xy - 6xy - 9y^2$$

$$\Rightarrow 4x^2 + 0 - 9y^2$$

$$\Rightarrow 4x^2 - 9y^2$$

$$(iii) (2x + 3y) (2x - 3y)$$

$$\Rightarrow 2x (2x - 3y) + 3y (2x - 3y)$$

$$\Rightarrow 2x \times 2x - 2x \times 3y + 3y \times 2x - 3y \times 3y$$

$$\Rightarrow 4x^2 - 6xy + 6xy - 9y^2$$

$$\Rightarrow 4x^2 - 0 - 9y^2$$

$$\Rightarrow 4x^2 - 9y^2$$

$$(iv) (2x - 3y) (2x - 3y)$$

$$\Rightarrow 2x (2x - 3y) - 3y (2x - 3y)$$

$$\Rightarrow 2x \times 2x - 2x \times 3y - 3y \times 2x + 3y \times 3y$$

$$\Rightarrow 4x^2 - 6xy - 6xy + 9y^2$$

$$\Rightarrow 4x^2 - 12xy + 9y^2$$

$$(v) (-2x + 3y) (2x - 3y)$$

$$\Rightarrow -2x (2x - 3y) + 3y (2x - 3y)$$

$$\Rightarrow -4x^2 + 6xy + 6xy - 9y^2$$

$$\Rightarrow -4x^2 + 12xy - 9y^2$$

$$(vi) (xy + 2b)(xy - 2b)$$

$$\Rightarrow xy(xy - 2b) + 2b(xy - 2b)$$

$$\Rightarrow x^2y^2 - 2bxy + 2bxy - 4b^2$$

$$\Rightarrow x^2y^2 - 4b^2$$

$$(vii) (x - a)(x + 3b)$$

$$\Rightarrow x(x + 3b) - a(x + 3b)$$

$$\Rightarrow x^2 + 3bx - ax - 3ab$$

$$(viii) (2x + 5y + 6)(3x + y - 8)$$

$$\Rightarrow 2x(3x + y - 8) + 5y(3x + y - 8) + 6(3x + y - 8)$$

$$\Rightarrow 6x^2 + 2xy - 16x + 15xy + 5y^2 - 40y + 18x + 6y - 48$$

$$\Rightarrow 6x^2 + 2xy + 15xy - 16x + 18x + 5y^2 - 40y + 6y - 48$$

$$\Rightarrow 6x^2 + 17xy + 2x + 5y^2 - 34y - 48$$

$$(ix) (3x - 5y + 2)(5x - 4y - 3)$$

$$\Rightarrow 3x(5x - 4y - 3) - 5y(5x - 4y - 3) + 2(5x - 4y - 3)$$

$$\Rightarrow 15x^2 - 12xy - 9x - 25xy + 20y^2 + 15y + 10x - 8y - 6$$

$$\Rightarrow 15x^2 - 12xy - 25xy - 9x + 10x + 20y^2 + 15y - 8y - 6$$

$$\Rightarrow 15x^2 - 37xy + x + 20y^2 + 7y - 6$$

$$(x) (6x - 2y)(3x - y)$$

$$\Rightarrow 6x(3x - y) - 2y(3x - y)$$

$$\Rightarrow 18x^2 - 6xy - 6xy + 2y^2$$

$$\Rightarrow 18x^2 - 12xy + 2y^2$$

$$(xi) (1 + 6x^2 - 4x^3)(-1 + 3x - 3x^2)$$

$$1(-1 + 3x - 3x^2) + 6x^2(-1 + 3x - 3x^2) - 4x^3(-1 + 3x - 3x^2)$$

$$-1 + 3x - 3x^2 - 6x^2 + 18x^3 - 18x^4 + 4x^3 - 12x^4 + 12x^5$$

$$-1 + 3x - 9x^2 + 22x^3 - 30x^4 + 12x^5$$

EXERCISE 11 (D)

Question 1.

Divide:

- (i) $-16ab^2c$ by $6abc$ (ii) $25x^2y$ by $-5y^2$
(iii) $8x + 24$ by 4 (iv) $4a^2 - a$ by $-a$
(v) $8m - 16$ by -8 (vi) $-50 + 40p$ by $10p$
(vii) $4x^3 - 2x^2$ by $-x$
(viii) $10a^3 - 15a^2b$ by $-5a^2$
(ix) $12x^3y - 8x^2y^2 + 4x^2y^3$ by $4xy$
(x) $9a^4b - 15a^3b^2 + 12a^2b^3$ by $-3a^2b$

Solution:

(i) $-16ab^2c$ by $6abc$

$$= -\frac{16ab^2c}{6abc} = -\frac{8}{3}b$$

(ii) $25x^2y$ by $-5y^2$

$$= \frac{25x^2y}{-5y^2} = -5\frac{x^2}{y}$$

(iii) $8x + 24$ by 4

$$= \frac{8x + 24}{4} = \frac{8x}{4} + \frac{24}{4} = 2x + 6$$

(iv) $4a^2 - a$ by $-a$

$$= \frac{4a^2 - a}{-a} = \frac{4a^2}{-a} - \frac{a}{-a} \\ = -4a + 1 \text{ Ans.}$$

(v) $8m - 16$ by -8

$$= \frac{8m - 16}{-8} = \frac{8m}{-8} - \frac{16}{-8} = -m + 2$$

(vi) $-50 + 40p$ by $10p$

$$= \frac{-50 + 40p}{10p} = \frac{-50}{10p} + \frac{40p}{10p}$$

$$= -\frac{5}{p} + 4$$

(vii) $4x^3 - 2x^2$ by $-x$

$$= \frac{4x^3 - 2x^2}{-x} = \frac{4x^3}{-x} - \frac{2x^2}{-x} \\ = -4x^2 + 2x$$

(viii) $10a^3 - 15a^2b$ by $-5a^2$

$$= \frac{10a^3 - 15a^2b}{-5a^2} = \frac{10a^3}{-5a^2} - \frac{15a^2b}{-5a^2}$$

$$= -2a + 3b \text{ Ans.}$$

(ix) $12x^3y - 8x^2y^2 + 4x^2y^3$ by $4xy$

$$= \frac{12x^3y - 8x^2y^2 + 4x^2y^3}{4xy}$$

$$= \frac{12x^3y}{4xy} - \frac{8x^2y^2}{4xy} + \frac{4x^2y^3}{4xy}$$

$$= 3x^2 - 2xy + xy^2$$

(x) $9a^4b - 15a^3b^2 + 12a^2b^3$ by $-3a^2b$

$$= \frac{9a^4b - 15a^3b^2 + 12a^2b^3}{-3a^2b}$$

$$= \frac{9a^4b}{-3a^2b} - \frac{15a^3b^2}{-3a^2b} + \frac{12a^2b^3}{-3a^2b}$$

$$= -3a^2 + 5ab - 4b^2$$

Question 2.

Divide :

(i) $n^2 - 2n + 1$ by $n - 1$

(ii) $m^2 - 2mn + n^2$ by $m - n$

(iii) $4a^2 + 4a + 1$ by $2a + 1$

(iv) $p^2 + 4p + 4$ by $p + 2$

(v) $x^2 + 4xy + 4y^2$ by $x + 2y$

(vi) $2a^2 - 11a + 12$ by $a - 4$

(vii) $6x^2 + 5x - 6$ by $2x + 3$

(viii) $8a^2 + 4a - 60$ by $2a - 5$

(ix) $9x^2 - 24xy + 16y^2$ by $3x - 4y$

(x) $15x^2 + 31xy + 14y^2$ by $5x + 7y$

(xi) $35a^3 + 3a^2b - 2ab^2$ by $5a - b$

(xii) $6x^3 + 5x^2 - 21x + 10$ by $3x - 2$

Solution:

(i) $n^2 - 2n + 1$ by $n - 1$

$$\begin{array}{r}
 n-1 \\
 \hline
 n-1 \overline{) n^2 - 2n + 1} \quad \cdot \\
 \underline{n^2 - + } \\
 - + \\
 \hline
 -n + 1 \\
 -n + 1 \\
 \hline
 + - \\
 \hline
 \times \\
 \hline
 = n - 1
 \end{array}$$

(ii) $m^2 - 2mn + n^2$ by $m - n$

$$\begin{array}{r}
 m - n \\
 \hline
 m - n \big) m^2 - 2mn + n^2 \big(\\
 \underline{m^2 - mn} \\
 - + \\
 \hline
 - mn + n^2 \\
 - mn + n^2 \\
 \hline
 + - \\
 \hline
 \times \\
 \hline
 = m - n
 \end{array}$$

(iii) $4a^2 + 4a + 1$ by $2a + 1$

$$\begin{array}{r}
 2a + 1 \\
 \hline
 2a + 1 \big) 4a^2 + 4a + 1 \big(\\
 \underline{4a^2 + 2a} \\
 - - \\
 \hline
 2a + 1 \\
 2a + 1 \\
 \hline
 - - \\
 \hline
 \times \\
 \hline
 = 2a + 1
 \end{array}$$

$$(iv) p^2 + 4p + 4 \text{ by } p + 2$$

$$\begin{array}{r}
 p + 2 \\
 \hline
 p + 2 \overline{) p^2 + 4p + 4} \left(\begin{array}{r} p^2 + 2p \\ - \quad - \\ \hline 2p + 4 \\ 2p + 4 \\ \hline \end{array} \right. \\
 \hline
 \begin{array}{r} - \quad - \\ \hline \times \\ \hline \end{array} \\
 = p + 2
 \end{array}$$

$$(v) x^2 + 4xy + 4y^2 \text{ by } x + 2y$$

$$\begin{array}{r}
 x + 2y \\
 \hline
 x + 2y \overline{) x^2 + 4xy + 4y^2} \left(\begin{array}{r} x^2 + 2xy \\ - \quad - \\ \hline 2xy + 4y^2 \\ 2xy + 4y^2 \\ \hline \end{array} \right. \\
 \hline
 \begin{array}{r} - \quad - \\ \hline \times \\ \hline \end{array} \\
 = x + 2y
 \end{array}$$

$$(vi) 2a^2 - 11a + 12 \text{ by } a - 4$$

$$\begin{array}{r}
 2a - 3 \\
 \hline
 a - 4 \overline{) 2a^2 - 11a + 12} \left(\begin{array}{r} 2a^2 - 8a \\ - \quad + \\ \hline -3a + 12 \\ -3a + 12 \\ \hline \end{array} \right. \\
 \hline
 \begin{array}{r} + \quad - \\ \hline \times \\ \hline \end{array} \\
 = 2a - 3
 \end{array}$$

(vii) $6x^2 + 5x - 6$ by $2x + 3$

$$\begin{array}{r}
 3x - 2 \\
 \hline
 2x + 3 \overline{) 6x^2 + 5x - 6} \\
 \underline{6x^2 + 9x} \\
 -4x - 6 \\
 \underline{-4x - 6} \\
 0
 \end{array}$$

(viii) $8a^2 + 4a - 60$ by $2a - 5$

$$\begin{array}{r}
 4a + 12 \\
 \hline
 2a - 5 \overline{) 8a^2 + 4a - 60} \\
 \underline{8a^2 - 20a} \\
 24a - 60 \\
 \underline{24a - 60} \\
 0
 \end{array}$$

(ix) $9x^2 - 24xy + 16y^2$ by $3x - 4y$

$$\begin{array}{r}
 3x - 4y \\
 \hline
 3x - 4y \overline{) 9x^2 - 24xy + 16y^2} \\
 \underline{9x^2 - 12xy} \\
 -12xy + 16y^2 \\
 \underline{-12xy + 16y^2} \\
 0
 \end{array}$$

$$(x) \ 15x^2 + 31xy + 14y^2 \text{ by } 5x + 7y$$

$$3x + 2y$$

$$5x + 7y \) \ 15x^2 + 31xy + 14y^2 \ ($$

$$15x^2 + 21xy$$

$$\begin{array}{r} - \quad - \end{array}$$

$$10xy + 14y^2$$

$$10xy + 14y^2$$

$$\begin{array}{r} - \quad - \end{array}$$

$$\times$$

$$= 3x + 2y \text{ Ans.}$$

$$(xi) \ 35a^3 + 3a^2b - 2ab^2 \text{ by } 5a - b$$

$$7a^2 + 2ab$$

$$5a - b \) \ 35a^3 + 3a^2b - 2ab^2 \ ($$

$$35a^3 - 7a^2b$$

$$\begin{array}{r} - \quad + \end{array}$$

$$10a^2b - 2ab^2$$

$$10a^2b - 2ab^2$$

$$\begin{array}{r} - \quad + \end{array}$$

$$\times$$

$$= 7a^2 + 2ab$$

$$(xii) \ 6x^3 + 5x^2 - 21x + 10 \text{ by } 3x - 2$$

$$2x^2 + 3x - 5$$

$$3x - 2 \) \ 6x^3 + 5x^2 - 21x + 10 \ ($$

$$6x^3 - 4x^2$$

$$\begin{array}{r} - \quad + \end{array}$$

$$9x^2 - 21x$$

$$9x^2 - 6x$$

$$\begin{array}{r} - \quad + \end{array}$$

$$-15x + 10$$

$$-15x + 10$$

$$\begin{array}{r} + \quad - \end{array}$$

$$\times$$

$$= 2x^2 + 3x - 5$$

Question 3.

The area of a rectangle is $6x^2 - 4xy - 10y^2$ square unit and its length is $2x + 2y$ unit. Find its breadth

Solution:

Area of a rectangle

$$= 6x^2 - 4xy - 10y^2 \text{ sq. units}$$

$$\text{Length} = 2x + 2y \text{ units}$$

$$\therefore \text{Breadth} = \frac{\text{Area}}{\text{Length}}$$

$$\begin{array}{r}
 = \frac{6x^2 - 4xy - 10y^2}{2x + 2y} \\
 \begin{array}{r}
 6x^2 - 4xy - 10y^2 \text{ (} \\
 6x^2 + 6xy \\
 \hline
 -10xy - 10y^2 \\
 -10xy - 10y^2 \\
 \hline
 + \quad + \\
 \hline
 \times \\
 \hline
 \end{array} \\
 = 3x - 5y \text{ units}
 \end{array}$$

$$\text{Hence breadth} = 3x - 5y \text{ units}$$

Question 4.

The area of a rectangular field is $25x^2 + 20xy + 3y^2$ square unit. If its length is $5x + 3y$ unit, find its breadth, Hence find its perimeter.

Solution:

Area of a rectangular field

$$= 25x^2 + 20xy + 3y^2$$

$$\text{Length} = (5x + 3y) \text{ unit}$$

$$\therefore \text{Breadth} = \frac{\text{Area}}{\text{Length}} = \frac{25x^2 + 20xy + 3y^2}{5x + 3y}$$

$$\begin{array}{r} 5x + y \\ \hline 5x + 3y \\ 25x^2 + 15xy \\ - - \\ \hline 5xy + 3y^2 \\ 5xy + 3y^2 \\ - - \\ \hline x \end{array}$$

Hence Breadth = $5x + y$

Hence perimeter of rectangular field

$$= 2(l + b)$$

$$= 2(5x + 3y + 5x + y)$$

$$= 2(10x + 4y)$$

$$= 20x + 8y$$

Question 5.

Divide:

- (i) $2m^3n^5$ by $-mn$
- (ii) $5x^2 - 3x$ by x
- (iii) $10x^3y - 9xy^2 - 4x^2y^2$ by xy
- (iv) $3y^3 - 9ay^2 - 6ab^2y$ by $-3y$
- (v) $x^5 - 15x^4 - 10x^2$ by $-5x^2$
- (vi) $12a^2 + ax - 6x^2$ by $3a - 2x$
- (vii) $6x^2 - xy - 35y^2$ by $2x - 5y$
- (viii) $x^3 - 6x^2 + 11x - 6$ by $x^2 - 4x + 3$
- (ix) $m^3 - 4m^2 + m + 6$ by $m^2 - m - 2$

Solution:

- (i) $2m^3n^5$ by $-mn$
$$= \frac{2m^3n^5}{-mn} = -2m^2n^4$$
- (ii) $5x^2 - 3x$ by x
$$= \frac{5x^2 - 3x}{x} = \frac{5x^2}{x} - \frac{3x}{x} = 5x - 3$$
- (iii) $10x^3y - 9xy^2 - 4x^2y^2$ by xy
$$= \frac{10x^3y - 9xy^2 - 4x^2y^2}{xy}$$
$$= \frac{10x^3y}{xy} - \frac{9xy^2}{xy} - \frac{4x^2y^2}{xy}$$
$$= 10x^2 - 9y - 4xy$$
- (iv) $3y^3 - 9ay^2 - 6ab^2y$ by $-3y$
$$= \frac{3y^3 - 9ay^2 - 6ab^2y}{-3y}$$
$$= \frac{3y^3}{-3y} - \frac{9ay^2}{-3y} - \frac{6ab^2y}{-3y}$$
$$= -y^2 + 3ay^2 + 2ab^2$$
- (v) $x^5 - 15x^4 - 10x^2$ by $-5x^2$
$$= \frac{x^5 - 15x^4 - 10x^2}{-5x^2}$$
$$= \frac{x^5}{-5x^2} - \frac{15x^4}{-5x^2} - \frac{10x^2}{-5x^2}$$

$$= \frac{-1}{5}x^3 + 3x^2 + 2$$

(vi) $12a^2 + ax - 6x^2$ by $3a - 2x$

$$\begin{array}{r} 3a - 2x \overline{) 12a^2 + ax - 6x^2} \\ \underline{12a^2 + 8ax} \\ -9ax - 6x^2 \\ \underline{-9ax - 6x^2} \\ 0 \end{array}$$

$$= 4a + 3x$$

(vii) $6x^2 - xy - 35y^2$ by $2x - 5y$

$$\begin{array}{r} 2x - 5y \overline{) 6x^2 - xy - 35y^2} \\ \underline{6x^2 - 15xy} \\ 14xy - 35y^2 \\ \underline{14xy - 35y^2} \\ 0 \end{array}$$

$$= 3x + 7y$$

(viii) $x^3 - 6x^2 + 11x - 6$ by $x^2 - 4x + 3$

$$\begin{array}{r} x^2 - 4x + 3 \overline{) x^3 - 6x^2 + 11x - 6} \\ \underline{x^3 - 4x^2 + 3x} \\ -2x^2 + 8x - 6 \\ \underline{-2x^2 + 8x - 6} \\ 0 \end{array}$$

$$= x - 2$$

(ix) $m^3 - 4m^2 + m + 6$ by $m^2 - m - 2$

$$\begin{array}{r}
 m^2 - m - 2 \overline{) m^3 - 4m^2 + m + 6} \quad (m - 3) \\
 \underline{m^3 - m^2 - 2m} \\
 -3m^2 + 3m + 6 \\
 \underline{-3m^2 + 3m + 6} \\
 + - \\
 \hline
 \times \\
 \hline
 = m - 3
 \end{array}$$

EXERCISE 11 (E)

Simplify

Question 1.

$$\frac{x}{2} + \frac{x}{4}$$

Solution:

$$\frac{x}{2} + \frac{x}{4}$$

$$\frac{2x + x}{4} = \frac{3x}{4}$$

Question 2.

$$\frac{a}{10} + \frac{2a}{5}$$

Solution:

$$\frac{a}{10} + \frac{2a}{5}$$

$$\frac{a + 4a}{10} = \frac{5a}{10} = \frac{a}{2}$$

Question 3.

$$\frac{y}{4} + \frac{3y}{5}$$

Solution:

$$\frac{y}{4} + \frac{3y}{5}$$

$$\frac{5y + 12y}{20} = \frac{17y}{20}$$

Question 4.

$$\frac{x}{2} - \frac{x}{8}$$

Solution:

$$\frac{x}{2} - \frac{x}{8}$$

$$\frac{4x - x}{8} = \frac{3x}{8}$$

Question 5.

$$\frac{3y}{4} - \frac{y}{5}$$

Solution:

$$\frac{3y}{4} - \frac{y}{5}$$

$$\frac{15y - 4y}{20} = \frac{11y}{20}$$

Question 6.

$$\frac{2p}{3} - \frac{3p}{5}$$

Solution:

$$\frac{2p}{3} - \frac{3p}{5}$$

$$\frac{10p - 9p}{15} = \frac{p}{15}$$

Question 7.

$$\frac{k}{2} + \frac{k}{3} + \frac{2k}{5}$$

Solution:

$$\frac{k}{2} + \frac{k}{3} + \frac{2k}{5}$$

$$\frac{15k + 10k + 12k}{30} = \frac{37k}{30}$$

(L.C.M. of 2, 3, 5 = 30)

Question 8.

$$\frac{2x}{5} + \frac{3x}{4} - \frac{3x}{5}$$

Solution:

$$\frac{2x}{5} + \frac{3x}{4} - \frac{3x}{5} \quad (\text{LCM of 5, 4} = 20)$$

$$\frac{8x + 15x - 12x}{20} = \frac{23x - 12x}{20} = \frac{11x}{20}$$

Question 9.

$$\frac{4a}{7} - \frac{2a}{3} + \frac{a}{7}$$

Solution:

$$\frac{4a}{7} - \frac{2a}{3} + \frac{a}{7}$$
$$\frac{12a - 14a + 3a}{21} = \frac{15a - 14a}{21} = \frac{a}{21}$$

(LCM of 7, 3 = 21)

Question 10.

$$\frac{2b}{5} - \frac{7b}{15} + \frac{13b}{3}$$

Solution:

$$\frac{2b}{5} - \frac{7b}{15} + \frac{13b}{3} \quad (\text{L.C.M. of 3, 5, 15} = 15)$$
$$\frac{6b - 7b + 65b}{15} = \frac{71b - 7b}{15} = \frac{64b}{15}$$

Question 11.

$$\frac{6k}{7} - \left(\frac{8k}{9} - \frac{k}{3} \right)$$

Solution:

$$\frac{6k}{7} - \left(\frac{8k}{9} - \frac{k}{3} \right) = \frac{54k - (56k - 21k)}{63}$$

(L.C.M. of 7, 9, 3 = 63)

$$= \frac{54k - (35k)}{63}$$
$$= \frac{54k - 35k}{63} = \frac{19k}{63}$$

Question 12.

$$\frac{3a}{8} + \frac{4a}{5} - \left(\frac{a}{2} + \frac{2a}{5} \right)$$

Solution:

$$\begin{aligned} & \frac{3a}{8} + \frac{4a}{5} - \left(\frac{a}{2} + \frac{2a}{5} \right) \\ &= \frac{15a + 32a - (20a + 16a)}{40} \\ & \quad \text{(L.C.M. of 8, 5, 2 = 40)} \\ &= \frac{47a - 36a}{40} = \frac{11a}{40} \end{aligned}$$

Question 13.

$$x + \frac{x}{2} + \frac{x}{3}$$

Solution:

$$\begin{aligned} & \frac{x}{1} + \frac{x}{2} + \frac{x}{3} \\ &= \frac{6x + 3x + 2x}{6} = \frac{11x}{6} \end{aligned}$$

Question 14.

$$\frac{y}{5} + y - \frac{19y}{15}$$

Solution:

$$\begin{aligned} & \frac{y}{5} + \frac{y}{1} - \frac{19y}{15} \\ &= \frac{3y + 15y - 19y}{15} = \frac{18y - 19y}{15} \\ &= \frac{-y}{15} \end{aligned}$$

Question 15.

$$\frac{x}{5} + \frac{x+1}{2}$$

Solution:

$$\frac{x}{5} + \frac{x+1}{2} = \frac{2x+5x+5}{10} = \frac{7x+5}{10}.$$

(L.C.M. of 5, 2 = 10)

Question 16.

$$x + \frac{x+2}{3}$$

Solution:

$$\begin{aligned} \frac{x}{1} + \frac{x+2}{3} \\ = \frac{3x+x+2}{3} = \frac{4x+2}{3} \end{aligned}$$

Question 17.

$$\frac{3y}{5} - \frac{y+2}{2}.$$

Solution:

$$\begin{aligned} \frac{3y}{5} - \frac{y+2}{2} \\ = \frac{6y - (5y+10)}{10} \\ = \frac{6y-5y-10}{10} = \frac{y-10}{10}, \end{aligned}$$

Question 18.

$$\frac{2a+1}{3} + \frac{3a-1}{2}$$

Solution:

$$\frac{2a+1}{3} + \frac{3a-1}{2}$$
$$\frac{4a+2+9a-3}{6} = \frac{13a-1}{6}$$

(L.C.M. of 3, 2 = 6)

Question 19.

$$\frac{k+1}{2} + \frac{2k-1}{3} - \frac{k+3}{4}$$

Solution:

$$\frac{k+1}{2} + \frac{2k-1}{3} - \frac{k+3}{4}$$
$$\frac{6k+6+8k-4-3k-9}{12}$$

(L.C.M. of 2, 3, 4 = 12)

$$= \frac{14k-3k+6-13}{12} = \frac{11k-7}{12}$$

Question 20.

$$\frac{m}{5} - \frac{m-2}{3} + m$$

Solution:

$$\frac{m}{5} - \frac{m-2}{3} + \frac{m}{1}$$
$$\frac{3m-5(m-2)+15m}{15}$$
$$= \frac{3m-5m+10+15m}{15}$$
$$= \frac{18m-5m+10}{15} = \frac{13m+10}{15}$$

Question 21.

$$\frac{5(x-4)}{3} + \frac{2(5x-3)}{5} + \frac{6(x-4)}{7}$$

Solution:

$$\begin{aligned} & \frac{5(x-4)}{3} + \frac{2(5x-3)}{5} + \frac{6(x-4)}{7} \\ & \frac{175(x-4) + 42(5x-3) + 90(x-4)}{105} \\ & \quad \quad \quad \text{105} \\ & \quad \quad \quad \text{(L.C.M. of 3, 5, 7 = 105)} \\ & = \frac{175x - 700 + 210x - 126 + 90x - 360}{105} \\ & = \frac{175x + 210x + 90x - 700 - 126 - 360}{105} \\ & = \frac{475x - 1186}{105} \end{aligned}$$

Question 22.

$$\left(p + \frac{p}{3}\right) \left(2p + \frac{p}{2}\right) \left(3p - \frac{2p}{3}\right)$$

Solution:

$$\begin{aligned} & \left(p + \frac{p}{3}\right) \left(2p + \frac{p}{2}\right) \left(3p - \frac{2p}{3}\right) \\ & = p \left(1 + \frac{1}{3}\right) p \left(2 + \frac{1}{2}\right) p \left(3 - \frac{2}{3}\right) \\ & = p^3 \left(\frac{3+1}{3}\right) \left(\frac{4+1}{2}\right) \left(\frac{9-2}{3}\right) \\ & = p^3 \times \frac{4}{3} \times \frac{5}{2} \times \frac{7}{3} = p^3 \times \frac{70}{9} = \frac{70p^3}{9} \end{aligned}$$

Question 23.

$$\frac{7}{30} \text{ of } \left(\frac{p}{3} + \frac{7p}{15} \right)$$

Solution:

$$\begin{aligned} & \frac{7}{30} \text{ of } \left(\frac{p}{3} + \frac{7p}{15} \right) \\ &= \frac{7}{30} \text{ of } \left(\frac{5p + 7p}{15} \right) = \frac{7}{30} \times \frac{12}{15} p = \frac{14p}{75} \end{aligned}$$

Question 24.

$$\left(2p + \frac{p}{7} \right) \div \left(\frac{9p}{10} + 4p \right)$$

Solution:

$$\begin{aligned} & \left(2p + \frac{p}{7} \right) \div \left(\frac{9p}{10} + 4p \right) \\ &= \frac{14p + p}{7} \div \frac{9p + 40p}{10} = \frac{15}{7} p \div \frac{49}{10} p \\ &= \frac{15}{7} p \times \frac{10}{49p} = \frac{150}{343} \end{aligned}$$

Question 25.

$$\left(\frac{5k}{8} - \frac{3k}{5} \right) \div \frac{k}{4}$$

Solution:

$$\begin{aligned} & \left(\frac{5k}{8} - \frac{3k}{5} \right) \div \frac{k}{4} \\ &= \frac{25k - 24k}{40} \div \frac{k}{4} = \frac{k}{40} \div \frac{k}{4} \\ &= \frac{k}{40} \times \frac{4}{k} = \frac{1}{10} \end{aligned}$$

Question 26.

$$\left(\frac{y}{6} + \frac{2y}{3}\right) \div \left(y + \frac{2y-1}{3}\right)$$

Solution:

$$\begin{aligned} &\left(\frac{y}{6} + \frac{2y}{3}\right) \div \left(y + \frac{2y-1}{3}\right) \\ &= \left(\frac{y+4y}{6}\right) \div \left(\frac{3y+2y-1}{3}\right) \\ &= \frac{5y}{6} \div \frac{5y-1}{3} = \frac{5y}{6} \times \frac{3}{5y-1} \\ &= \frac{5y}{2(5y-1)} = \frac{5y}{10y-2} \end{aligned}$$

EXERCISE 11 (F)

Enclose the given terms in brackets as required :

Question 1.

$$x - y - z = x - \{\dots\dots\}$$

Solution:

$$x - y - z = x - (y + z)$$

Question 2.

$$x^2 - xy^2 - 2xy - y^2 = x^2 - (\dots\dots\dots)$$

Solution:

$$\begin{aligned} &x^2 - xy^2 - 2xy - y^2 \\ &= x^2 - (xy^2 + 2xy + y^2) \end{aligned}$$

Question 3.

$$4a - 9 + 2b - 6 = 4a - (\dots\dots\dots)$$

Solution:

$$\begin{aligned} &4a - 9 + 2b - 6 \\ &= 4a - (9 - 2b + 6) \end{aligned}$$

Question 4.

$$x^2 - y^2 + z^2 + 3x - 2y = x^2 - (\dots\dots\dots)$$

Solution:

$$\begin{aligned} x^2 - y^2 + z^2 + 3x - 2y \\ = x^2 - (y^2 - z^2 - 3x + 2y) \end{aligned}$$

Question 5.

$$-2a^2 + 4ab - 6a^2b^2 + 8ab^2 = -2a (\dots\dots\dots)$$

Solution:

$$\begin{aligned} -2a^2 + 4ab - 6a^2b^2 + 8ab^2 \\ = -2a (a - 2b + 3ab^2 - 4b^2) \end{aligned}$$

Simplify :

Question 6.

$$2x - (x + 2y - z)$$

Solution:

$$\begin{aligned} 2x - (x + 2y - z) &= 2x - x - 2y + z \\ &= x - 2y + z \end{aligned}$$

Question 7.

$$p + q - (p - q) + (2p - 3q)$$

Solution:

$$\begin{aligned} p + q - (p - q) + (2p - 3q) \\ = p + q - p + q + 2p - 3q = 2p - q \end{aligned}$$

Question 8.

$$9x - (-4x + 5)$$

Solution:

$$\begin{aligned} 9x - (-4x + 5) &= 9x + 4x - 5 \\ &= 13x - 5 \end{aligned}$$

Question 9.

$$6a - (-5a - 8b) + (3a + b)$$

Solution:

$$6a - (-5a - 8b) + (3a + b)$$

$$\begin{aligned}
 &= 6a + 5a + 8b + 3a + b \\
 &= 6a + 5a + 3a + 8b + b \\
 &= 14a + 9b
 \end{aligned}$$

Question 10.

$$(p - 2q) - (3q - r)$$

Solution:

$$(p - 2q) - (3q - r) = p - 2q - 3q + r = p - 5q + r$$

Question 11.

$$9a(2b - 3a + 7c)$$

Solution:

$$\begin{aligned}
 &9a(2b - 3a + 7c) \\
 &= 18ab - 27a^2 + 63ca
 \end{aligned}$$

Question 12.

$$-5m(-2m + 3n - 7p)$$

Solution:

$$\begin{aligned}
 &-5m(-2m + 3n - 7p) \\
 &= -5m \times (-2m) + (-5m)(3n) - (-5m)(7p) \\
 &= 10m^2 - 15mn + 35mp.
 \end{aligned}$$

Question 13.

$$-2x(x + y) + x^2$$

Solution:

$$\begin{aligned}
 &-2x(x + y) + x^2 \\
 &= -2x \times x + (-2x)y + x^2 \\
 &= -2x^2 - 2xy + x^2 \\
 &= -2x^2 + x^2 - 2xy = -x^2 - 2xy
 \end{aligned}$$

Question 14.

$$b\left(2b - \frac{1}{b}\right) - 2b\left(b - \frac{1}{b}\right)$$

Solution:

$$\begin{aligned} & b\left(2b - \frac{1}{b}\right) - 2b\left(b - \frac{1}{b}\right) \\ &= b \times 2b - b \times \frac{1}{b} - 2b \times b + 2b \times \frac{1}{b} \\ &= 2b^2 - 1 - 2b^2 + 2 \\ &= 2b^2 - 2b^2 - 1 + 2 = 1 \end{aligned}$$

Question 15.

$$8(2a + 3b - c) - 10(a + 2b + 3c)$$

Solution:

$$\begin{aligned} & 8(2a + 3b - c) - 10(a + 2b + 3c) \\ &= 16a + 24b - 8c - 10a - 20b - 30c \\ &= 16a - 10a + 24b - 20b - 8c - 30c \\ &= 6a + 4b - 38c \end{aligned}$$

Question 16.

$$a\left(a + \frac{1}{a}\right) - b\left(b - \frac{1}{b}\right) - c\left(c + \frac{1}{c}\right)$$

Solution:

$$\begin{aligned} & a\left(a + \frac{1}{a}\right) - b\left(b - \frac{1}{b}\right) - c\left(c + \frac{1}{c}\right) \\ &= a^2 + 1 - b^2 + 1 - c^2 - 1 \\ &= a^2 - b^2 - c^2 + 1 \end{aligned}$$

Question 17.

$$5x(2x + 3y) - 2x(x - 9y)$$

Solution:

$$\begin{aligned} & 5x(2x + 3y) - 2x(x - 9y) \\ &= 10x^2 + 15xy - 2x^2 + 18xy \\ &= 10x^2 - 2x^2 + 15xy + 18xy \\ &= 8x^2 + 33xy \end{aligned}$$

Question 18.

$$a + (b + c - d)$$

Solution:

$$\begin{aligned} a + (b + c - d) &= a + (b + c - d) \\ &= a + b + c - d \end{aligned}$$

Question 19.

$$5 - 8x - 6 - x$$

Solution:

$$\begin{aligned} 5 - 8x - 6 - x \\ &= 5 - 6 - 8x - x \\ &= -1 - 7x \end{aligned}$$

Question 20.

$$2a + (6 - \overline{a - b})$$

Solution:

$$\begin{aligned} 2a + (6 - \overline{a - b}) \\ &= 2a + (b - a + b) \\ &= 2a + b - a + b \\ &= a + 2b \end{aligned}$$

Question 21.

$$3x + [4x - (6x - 3)]$$

Solution:

$$\begin{aligned} 3x + [4x - (6x - 3)] \\ &= 3x + [4x - 6x + 3] \\ &= 3x + 4x - 6x + 3 \\ &= 3x + 4x - 6x + 3 \\ &= 7x - 6x + 3 = x + 3 \end{aligned}$$

Question 22.

$$5b - \{6a + (8 - b - a)\}$$

Solution:

$$\begin{aligned} 5b - \{6a + 8 - 6a\} \\ &= 5b - 6a - 8 + b + a \\ &= -6a + a + 5b + b - 8 \\ &= -5a + 6b - 8 \end{aligned}$$

Question 23.

$$2x - [5y - (3x - y) + x]$$

Solution:

$$\begin{aligned} & 2x - [5y - (3x - y) + x] \\ &= 2x - \{5y - 3x + y + x\} \\ &= 2x - 5y + 3x - y - x \\ &= 2x + 3x - x - 5y - y \\ &= 4x - 6y \end{aligned}$$

Question 24.

$$6a - 3(a + b - 2)$$

Solution:

$$\begin{aligned} & 6a - 3(a + b - 2) \\ &= 6a - 3a - 3b + 6 \\ &= 3a - 3b + 6 \end{aligned}$$

Question 25.

$$8[m + 2n - p - 7(2m - n + 3p)]$$

Solution:

$$\begin{aligned} & 8[m + 2n - p - 7(2m - n + 3p)] \\ & 8[m + 2n - p - 14m + 7n - 21p] \\ &= 8m + 16n - 8p - 112m + 56n - 168p \\ &= 8m - 112m + 16n + 56n - 8p - 168p \\ &= -104m + 72n - 176p \end{aligned}$$

Question 26.

$$\{9 - (4p - 6q)\} - \{3q - (5p - 10)\}$$

Solution:

$$\begin{aligned} & \{9 - (4p - 6q)\} - \{3q - (5p - 10)\} \\ & \{9 - 4p + 6q\} - \{3q - 5p + 10\} \\ &= 9 - 4p + 6q - 3q + 5p - 10 \\ &= 9 - 4p + 5p + 6q - 3q - 10 \\ &= p + 3q - 1 \end{aligned}$$

Question 27.

$$2[a - 3\{a + 5\{a - 2\} + 7\}]$$

Solution:

$$2[a - 3\{a + 5\{a - 2\} + 7\}]$$

$$\begin{aligned}
&= 2 [a - 3 \{a + 5a - 10 + 7\}] \\
&= 2 [a - 3a - 15a + 30 - 21] \\
&= 2a - 6a - 30a + 60 - 42 \\
&= 2a - 36a + 60 - 42 \\
&= -34a + 18
\end{aligned}$$

Question 28.

$$5a - [6a - \{9a - (10a - \overline{4a - 3a})\}]$$

Solution:

$$\begin{aligned}
&5a - [6a - \{9a - (10a - 4a + 3a)\}] \\
&= 5a - [6a - \{9a - (10a - 4a + 3a)\}] \\
&= 5a - [6a - \{9a - 10a + 4a - 3a\}] \\
&= 5a - [6a - 9a + 10a - 4a + 3a] \\
&= 5a - 6a + 9a - 10a + 4a - 3a \\
&= 5a + 9a + 4a - 6a - 10a - 3a \\
&= 18a - 19a = -a
\end{aligned}$$

Question 29.

$$9x + 5 - [4x - \{3x - 2(4x - 3)\}]$$

Solution:

$$\begin{aligned}
&9x + 5 - [4x - \{3x - 2(4x - 3)\}] \\
&= 9x + 5 - [4x - \{3x - 8x + 6\}] \\
&= 9x + 5 - [4x - 3x + 8x - 6] \\
&= 9x + 5 - 4x + 3x - 8x + 6 \\
&= 9x + 3x - 4x - 8x + 5 + 6 \\
&= 12x - 12x + 11 = 11
\end{aligned}$$

Question 30.

$$(x + y - z)x + (z + x - y)y - (x + y - z)z$$

Solution:

$$\begin{aligned}
&(x + y - z)x + (z + x - y)y - (x + y - z)z \\
&= x^2 + xy - zx + yz + xy - y^2 - zx - yz + z^2 \\
&= x^2 - y^2 + z^2 + 2xy - 2zx
\end{aligned}$$

Question 31.

$$-1 [a - 3 \{b - 4(a - b - 8) + 4a\} + 10]$$

Solution:

$$\begin{aligned}
&-1 [a - 3 \{b - 4(a - b - 8) + 4a\} + 10] \\
&= -1 [a - 3 \{b - 4(a - b - 8) + 4a\} + 10]
\end{aligned}$$

$$\begin{aligned}
&= -1[a-3\{b-4a+Ab+32+4a\}+10] \\
&= -1[a-3b+12a-126-96-12a+10] \\
&= -a+3b-12a+12b+96+12a-10 \\
&= -a-12a+12a+3b+12b-96-10 \\
&= -a+15b-106
\end{aligned}$$

Question 32.

$$p^2 - [x^2 - \{x^2 - (q^2 - \overline{x^2 - q^2}) - 2y^2\}]$$

Solution:

$$\begin{aligned}
&p^2 - [x^2 - \{x^2 - (q^2 - \overline{x^2 - q^2}) - 2y^2\}] \\
&= p^2 - [x^2 - \{x^2 - (q^2 - x^2 + q^2) - 2y^2\}] \\
&= p^2 - [x^2 - \{x^2 - (2q^2 - x^2) - 2y^2\}] \\
&= p^2 - [x^2 - \{x^2 - 2q^2 + x^2 - 2y^2\}] \\
&= p^2 - x^2 + 2x^2 - 2q^2 - 2y^2 \\
&= p^2 + x^2 - 2q^2 - 2y^2
\end{aligned}$$

Question 33.

$$10 - \{4a - (7 - \overline{a - 5}) - (5a - \overline{1 + a})\}$$

Solution:

$$\begin{aligned}
&10 - \{4a - (7 - \overline{a - 5}) - (5a - \overline{1 + a})\} \\
&= 10 - \{4a - (7 - a + 5) - (5a - 1 - a)\} \\
&= 10 - \{4a - (12 - a) - (4a - 1)\} \\
&= 10 - \{4a - 12 + a - 4a + 1\} \\
&= 10 - 4a + 12 - a + 4a - 1 \\
&= 10 + 12 - 1 - 4a - a + 4a \\
&= 21 - a
\end{aligned}$$

Question 34.

$$7a - [8a - \{11a - (12a - \overline{6a - 5a})\}]$$

Solution:

$$\begin{aligned}
&7a - [8a - \{11a - (12a - \overline{6a - 5a})\}] \\
&= 7a - [8a - \{11a - (12a - 6a + 5a)\}] \\
&= 7a - [8a - \{11a - (17a - 6a)\}] \\
&= 7a - [8a - \{11a - (11a)\}]
\end{aligned}$$

$$= 7a - [8a - \{11a - 11a\}]$$

$$= 7a - 8a = -a$$

Question 35.

$$8x - [4y - \{4x + (2x - \overline{2y - 2x})\}]$$

Solution:

$$\begin{aligned} & 8x - [4y - \{4x + (2x - \overline{2y - 2x})\}] \\ &= 8x - [4y - \{4x + (2x - 2y + 2x)\}] \\ &= 8x - [4y - \{4x + (4x - 2y)\}] \\ &= 8x - [4y - \{4x + 4x - 2y\}] \\ &= 8x - [4y - 4x - 4x + 2y] \\ &= 8x - [-8x + 6y] \\ &= 8x + 8x - 6y \\ &= 16x - 6y \end{aligned}$$

Question 36.

$$x - (3y - \overline{4z - 3x} + 2z - \overline{5y - 7x})$$

Solution:

$$\begin{aligned} & x - (3y - \overline{4z - 3x} + 2z - \overline{5y - 7x}) \\ &= x - (3y - 4z + 3x + 2z - 5y + 7x) \\ &= x - (-2y - 2z + 10x) \\ &= x + 2y + 2z - 10x \\ &= -9x + 2y + 2z \end{aligned}$$