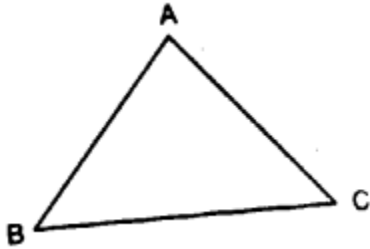


Triangles

POINTS TO REMEMBER

1. Definition of a triangle : A closed figure, having 3 sides, is called a triangle and is usually denoted by the Greek letter Δ (delta).



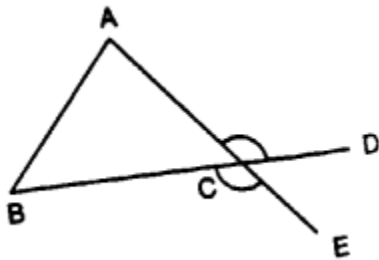
The figure, given alongside, shows a triangle ABC (ΔABC) bounded by three sides AB, BC and CA.

Hence it has six elements : 3 angles and 3 sides.

2. Vertex : The point, where any two sides of a triangle meet, is called a vertex. Clearly, the given triangle has three vertices; namely : A, B and C. [Vertices is the plural of vertex]

3. Interior angles : In ΔABC (given above), the angles BAC, ABC and ACB are called its interior angles as they lie inside the ΔABC . The sum of interior angles of a triangle is always 180° .

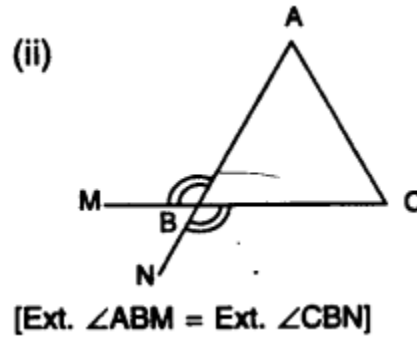
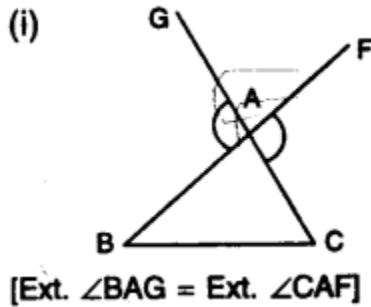
4. Exterior angles : When any side of a triangle is produced the angle so formed, outside the triangle and at its vertex, is called its **exterior angle**.



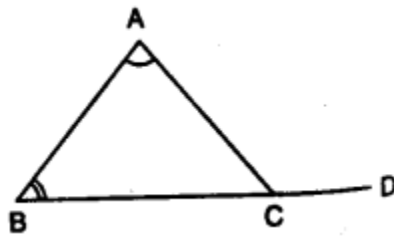
e.g. if side BC is produced to the point D; then $\angle ACD$ is its exterior angle. And, if side AC is produced to the point E, then the exterior angle would be $\angle BCE$.

Thus, at every vertex, two exterior angles can be formed and that these two angles being vertically opposite angles, are always equal.

Make the following figures clear :



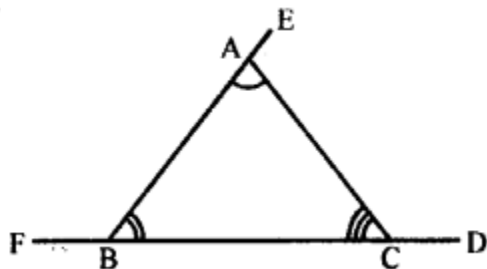
5. **Interior opposite angles** : When any side of a triangle is produced; an exterior angle is formed. The two interior angles of this triangle, that are opposite to the exterior angle formed; are called its **interior opposite angles**.



In the given figure, side BC of $\triangle ABC$ is produced to the point D, so that the exterior $\angle ACD$ is formed. Then the two interior opposite angles are $\angle B$ and $\angle A$.

6. **Relation between exterior angle and interior opposite angles** :

Exterior angle of a triangle is always equal to the sum of its two interior opposite angles.

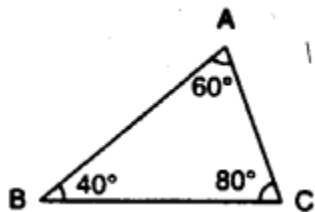


In $\triangle ABC$,
Ext. $\angle ACD = \angle A + \angle B$

7. CLASSIFICATION OF TRIANGLES

(A) With regard to their angles :

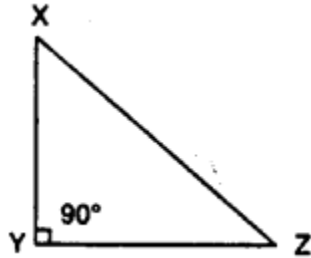
1. **Acute angled triangle** : It is a triangle, whose each angle is acute i.e. each angle is less than 90° .



2. Right angled triangle : It is a triangle, whose one angle is a right angle i.e. equal to 90° .

The figure, given alongside, shows a right angled triangle XYZ as $\angle XYZ = 90^\circ$

Note : (i) One angle of a right triangle is 90° and the other two angles of it are acute; such that their sum is always 90° .

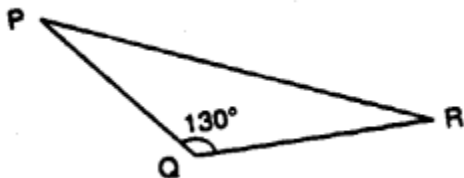


In $\triangle XYZ$, given above, $\angle Y = 90^\circ$ and each of $\angle X$ and $\angle Z$ is acute such that $\angle X + \angle Z = 90^\circ$.

(ii) In a right triangle, the side opposite to the right angle is largest of all its sides and is called the **hypotenuse**. In given right angled $\triangle XYZ$ side XZ is its hypotenuse

3. Obtuse angled triangle : If one angle of a triangle is obtuse, it is called an obtuse angled triangle.

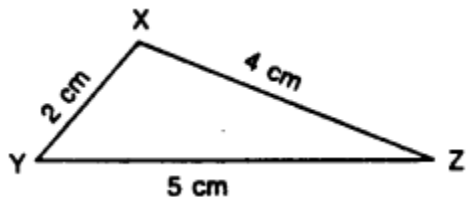
Note : In case of an obtuse angled triangle, each of the other two angles is always acute and their sum is less than 90° .



(B) With regard to their sides :

(1) Scalene triangle: If all the sides of a triangle are unequal, it is called a **scalene triangle**.

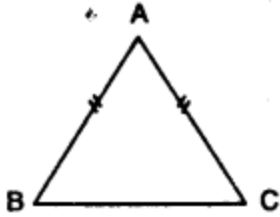
In a scalene triangle; all its angles are also unequal.



(2) Isosceles triangle : If atleast two sides of a triangle are equal, it is called an **isosceles triangle**.

In $\triangle ABC$, shown alongside, side AB = side AC.

$\therefore \triangle ABC$ is an isosceles triangle.



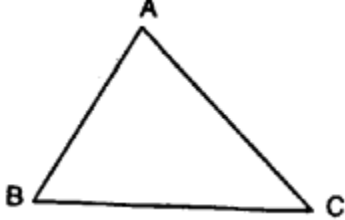
Note : (i) The angle contained by equal sides i.e. $\angle BAC$ is called the **vertical angle** or the **angle of vertex**.

(ii) The third side (i.e. the unequal side) is called the **base** of the isosceles triangle.

(iii) The two other angles (i.e. other than the angle of vertex) are called the **base angles** of the triangle.

IMPORTANT PROPERTIES OF AN ISOSCELES TRIANGLE

The base angles i.e. the angles opposite to equal sides of an isosceles triangle are always equal.



In given triangle ABC,

(i) If side $AB =$ side BC ; then angle opposite to $AB =$ angle opposite to BC i.e. $\angle C = \angle A$.

(ii) If side $BC =$ side AC ; then angle opposite to $BC =$ angle opposite to AC i.e. $\angle A = \angle B$ and so on.

Conversely : If any two angles of a triangle are equal; the sides opposite to these angles are also equal i.e. the triangle is isosceles.

Thus in $\triangle ABC$,

(i) If $\angle B = \angle C \Rightarrow$ side opposite to $\angle B =$ side opposite to $\angle C$ i.e. side $AC =$ side AB .

(ii) If $\angle A = \angle B \Rightarrow$ side $BC =$ side AC and so on.

(3) Equilateral triangle :

If all the sides of a triangle are equal, it is called an equilateral triangle.

In the given figure, $\triangle ABC$ is equilateral, because $AB = BC = CA$.

Also, all the angles of an equilateral triangle are equal to each other and so each angle $= 60^\circ$. [$\because 60^\circ + 60^\circ + 60^\circ = 180^\circ$]



Since, all the angles of an equilateral triangle are equal, it is also known as equiangular

triangle. Note : An equilateral triangle is always an isosceles triangle, but its converse is not always true.

(4) Isosceles right angled triangle : If one angle of an isosceles triangle is 90° , it is called an isosceles right angled triangle.

In the given figure, $\triangle ABC$ is an isosceles right angled triangle, because : $\angle ACB = 90^\circ$ and $AC = BC$.

Here, the base is AB , the vertex is C and the base angles are $\angle BAC$ and $\angle ABC$, which are equal.

Since, the sum of the angles of a triangle = 180°

$$\therefore \angle ABC = \angle BAC = 45^\circ [\because 45^\circ + 45^\circ + 90^\circ = 180^\circ]$$

EXERCISE 15 (A)

Question 1.

State, if the triangles are possible with the following angles :

(i) 20° , 70° and 90°

(ii) 40° , 130° and 20°

(iii) 60° , 60° and 50°

(iv) 125° , 40° and 15°

Solution:

We know that, the sum of three angles of a triangle is 180° , therefore

(i) Sum of 20° , 70° and 90°

$$= 20^\circ + 70^\circ + 90^\circ = 180^\circ.$$

Since the sum is 180° . Hence it is possible.

(ii) Sum of 40° , 130° and 20°

$$= 40^\circ + 130^\circ + 20^\circ = 190^\circ.$$

Since the sum is not 180° , therefore it is not possible.

(iii) Sum of 60° , 60° and 50°

$$= 60^\circ + 60^\circ + 50^\circ = 170^\circ.$$

Since the sum is not 180° , therefore it is not possible.

(iv) Sum of 125° , 40° and 15°

$$= 125^\circ + 40^\circ + 15^\circ = 180^\circ.$$

Since the sum is 180° , therefore it is possible.

Question 2.

If the angles of a triangle are equal, find its angles.

Solution:

Since the three angles of a triangle are equal and their sum is 180° , therefore each angle

$$\text{will be } \frac{180^\circ}{3} = 60^\circ.$$

Question 3.

In a triangle ABC, $\angle A = 45^\circ$ and $\angle B = 75^\circ$, find $\angle C$.

Solution:

Since the sum of angles of a triangle is 180°

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 45^\circ + 75^\circ + \angle C = 180^\circ$$

$$\Rightarrow 120^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - 120^\circ = 60^\circ$$

Question 4.

In a triangle PQR, $\angle P = 60^\circ$ and $\angle Q = \angle R$, find $\angle R$.

Solution:

$$\text{Let } \angle Q = \angle R = x, \angle P = 60^\circ$$

$$\text{But } \angle P + \angle Q + \angle R = 180^\circ$$

$$\therefore 60^\circ + x + x = 180^\circ$$

$$\Rightarrow 60^\circ + 2x = 180^\circ$$

$$\Rightarrow 2x = 180^\circ - 60^\circ = 120^\circ$$

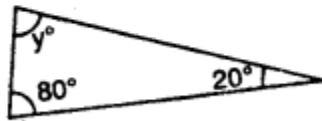
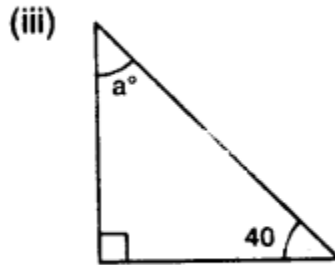
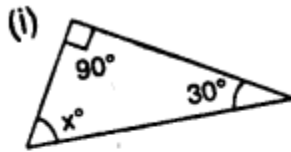
$$\Rightarrow x = \frac{120^\circ}{2} = 60^\circ$$

$$\therefore \angle Q = \angle R = 60^\circ$$

$$\text{Hence, } \angle R = 60^\circ$$

Question 5.

Calculate the unknown marked angles in each figure :



Solution:

We know that, the sum of three angles of a triangle is 180° , therefore

(i) In figure (i),

$$90^\circ + 30^\circ + x = 180^\circ$$

$$\Rightarrow 120^\circ + x = 180^\circ$$

$$\Rightarrow x = 180^\circ - 120^\circ = 60^\circ$$

Hence $x = 60^\circ$

(ii) In figure (ii),

$$y + 80^\circ + 20^\circ = 180^\circ$$

$$\Rightarrow y + 100^\circ = 180^\circ$$

$$\Rightarrow y = 180^\circ - 100^\circ = 80^\circ$$

Hence $y = 80^\circ$

(iii) In figure (iii),

$$a + 90^\circ + 40^\circ = 180^\circ$$

$$\Rightarrow a + 130^\circ = 180^\circ$$

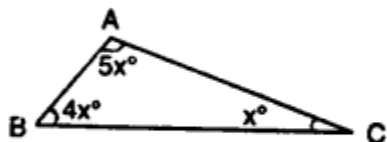
$$\Rightarrow a = 180^\circ - 130^\circ = 50^\circ$$

Hence $a = 50^\circ$

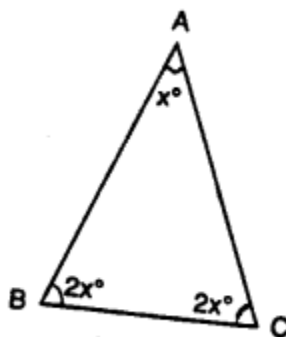
Question 6.

Find the value of each angle in the given figures:

(i)



(ii)



Solution:

(i) In the figure (i),

$$\angle A + \angle B + \angle C = 180^\circ$$

(Sum of angles of a triangle)

$$\Rightarrow 5x^\circ + 4x^\circ + x^\circ = 180^\circ$$

$$\Rightarrow 10x^\circ = 180^\circ \Rightarrow x = \frac{180^\circ}{10} = 18^\circ$$

$$\therefore \angle A = 5x^\circ = 5 \times 18^\circ = 90^\circ$$

$$\angle B = 4x = 4 \times 18^\circ = 72^\circ$$

$$\text{and } \angle C = x = 18^\circ$$

(ii) In figure (ii),

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow x^\circ + 2x^\circ + 2x^\circ = 180^\circ$$

$$\Rightarrow 5x^\circ = 180^\circ \Rightarrow x^\circ = \frac{180^\circ}{5} = 36^\circ$$

$$\therefore \angle A = x^\circ = 36^\circ$$

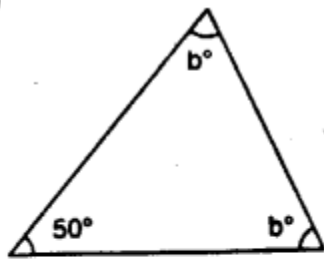
$$\angle B = 2x^\circ = 2 \times 36^\circ = 72^\circ$$

$$\text{and } \angle C = 2x^\circ = 2 \times 36^\circ = 72^\circ$$

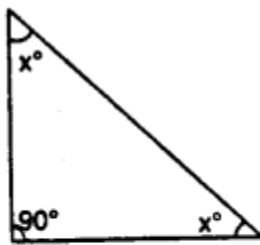
Question 7.

Find the unknown marked angles in the given figure:

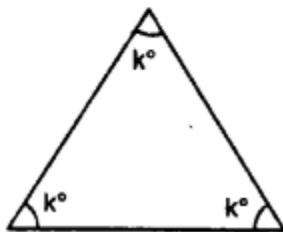
(i)



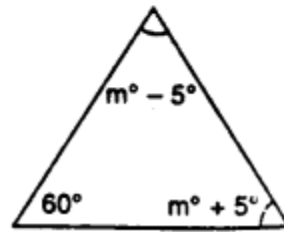
(ii)



(iii)



(iv)



Solution:

(i) In the figure (i),

$$\angle A + \angle B + \angle C = 180^\circ$$

(Sum of angles of a triangle)

$$b^\circ + 50^\circ + b^\circ = 180^\circ$$

$$\Rightarrow 2b^\circ + 50^\circ = 180^\circ$$

$$\Rightarrow 2b^\circ = 180^\circ - 50^\circ = 130^\circ$$

$$\Rightarrow b^\circ = \frac{130^\circ}{2} = 65^\circ$$

$$\text{Hence } \angle A = b^\circ = 65^\circ$$

$$\text{and } \angle C = b^\circ = 65^\circ$$

(ii) In the figure (ii)

$$\angle A + \angle B + \angle C = 180^\circ$$

(Sum of angles of a triangle)

$$x^\circ + 90^\circ + x^\circ = 180^\circ$$

$$2x^\circ + 90^\circ = 180^\circ$$

$$2x^\circ = 180^\circ - 90^\circ$$

$$2x^\circ = 90^\circ$$

$$x^\circ = \frac{90^\circ}{2} = 45^\circ$$

$$\text{Hence } \angle A = x^\circ = 45^\circ$$

$$\text{and } \angle C = x^\circ = 45^\circ$$

(iii) In the figure (iii)

$$\angle A + \angle B + \angle C = 180^\circ$$

(Sum of angles of a triangle)

$$k^\circ + k^\circ + k^\circ = 180^\circ$$

$$3k^\circ = 180^\circ$$

$$k^\circ = \frac{180^\circ}{3} = 60^\circ$$

$$\text{Hence } \angle A = k^\circ = 60^\circ, \angle B = k^\circ = 60^\circ$$

$$\text{and } \angle C = k^\circ = 60^\circ$$

(iv) In the figure (iv)

$$\angle A + \angle B + \angle C = 180^\circ$$

(Sum of Angles of a triangle)

$$(m^\circ - 5^\circ) + 60^\circ + (m^\circ + 5^\circ) = 180^\circ$$

$$m^\circ - 5^\circ + 60^\circ + m^\circ + 5^\circ = 180^\circ$$

$$2m^\circ = 180^\circ - 65 + 5$$

$$2m^\circ = 120^\circ$$

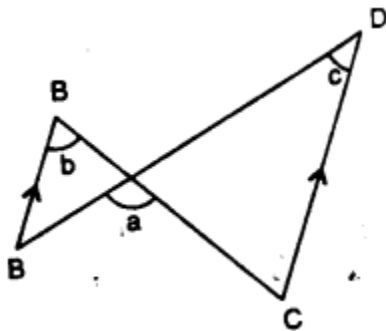
$$\therefore m^\circ = \frac{120^\circ}{2} = 60^\circ$$

$$\text{Hence } \angle A = m^\circ - 5^\circ = 60^\circ - 5^\circ = 55^\circ$$

$$\text{and } \angle C = m^\circ + 5^\circ = 60^\circ + 5^\circ = 65^\circ$$

Question 8.

In the given figure, show that: $\angle a = \angle b + \angle c$



- (i) If $\angle b = 60^\circ$ and $\angle c = 50^\circ$; find $\angle a$.
(ii) If $\angle a = 100^\circ$ and $\angle b = 55^\circ$: find $\angle c$.
(iii) If $\angle a = 108^\circ$ and $\angle c = 48^\circ$; find $\angle b$.

Solution:

$\therefore AB \parallel CD$

$\therefore b = c$ and $\angle A = \angle C$ (Alternate angles)

Now in $\triangle PCD$,

$$\text{Ext. } \angle APC = \angle C + \angle D$$

$$\Rightarrow a = b + c$$

(i) If $b = 60^\circ$, $c = 50^\circ$, then

$$a = b + c = 60^\circ + 50^\circ = 110^\circ$$

(ii) If $a = 100^\circ$ and $b = 55^\circ$,

$$\text{then } a = b + c \Rightarrow 100^\circ = 55^\circ + c$$

$$\Rightarrow c = 100^\circ - 55^\circ = 45^\circ$$

(iii) If $a = 108^\circ$ and $c = 48^\circ$,

$$\text{then } a = b + c \Rightarrow 108^\circ = b + 48^\circ$$

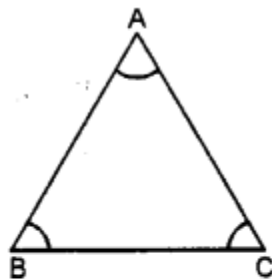
$$\Rightarrow b = 108^\circ - 48^\circ = 60^\circ$$

Question 9.

Calculate the angles of a triangle if they are in the ratio 4 : 5 : 6.

Solution:

We know that sum of angles of a triangle is 180°



$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

But $\angle A : \angle B : \angle C = 4 : 5 : 6$

Let $\angle A = 4x$, $\angle B = 5x$ and $\angle C = 6x$,

$$\text{then } 4x + 5x + 6x = 180^\circ$$

$$\Rightarrow 15x = 180^\circ \Rightarrow x = \frac{180^\circ}{15} = 12^\circ$$

$$\therefore \angle A = 4x = 4 \times 12^\circ = 48^\circ$$

$$\angle B = 5x = 5 \times 12^\circ = 60^\circ$$

$$\angle C = 6x = 6 \times 12^\circ = 72^\circ$$

Question 10.

One angle of a triangle is 60° . The other two angles are in the ratio of 5 : 7. Find the two angles.

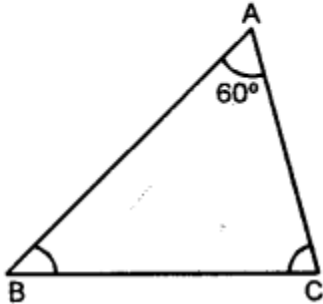
Solution:

In $\triangle ABC$,

Let $\angle A = 60^\circ$ and then $\angle B : \angle C = 5 : 7$

But $\angle A + \angle B + \angle C = 180^\circ$

(Angles of a triangle)



$$\Rightarrow 60^\circ + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle B + \angle C = 180^\circ - 60^\circ = 120^\circ$$

Let $\angle B = 5x$ and $\angle C = 7x$

$$\therefore 5x + 7x = 120^\circ$$

$$\Rightarrow 12x = 120^\circ \quad \Rightarrow x = \frac{120^\circ}{12} = 10^\circ$$

$$\therefore \angle B = 5x = 5 \times 10^\circ = 50^\circ$$

$$\angle C = 7x = 7 \times 10 = 70^\circ$$

Question 11.

One angle of a triangle is 61° and the other two angles are in the ratio $1\frac{1}{2} : 1\frac{1}{3}$. Find these angles.

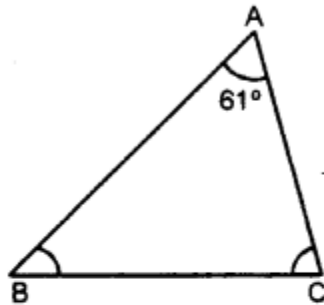
Solution:

In $\triangle ABC$,

Let $\angle A = 61^\circ$

But $\angle A + \angle B + \angle C = 180^\circ$

(Angles of a triangle)



$$\Rightarrow 61^\circ + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle B + \angle C = 180^\circ - 61^\circ = 119^\circ$$

$$\text{But } \angle B : \angle C = 1\frac{1}{2} : 1\frac{1}{3} = \frac{3}{2} : \frac{4}{3}$$

$$= \frac{9:8}{6} = 9:8$$

Let $\angle B = 9x$ and $\angle C = 8x$,

then, $9x + 8x = 119^\circ$

$$\Rightarrow 17x = 119^\circ$$

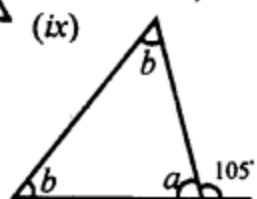
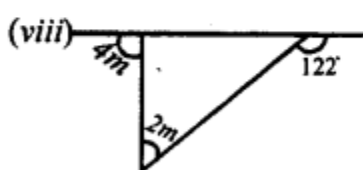
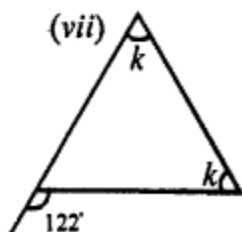
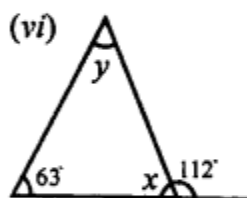
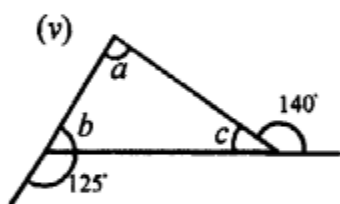
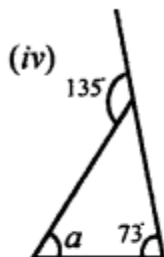
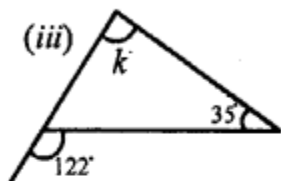
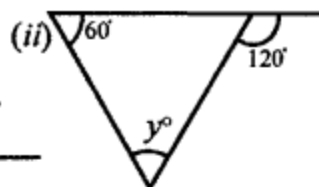
$$\Rightarrow x = \frac{119^\circ}{17} = 7^\circ$$

$$\therefore \angle B = 9x = 9 \times 7^\circ = 63^\circ$$

$$\angle C = 8x = 8 \times 7^\circ = 56^\circ$$

Question 12.

Find the unknown marked angles in the given figures :



Solution:

We know that in a triangle, if one side of it is produced, then

Exterior angle = sum of its interior opposite angles.

(i) In Fig. (i), $110^\circ = x^\circ + 30^\circ$

$$\Rightarrow x^\circ = 110^\circ - 30^\circ = 80^\circ$$

(ii) In Fig. (ii), $120^\circ = y^\circ + 60^\circ$

$$\Rightarrow y^\circ = 120^\circ - 60^\circ = 60^\circ$$

(iii) In Fig. (iii), $122^\circ = k^\circ + 35^\circ$

$$\Rightarrow k^\circ = 122^\circ - 35^\circ = 87^\circ$$

(iv) In Fig. (iv), $135^\circ = a^\circ + 73^\circ$

$$\Rightarrow a^\circ = 135^\circ - 73^\circ = 62^\circ$$

(v) In Fig. (v), $125^\circ = a + c$... (i)

and $140^\circ = a + b$... (ii)

Adding, we get

$$a + c + a + b = 125^\circ + 140^\circ$$

$$\Rightarrow a + a + b + c = 265^\circ$$

But $a + b + c = 180^\circ$

(Sum of angles of a triangle)

$$\therefore a + 180^\circ = 265^\circ$$

$$\Rightarrow a = 265^\circ - 180^\circ = 85^\circ$$

But $a + b = 140^\circ$

$$\Rightarrow 85^\circ + b = 140^\circ$$

$$\Rightarrow b = 140^\circ - 85^\circ = 55^\circ$$

and $a + c = 125^\circ \Rightarrow 85^\circ + c = 125^\circ$

$$\Rightarrow c = 125^\circ - 85^\circ = 40^\circ$$

Hence $a = 85^\circ$, $b = 55^\circ$ and $c = 40^\circ$

(vi) In Fig. (vi),

$$112^\circ + x^\circ = 180^\circ \quad (\text{Linear pair})$$

$$\Rightarrow x = 180^\circ - 112^\circ = 68^\circ$$

and $112^\circ = y + 63^\circ$

$$\Rightarrow y = 112^\circ - 63^\circ = 49^\circ$$

Hence $x = 68^\circ$, $y = 49^\circ$

(vii) In fig. (vii),

$$120^\circ = a + a \Rightarrow 2a = 120^\circ$$

$$\Rightarrow a = \frac{120^\circ}{2} = 60^\circ$$

$\therefore a = 60^\circ$ Ans.

(viii) In fig. (viii),

$$140^\circ + a = 180^\circ \quad (\text{Linear pair})$$

$$\Rightarrow a = 180^\circ - 140^\circ = 40^\circ$$

Now $4m = 2m = a \Rightarrow 4m - 2m = a$

$$\Rightarrow 2m = 40^\circ \Rightarrow m = \frac{40^\circ}{2} = 20^\circ$$

Hence $m = 20^\circ$

(ix) In fig. (ix),

$$105^\circ = b + b \Rightarrow 2b = 105^\circ$$

$$\Rightarrow b = \frac{105^\circ}{2} = 52.5^\circ$$

But $a + 105^\circ = 180^\circ$ (Linear pair)

$$\Rightarrow a = 180^\circ - 105^\circ = 75^\circ$$

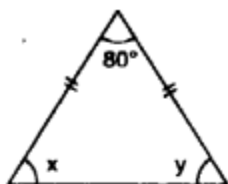
Hence $a = 75^\circ$, $b = 52.5^\circ$

EXERCISE 15 (B)

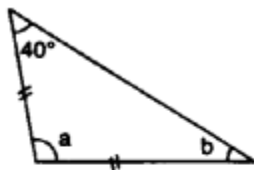
Question 1.

Find the unknown angles in the given figures:

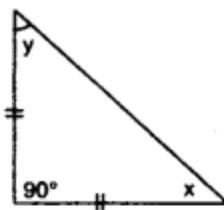
(i)



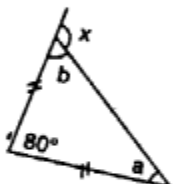
(ii)



(iii)

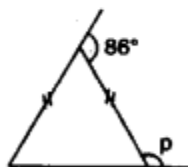


(iv)

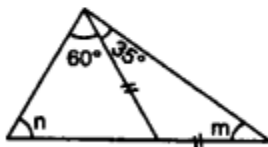


(v)

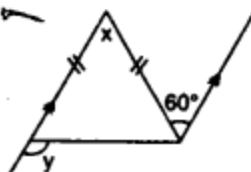
(v)



(vi)



(vii)



Solution:

(i) In Fig (i),

$$x = y \quad (\text{Angles opposite to equal sides})$$

$$\text{But } x + y + 80^\circ = 180^\circ \quad (\text{Angles of a triangle})$$

$$\Rightarrow x + x + 80^\circ = 180^\circ$$

$$\Rightarrow 2x + 80^\circ = 180^\circ$$

$$\Rightarrow 2x = 180^\circ - 80^\circ = 100^\circ$$

$$\Rightarrow x = \frac{100^\circ}{2} = 50^\circ \quad \therefore y = x = 50^\circ$$

$$\text{Hence } x = 50^\circ, y = 50^\circ$$

(ii) In Fig. (ii),

$$b = 40^\circ \quad (\text{Angles opposite to equal sides})$$

$$\text{But } a + b + 40^\circ = 180^\circ$$

(Angles of a triangle)

$$\Rightarrow a + 40^\circ + 40^\circ = 180^\circ$$

$$\Rightarrow a + 80^\circ = 180^\circ$$

$$\Rightarrow a = 180^\circ - 80^\circ = 100^\circ$$

$$\text{Hence } a = 100^\circ, b = 40^\circ$$

(iii) In Fig. (iii),

$$x = y \quad (\text{Angles opposite to equal sides})$$

$$\text{But } x + y + 90^\circ = 180^\circ$$

(Angles of a triangle)

$$\Rightarrow x + x + 90^\circ = 180^\circ$$

$$\Rightarrow 2x + 90^\circ = 180^\circ$$

$$\Rightarrow 2x = 180^\circ - 90^\circ = 90^\circ$$

$$\therefore x = \frac{90^\circ}{2} = 45^\circ \quad \therefore y = x = 45^\circ$$

$$\text{Hence } x = 45^\circ, y = 45^\circ$$

(iv) In Fig. (iv),

$$a = b \quad (\text{Angles opposite to equal sides})$$

$$\text{But } a + b + 80^\circ = 180^\circ$$

(Angles of a triangle)

$$\Rightarrow a + a + 80^\circ = 180^\circ$$

$$\Rightarrow 2a + 80^\circ = 180^\circ$$

$$\Rightarrow 2a = 180^\circ - 80^\circ = 100^\circ$$

$$\Rightarrow a = \frac{100^\circ}{2} = 50^\circ \quad \therefore b = a = 50^\circ$$

$$x = a + 80^\circ$$

(Exterior angle of a triangle is equal to sum of its opposite interior angles)

$$= 50^\circ + 80^\circ = 130^\circ$$

$$\text{Hence } a = 50^\circ, b = 50^\circ \text{ and } x = 130^\circ$$

(v) In Fig. (v),

Let each equal angle of an isosceles triangle be x ,

$$\text{then } x + x = 86^\circ \Rightarrow 2x = 86^\circ$$

$$\Rightarrow x = \frac{86^\circ}{2} = 43^\circ$$

$$\text{But } p + x = 180^\circ \quad (\text{Linear pair})$$

$$p + 43^\circ = 180^\circ$$

$$\Rightarrow p = 180^\circ - 43^\circ \Rightarrow p = 137^\circ$$

$$\text{Hence } p = 137^\circ$$

(vi) In Fig. (vi),

$$m = 35^\circ \quad (\text{Angles opposite to equal sides})$$

$$\text{But } m + n + (60^\circ + 35^\circ) = 180^\circ$$

(Angles of a triangle)

$$\Rightarrow m + n + 95^\circ = 180^\circ$$

$$\Rightarrow 35^\circ + n + 95^\circ = 180^\circ$$

$$\Rightarrow n + 130^\circ = 180^\circ$$

$$\Rightarrow n = 180^\circ - 130^\circ = 50^\circ$$

$$\text{Hence } m = 35^\circ, n = 50^\circ$$

(vii) In Fig. (vii),

$$x = 60^\circ \quad (\text{Alternate angles})$$

Let each equal angle of an isosceles triangle

$$\text{be } a \text{ then } a + a + x = 180^\circ$$

(Angles of a triangle)

$$2a + x = 180^\circ \Rightarrow 2a + 60^\circ = 180^\circ$$

$$\Rightarrow 2a = 180^\circ - 60^\circ = 120^\circ$$

$$\Rightarrow a = \frac{120^\circ}{2} = 60^\circ$$

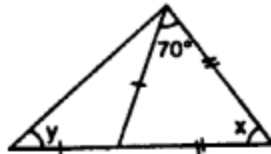
$$\therefore y = x + a = 60^\circ + 60^\circ = 120^\circ$$

$$\text{Hence } x = 60^\circ \text{ and } y = 120^\circ$$

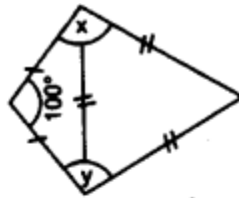
Question 2.

Apply the properties of isosceles and equilateral triangles to find the unknown angles in the given figures :

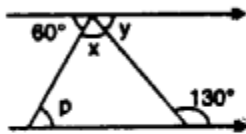
(i)



(ii)



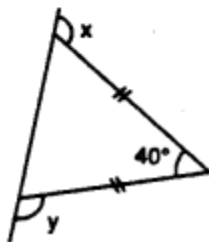
(iii)



(iv)



(v)



(vi)



Solution:

$$a = 70^\circ \quad (\text{Angles opposite to equal sides})$$

$$\text{But } a + 70^\circ + x = 180^\circ \quad (\text{Angles of a triangle})$$

$$\Rightarrow 70^\circ + 70^\circ + x = 180^\circ$$

$$\Rightarrow 140^\circ + x = 180^\circ$$

$$\Rightarrow x = 180^\circ - 140^\circ = 40^\circ$$

$$y = b \quad (\text{Angles opposite to equal sides})$$

$$\text{But } a = y + b$$

(Exterior angle of a triangle is equal to sum of its interior opposite angles)

$$\Rightarrow 70^\circ = y + y \quad \Rightarrow 2y = 70^\circ$$

$$\Rightarrow y = \frac{70^\circ}{2} = 35^\circ$$

$$\text{Hence } x = 40^\circ, y = 35^\circ$$

(ii) In Fig. (ii),

In an equilateral triangle.

$$\text{each angle} = 60^\circ$$

In isosceles triangle.,

$$\text{Let each base angle} = a$$

$$\therefore a + a + 100^\circ = 180^\circ$$

$$\Rightarrow 2a + 100^\circ = 180^\circ$$

$$\Rightarrow 2a = 180^\circ - 100^\circ = 80^\circ$$

$$\therefore a = \frac{80^\circ}{2} = 40^\circ \quad \therefore x = 60^\circ + 40^\circ = 100^\circ$$

$$\text{and } y = 60^\circ + 40^\circ = 100^\circ$$

(iii) In Fig. (iii), $130^\circ = x + p$

(Exterior angle of a triangle is equal to the sum of its interior opposite angles)

$$\therefore \text{Lines are parallel} \quad (\text{Given})$$

$$\therefore p = 60^\circ \quad (\text{Alternate angle})$$

$$\text{and } y = a$$

$$\text{But } a + 130^\circ = 180^\circ \quad (\text{Linear pair})$$

$$\Rightarrow a = 180^\circ - 130^\circ = 50^\circ \quad \therefore y = 50^\circ$$

$$\text{and } x + p = 130^\circ$$

$$\Rightarrow x + 60^\circ = 130^\circ \Rightarrow x = 130^\circ - 60^\circ = 70^\circ$$

$$\text{Hence } x = 70^\circ, y = 50^\circ \text{ and } p = 60^\circ$$

(iv) In Fig. (iv),

$$x = a + b$$

But $b = y$ (Angles opposite to equal sides)

Similarly $a = c$

$$\text{But } a + c + 30^\circ = 180^\circ$$

$$\Rightarrow a + a + 30^\circ = 180^\circ \Rightarrow 2a + 30^\circ = 180^\circ$$

$$\Rightarrow 2a = 180^\circ - 30^\circ = 150^\circ$$

$$\Rightarrow a = \frac{150^\circ}{2} = 75^\circ \text{ and } b + y = 90^\circ$$

$$\Rightarrow y + y = 90^\circ \quad \Rightarrow 2y = 90^\circ$$

$$\Rightarrow y = \frac{90^\circ}{2} = 45^\circ \quad \Rightarrow b = 45^\circ$$

$$\text{Hence } x = a + b = 75^\circ + 45^\circ = 120^\circ$$

$$\text{and } y = 45^\circ$$

(v) In Fig. (v),

$$a + b + 40^\circ = 180^\circ \quad (\text{Angles of a triangle})$$

$$\Rightarrow a + b = 180^\circ - 40^\circ = 140^\circ$$

But $a = b$ (Angles opposite to equal sides)

$$\therefore a = b = \frac{140^\circ}{2} = 70^\circ$$

$$\therefore x = b + 40^\circ = 70^\circ + 40^\circ = 110^\circ$$

(Exterior angle of a triangle is equal to the sum of its interior opposite angles)

$$\begin{aligned} \text{Similarly } y &= a + 40^\circ \\ &= 70^\circ + 40^\circ = 110^\circ \end{aligned}$$

$$\text{Hence } x = 110^\circ, y = 110^\circ$$

(vi) In the Fig. (vi),

$$a = b \quad (\text{Angles opposite to equal sides})$$

$$\therefore y = 120^\circ$$

$$\text{But } a + 120^\circ = 180^\circ \quad (\text{Linear pair})$$

$$\Rightarrow a = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore b = 60^\circ$$

$$\text{But } x + a + b = 180^\circ \quad (\text{Angles of a triangle})$$

$$\Rightarrow x + 60^\circ + 60^\circ = 180^\circ$$

$$\Rightarrow x + 120^\circ = 180^\circ$$

$$\therefore x = 180^\circ - 120^\circ = 60^\circ$$

$$b = z + 25^\circ$$

(Exterior angle of a triangle is equal to the sum of its interior opposite angles)

$$\Rightarrow 60^\circ = z + 25^\circ$$

$$\Rightarrow z = 60^\circ - 25^\circ = 35^\circ$$

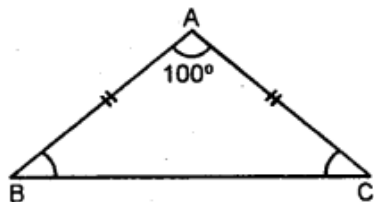
$$\text{Hence } x = 60^\circ, y = 120^\circ \text{ and } z = 35^\circ$$

Question 3.

The angle of vertex of an isosceles triangle is 100° . Find its base angles.

Solution:

In ΔABC ,



$$\therefore AB = AC.$$

$$\therefore \angle B = \angle C$$

$$\text{But } \angle A = 100^\circ$$

$$\text{and } \angle A + \angle B + \angle C = 180^\circ$$

(Angles of a triangle)

$$\Rightarrow 100^\circ + \angle B + \angle B = 180^\circ$$

$$\Rightarrow 2 \angle B = 180^\circ - 100^\circ \Rightarrow 2 \angle B = 80^\circ$$

$$\therefore \angle B = \frac{80^\circ}{2} = 40^\circ$$

$$\text{Hence } \angle B = \angle C = 40^\circ$$

Question 4.

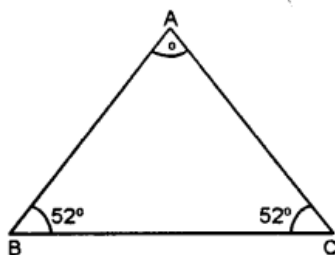
One of the base angles of an isosceles triangle is 52° . Find its angle of vertex.

Solution:

Each of the base angles of an isosceles

$$\Delta ABC = 52^\circ$$

$$\therefore \angle B = \angle C = 52^\circ$$



$$\text{But } \angle A + \angle B + \angle C = 180^\circ$$

(Angles of a triangle)

$$\Rightarrow \angle A + 52^\circ + 52^\circ = 180^\circ$$

$$\Rightarrow \angle A + 104^\circ = 180^\circ$$

$$\Rightarrow \angle A = 180^\circ - 104^\circ = 76^\circ$$

$$\text{Hence } \angle A = 76^\circ$$

Question 5.

In an isosceles triangle, each base angle is four times of its vertical angle. Find all the angles of the triangle.

Solution:

Let vertical angle of an isosceles triangle = x

$\therefore$ Each base angle = $4x$

$\therefore x + 4x + 4x = 180^\circ$

(Sum of angles of a triangle)

$$\Rightarrow 9x = 180^\circ \quad \Rightarrow \quad x = \frac{180^\circ}{9} = 20^\circ$$

$\therefore$ Vertical angle = 20°

and each of the base angle = $4x$

$$= 4 \times 20^\circ = 80^\circ$$

Question 6.

The vertical angle of an isosceles triangle is 15° more than each of its base angles. Find each angle of the triangle.

Solution:

Let each angle of the base of the isosceles triangle = x°

Then vertical angle = $x + 15^\circ$

Now $x + x + x + 15^\circ = 180^\circ$

(Sum of angles of a triangle)

$$\Rightarrow 3x + 15^\circ = 180^\circ$$

$$\Rightarrow 3x = 180^\circ - 15^\circ = 165^\circ$$

$$\therefore x = \frac{165^\circ}{3} = 55^\circ$$

Hence each base angle = 55°

and vertical angle = $55^\circ + 15^\circ = 70^\circ$

Question 7.

The base angle of an isosceles triangle is 15° more than its vertical angle. Find its each angle.

Solution:

Let vertical angle of the isosceles triangle
 $= x^\circ$

$\therefore$ Each base angle $= x + 15^\circ$

$$\therefore x + 15^\circ + x + 15^\circ + x^\circ = 180^\circ$$

(Sum of angles of a triangle)

$$\Rightarrow 3x + 30^\circ = 180^\circ$$

$$\Rightarrow 3x = 180^\circ - 30^\circ = 150^\circ$$

$$\therefore x = \frac{150^\circ}{3} = 50^\circ$$

Hence vertical angle $= 50^\circ$

and each base angle $= 50^\circ + 15^\circ = 65^\circ$

Question 8.

The vertical angle of an isosceles triangle is three times the sum of its base angles. Find each angle.

Solution:

Let each base angle of an isosceles triangle
 $= x$

then its vertical $= 3(x + x) = 3 \times 2x = 6x$.

$$\therefore 6x + x + x = 180^\circ$$

(Sum of angles of a triangle)

$$\Rightarrow 8x = 180^\circ \quad \Rightarrow x = \frac{180^\circ}{8} = 22.5^\circ$$

$\therefore$ Each base angle $= 22.5^\circ$

and vertical angle $= 3 \times (22.5 + 22.5)$
 $= 3 \times 45 = 135^\circ$

Question 9.

The ratio between a base angle and the vertical angle of an isosceles triangle is 1 : 4. Find each angle of the triangle.

Solution:

Ratio between base angle and vertical angle
of an isosceles triangle = 1 : 4

Let each base angle = x

then vertical angle = $4x$

$$\therefore x + x + 4x = 180^\circ$$

(Sum of angles of a triangle)

$$\Rightarrow 6x = 180^\circ \quad \Rightarrow$$

$$x = \frac{180^\circ}{6} = 30^\circ$$

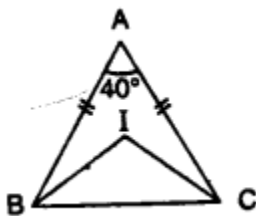
$$\therefore \text{Each base angle} = x = 30^\circ$$

and vertical angle = $4x$

$$= 4 \times 30^\circ = 120^\circ$$

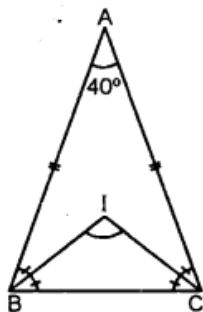
Question 10.

In the given figure, BI is the bisector of $\angle ABC$ and CI is the bisector of $\angle ACB$. Find $\angle BIC$.

**Solution:**

In $\triangle ABC$,

BI is the bisector of $\angle ABC$ and CI is the
bisector of $\angle ACB$.



$$\therefore AB = AC$$

$$\therefore \angle B = \angle C$$

(Angles opposite to equal sides)

$$\text{But } \angle A = 40^\circ$$

$$\text{and } \angle A + \angle B + \angle C = 180^\circ$$

(Angles of a triangle)

$$\Rightarrow 40^\circ + \angle B + \angle B = 180^\circ$$

$$\Rightarrow 40^\circ + 2\angle B = 180^\circ$$

$$\Rightarrow 2\angle B = 180^\circ - 40^\circ = 140^\circ$$

$$\Rightarrow \angle B = \frac{140^\circ}{2} = 70^\circ$$

$$\therefore \angle ABC = \angle ACB = 70^\circ$$

But BI and CI are the bisectors of $\angle ABC$ and $\angle ACB$ respectively.

$$\therefore \angle IBC = \frac{1}{2} \angle ABC = \frac{1}{2} (70^\circ) = 35^\circ$$

$$\text{and } \angle ICB = \frac{1}{2} \angle ACB = \frac{1}{2} \times 70^\circ = 35^\circ$$

Now in $\triangle IBC$,

$$\angle BIC + \angle IBC + \angle ICB = 180^\circ$$

(Angles of a triangle)

$$\Rightarrow \angle BIC + 35^\circ + 35^\circ = 180^\circ$$

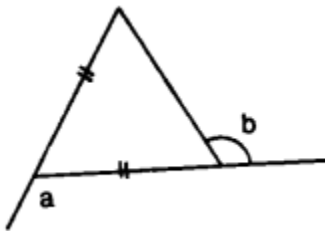
$$\Rightarrow \angle BIC + 70^\circ = 180^\circ$$

$$\Rightarrow \angle BIC = 180^\circ - 70^\circ = 110^\circ$$

$$\text{Hence } \angle BIC = 110^\circ$$

Question 11.

In the given figure, express a in terms of b .



Solution:

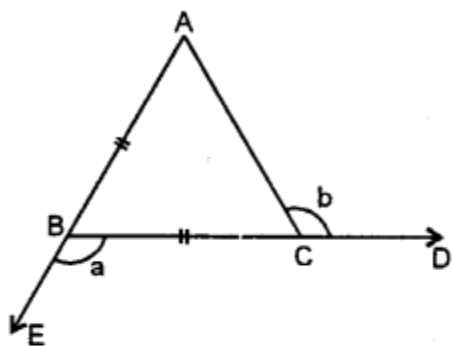
In $\triangle ABC$,

$$BC = BA$$

$$\therefore \angle BCA = \angle BAC$$

$$\text{and Ext. } \angle CBE = \angle BCA + \angle BAC$$

$$\Rightarrow a = \angle BCA + \angle BCA$$



$$\Rightarrow a = 2 \angle BCA \quad \dots(i)$$

$$\text{But } \angle ACB = 180^\circ - b$$

($\because \angle ACD$ and $\angle ACB$ are linear pair)

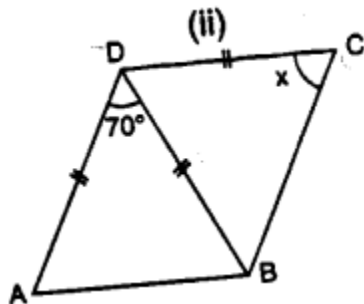
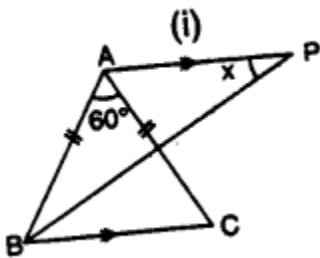
$$\Rightarrow \angle BCA = 180^\circ - b \quad \dots(ii)$$

$$\begin{aligned} \therefore a &= 2 \angle BCA = 2 (180^\circ - b) \\ &= 360^\circ - 2b \end{aligned}$$

Question 12.

(a) In Figure (i) BP bisects $\angle ABC$ and $AB = AC$. Find x .

(b) Find x in Figure (ii) Given: $DA = DB = DC$, BD bisects $\angle ABC$ and $\angle ADB = 70^\circ$.



Solution:

(a) In figure (i),

$AB = AC$, and BP bisects $\angle ABC$

AP||BC is drawn.

Now $\angle PBC = \angle PBA$

($\because$ PB is the bisector of $\angle ABC$)

$\therefore AP \parallel BC$

$\therefore \angle APB = \angle PBC$ (Alternate angles)

$$\Rightarrow x = \angle PBC \quad \dots(i)$$

In $\triangle ABC$, $\angle A = 60^\circ$

and $\angle B = \angle C$ ($\because AB = AC$)

But $\angle A + \angle B + \angle C = 180^\circ$

(Angles of a triangle)

$$\Rightarrow 60^\circ + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 60^\circ + \angle B + \angle B = 180^\circ$$

$$\Rightarrow 2 \angle B = 180^\circ - 60^\circ = 120^\circ$$

$$\therefore \angle B = \frac{120^\circ}{2} = 60^\circ$$

$$\Rightarrow \frac{1}{2} \angle B = \frac{60^\circ}{2} = 30^\circ \Rightarrow \angle PBC = 30^\circ$$

$\therefore$ From (i),

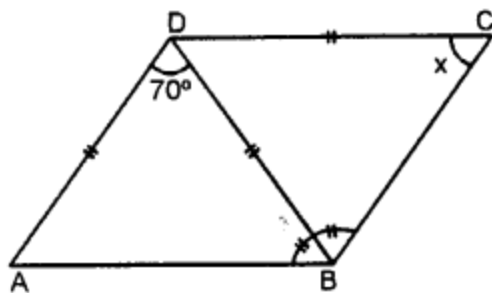
$$x = 30^\circ$$

(b) In the figure (ii),

$$DA = DB = DC$$

BD bisects $\angle ABC$

$$\text{and } \angle ADB = 70^\circ$$



$$\text{But } \angle ADB + \angle DAB + \angle DBA = 180^\circ$$

(Angles of a triangle)

$$\Rightarrow 70^\circ + \angle DBA + \angle DBA = 180^\circ$$

$$(\because DA = DB)$$

$$\Rightarrow 70^\circ + 2 \angle DBA = 180^\circ$$

$$\Rightarrow 2 \angle DBA = 180^\circ - 70^\circ$$

$$= 110^\circ$$

$$\therefore \angle DBA = \frac{110^\circ}{2} = 55^\circ$$

$\therefore$ BD is the bisector of $\angle ABC$,

$$\therefore \angle DBA = \angle DBC, = 55^\circ$$

But in $\triangle DBC$,

$$DB = DC$$

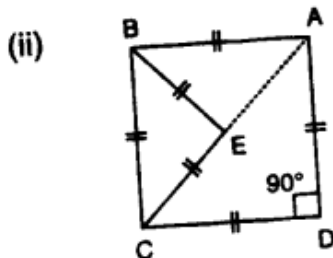
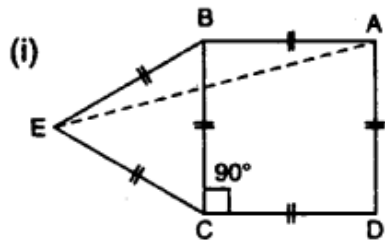
$$\therefore \angle DCB = \angle DBC$$

$$\Rightarrow x = 55^\circ$$

Question 13.

In each figure, given below, ABCD is a square and $\triangle BEC$ is an equilateral triangle.

Find, in each case : (i) $\angle ABE$ (ii) $\angle BAE$

**Solution:**

We know that the sides of a square are equal and each angle is of 90°

Three sides of an equilateral triangle are equal and each angle is of 60° . Therefore,

In fig. (i), ABCD is a square and $\triangle BEC$ is an equilateral triangle.

$$\begin{aligned} (i) \quad \angle ABE &= \angle ABC + \angle CBE \\ &= 90^\circ + 60^\circ = 150^\circ \end{aligned}$$

(ii) But in $\triangle ABE$

$$\begin{aligned} \angle ABE + \angle BEA + \angle BAE &= 180^\circ \\ &\quad \text{(Angles of a triangle)} \\ \Rightarrow 150^\circ + \angle BAE + \angle BAE &= 180^\circ \\ &\quad (\because AB = BE) \end{aligned}$$

$$\begin{aligned} \Rightarrow 150^\circ + 2 \angle BAE &= 180^\circ \\ \Rightarrow 2 \angle BAE &= 180^\circ - 150^\circ = 30^\circ \end{aligned}$$

$$\therefore \angle BAE = \frac{30^\circ}{2} = 15^\circ$$

In figure (ii),

$\therefore$ ABCD is a square and $\triangle BEC$ is an equilateral triangle,

$$\begin{aligned} (i) \quad \therefore \angle ABE &= \angle ABC - \angle CBE \\ &= 90^\circ - 60^\circ = 30^\circ \end{aligned}$$

$$\begin{aligned} (ii) \quad \text{In } \triangle ABE, \angle ABE + \angle AEB + \angle BAE &= 180^\circ \\ &\quad \text{(Angles of a triangle)} \end{aligned}$$

$$\begin{aligned} \Rightarrow 30^\circ + \angle BAE + \angle BAE &= 180^\circ \\ &\quad (\because AB = BE) \end{aligned}$$

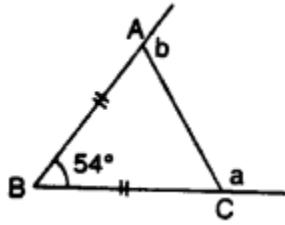
$$\Rightarrow 30^\circ + 2\angle BAE = 180^\circ$$

$$\Rightarrow 2\angle BAE = 180^\circ - 30^\circ = 150^\circ$$

$$\Rightarrow \angle BAE = \frac{150^\circ}{2} = 75^\circ$$

Question 14.

In $\triangle ABC$, BA and BC are produced. Find the angles a and h. if $AB = BC$.



Solution:

In $\triangle ABC$, sides BA and BC are produced

$$\angle ABC = 54^\circ ; AB = BC$$

Now in $\triangle ABC$,

$$\angle BAC + \angle BCA + \angle ABC = 180^\circ$$

(Angles of a triangle)

$$\Rightarrow \angle BAC + \angle BAC + 54^\circ = 180^\circ \quad (\because AB = BC)$$

$$\Rightarrow 2\angle BAC = 180^\circ - 54^\circ$$

$$\Rightarrow 2\angle BAC = 126^\circ$$

$$\therefore \angle BAC = \frac{126^\circ}{2} = 63^\circ \text{ and } \angle BCA = 63^\circ$$

$$\angle BAC + b = 180^\circ \quad (\text{Linear pair})$$

$$\Rightarrow 63^\circ + b = 180^\circ$$

$$\Rightarrow b = 180^\circ - 63^\circ = 117^\circ$$

$$\text{and } \angle BCA + a = 180^\circ \quad (\text{Linear pair})$$

$$\therefore 63^\circ + a = 180^\circ$$

$$\Rightarrow a = 180^\circ - 63^\circ = 117^\circ$$

$$\text{Hence } a = 117^\circ, b = 117^\circ$$

EXERCISE 15 (C)

Question 1.

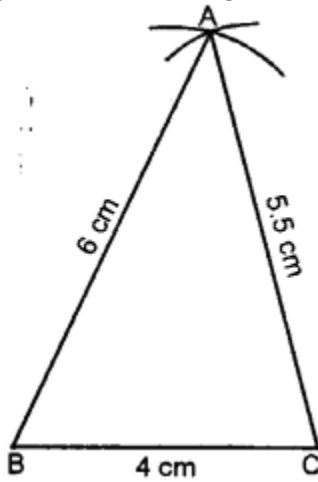
Construct a $\triangle ABC$ such that:

- (i) $AB = 6$ cm, $BC = 4$ cm and $CA = 5.5$ cm
- (ii) $CB = 6.5$ cm, $CA = 4.2$ cm and $BA = 5.1$ cm
- (iii) $BC = 4$ cm, $AC = 5$ cm and $AB = 3.5$ cm

Solution:

(i) Steps of Construction :

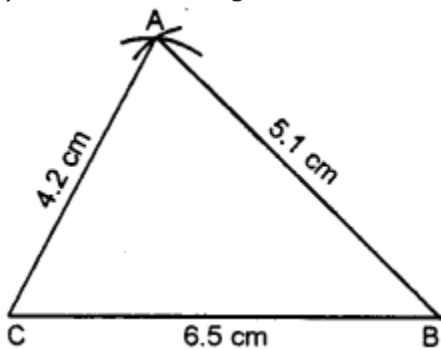
- (i) Draw a line segment $BC = 4$ cm.



- (ii) With centre B and radius 6 cm draw an arc.
- (iii) With centre C and radius 5.5 cm, draw another arc intersecting the first arc at A.
- (iv) Join AB and AC. $\triangle ABC$ is the required triangle.

(ii) Steps of Construction :

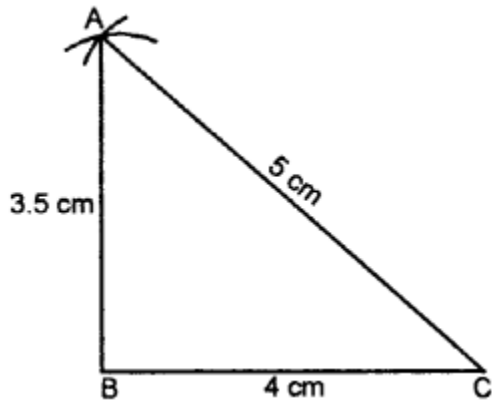
- (i) Draw a line segment $CB = 6.5$ cm



- (ii) With centre C and radius 4.2 cm draw an arc.
 - (iii) With centre B and radius 5.1 cm draw another arc intersecting the first arc at A.
 - (iv) Join AC and AB.
- $\triangle ABC$ is the required triangle.

(iii) Steps of Construction :

- (i) Draw a line segment $BC = 4$ cm.
- (ii) With centre B and radius 3.5 cm, draw an arc
- (iii) With centre C and radius 5 cm, draw another arc which intersects the first arc at A.



(iv) Join AB and AC.
 $\triangle ABC$ is the required triangle.

Question 2.

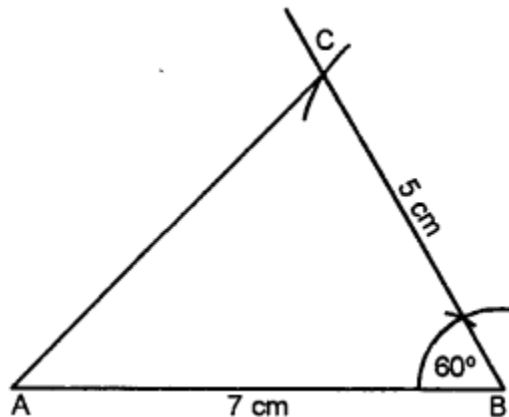
Construct a $\triangle ABC$ such that:

- (i) $AB = 7$ cm, $BC = 5$ cm and $\angle ABC = 60^\circ$
- (ii) $BC = 6$ cm, $AC = 5.7$ cm and $\angle ACB = 75^\circ$
- (iii) $AB = 6.5$ cm, $AC = 5.8$ cm and $\angle A = 45^\circ$

Solution:

(i) Steps of Construction :

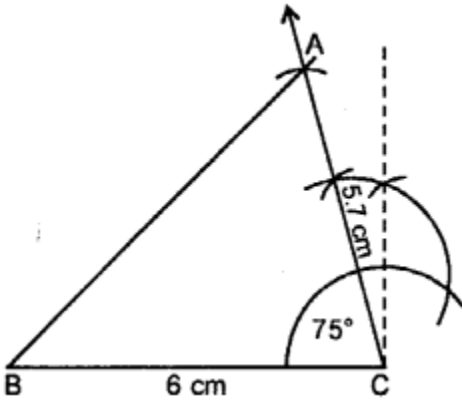
- (i) Draw a line segment $AB = 7$ cm.



- (ii) At B, draw a ray making an angle of 60° and cut off $BC = 5$ cm
- (iii) Join AC,
 $\triangle ABC$ is the required triangle.

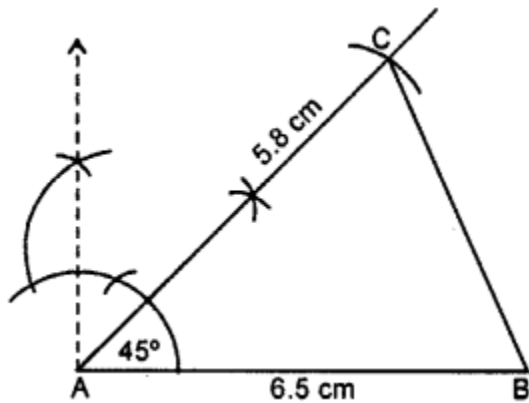
(ii) Steps of Construction :

- (i) Draw a line segment $BC = 6$ cm.
- (ii) At C, draw a ray making an angle of 75° and cut off $CA = 5.7$ cm.
- (iii) Join AB
 $\triangle ABC$ is the required triangle.



(iii) Steps of Construction :

- (i) Draw a line segment $AB = 6.5 \text{ cm}$



- (ii) At A, draw a ray making an angle of 45° and cut off $AC = 5.8 \text{ cm}$

- (iii) Join CB.

$\triangle ABC$ is the required triangle.

Question 3.

Construct a $\triangle PQR$ such that :

- (i) $PQ = 6 \text{ cm}$, $\angle Q = 60^\circ$ and $\angle P = 45^\circ$. Measure $\angle R$.
 (ii) $QR = 4.4 \text{ cm}$, $\angle R = 30^\circ$ and $\angle Q = 75^\circ$. Measure PQ and PR .
 (iii) $PR = 5.8 \text{ cm}$, $\angle P = 60^\circ$ and $\angle R = 45^\circ$.

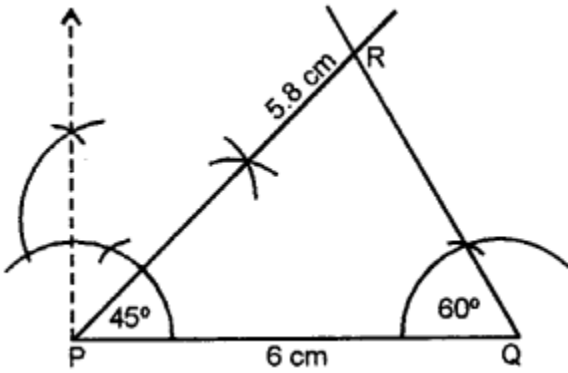
Measure $\angle Q$ and verify it by calculations

Solution:

(i) Steps of Construction:

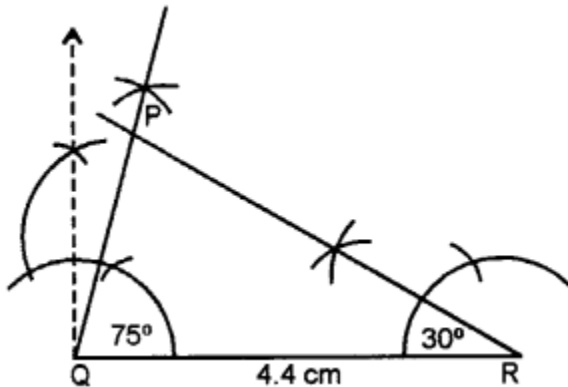
- (i) Draw a line segment $PQ = 6 \text{ cm}$.
 (ii) At P, draw a ray making an angle of 45°
 (iii) At Q, draw another ray making an angle of 60° which intersects the first ray at R.
 $\triangle PQR$ is the required triangle.

On measuring $\angle R$, it is 75° .



(ii) Steps of Construction :

(i) Draw a line segment QR = 4.4 cm.



(ii) At Q, draw a ray making an angle of 75°

(iii) At R, draw another arc making an angle of 30° ; which intersects the first ray at R
 $\triangle PQR$ is the required triangle.

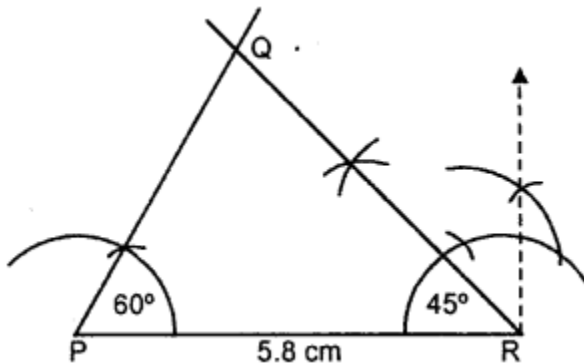
On measuring the lengths of PQ and PR, PQ = 2.1 cm and PR = 4.4 cm.

(iii) Steps of Construction :

(i) Draw a line segment PR = 5.8 cm

(ii) At P, construct an angle of 60°

(iii) At R, draw another angle of 45° meeting each other at Q.



$\triangle PQR$ is the required triangle. On measuring $\angle Q$, it is 75°

Verification : We know that sum of angles of a triangle is 180°

$$\therefore \angle P + \angle Q + \angle R = 180^\circ$$

$$\Rightarrow 60^\circ + \angle Q + 45^\circ = 180^\circ$$

$$\Rightarrow \angle Q + 105^\circ = 180^\circ$$

$$\Rightarrow \angle Q = 180^\circ - 105^\circ = 75^\circ.$$

Question 4.

Construct an isosceles $\triangle ABC$ such that:

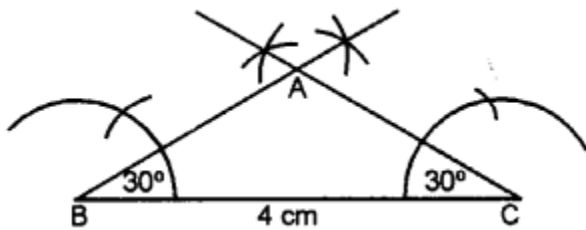
- (i) base $BC = 4$ cm and base angle $= 30^\circ$
- (ii) base $AB = 6.2$ cm and base angle $= 45^\circ$
- (iii) base $AC = 5$ cm and base angle $= 75^\circ$.

Measure the other two sides of the triangle.

Solution:

(i) Steps of Construction :

We know that in an isosceles triangle base angles are equal.

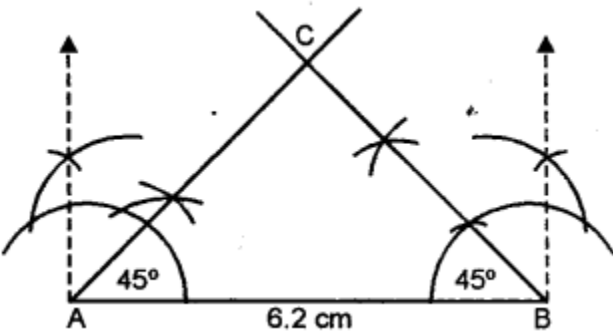


- (i) Draw a line segment $BC = 4$ cm.
- (ii) At B and C , draw rays making an angle of 30° each intersecting each other at A . $\triangle ABC$ is the required triangle.

On measuring the equal sides each is 2.5 cm (approx.) in length.

(ii) Steps of Construction :

We know that in an isosceles triangle, base angles are equal.

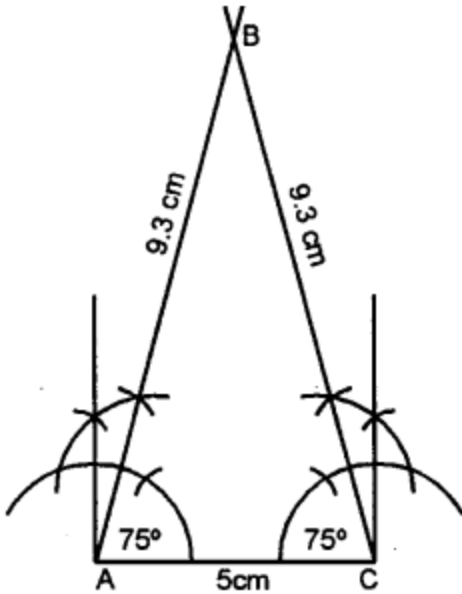


- (i) Draw a line segment $AB = 6.2$ cm
- (ii) At A and B , draw rays making an angle of 45° each which intersect each other at C . $\triangle ABC$ is the required triangle.

On measuring the equal sides, each is 4.3 cm (approx.) in length.

(iii) Steps of Construction :

We know that base angles of an isosceles triangles are equal.



- (i) Draw a line segment $AC = 5\text{ cm}$.
 - (ii) At A and C , draw rays making an angle of 75° each which intersect each other at B . $\triangle ABC$ is the required triangle.
- On measuring the equal sides, each is 9.3 cm in length.

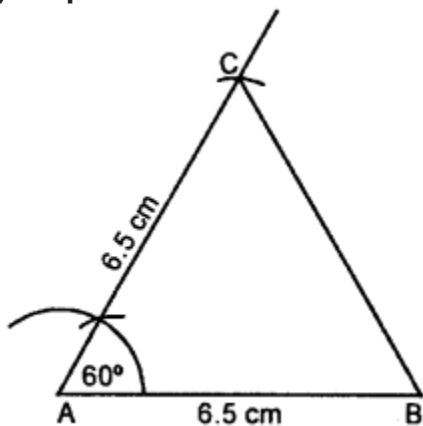
Question 5.

Construct an isosceles $\triangle ABC$ such that:

- (i) $AB = AC = 6.5\text{ cm}$ and $\angle A = 60^\circ$
- (ii) One of the equal sides = 6 cm and vertex angle = 45° . Measure the base angles.
- (iii) $BC = AB = 5.8\text{ cm}$ and $\angle B = 30^\circ$. Measure $\angle A$ and $\angle C$.

Solution:

(i) Steps of Construction :

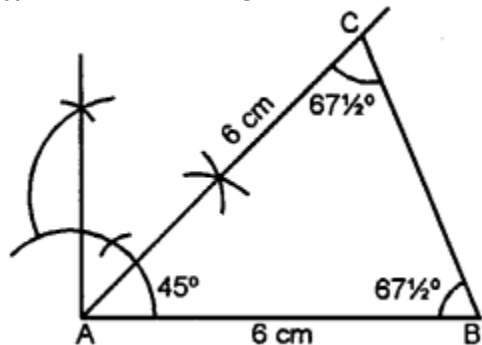


- (i) Draw a line segment $AB = 6.5\text{ cm}$.
- (ii) At A , draw a ray making an angle of 60° .
- (iii) Cut off $AC = 6.5\text{ cm}$
- (iv) Join BC .

$\triangle ABC$ is the required triangle.

(ii) Steps of Construction :

- (i) Draw a line segment $AB = 6$ cm



- (ii) At A, construct an angle equal to 45°

- (iii) Cut off $AC = 6$ cm

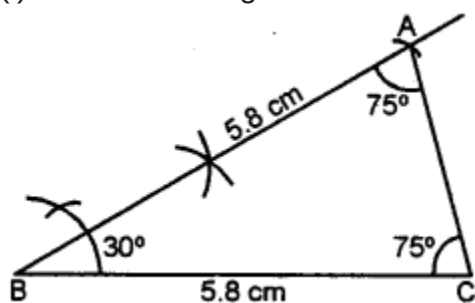
- (iv) Join BC.

$\triangle ABC$ is the required triangle.

On measuring, $\angle B$ and $\angle C$, each is equal 1° to, $67\frac{1}{2}^\circ$

(iii) Steps of Construction :

- (i) Draw a line segment $BC = 5.8$ cm



- (ii) At B, draw a ray making an angle of 30° .

- (iii) Cut off $BA = 5.8$ cm

- (iv) Join AC.

$\triangle ABC$ is the required triangle On measuring $\angle C$ and $\angle A$, each is equal to 75° .

Question 6.

Construct an equilateral $\triangle ABC$ such that:

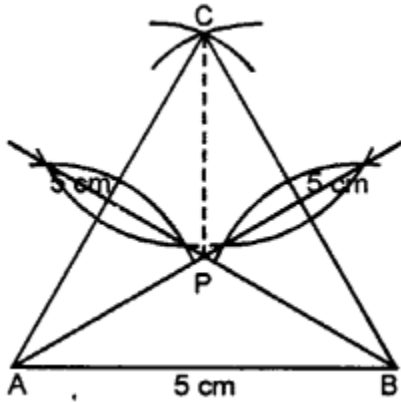
- (i) $AB = 5$ cm. Draw the perpendicular bisectors of BC and AC. Let P be the point of intersection of these two bisectors. Measure PA, PB and PC.

- (ii) Each side is 6 cm.

Solution:

(i) Steps of Construction :

- (i) Draw a line segment $AB = 5$ cm.



(ii) With centres A and B and radius 5 cm each, draw two arcs intersecting each other at C.

(iii) Join AC and BC $\triangle ABC$ is the required triangle.

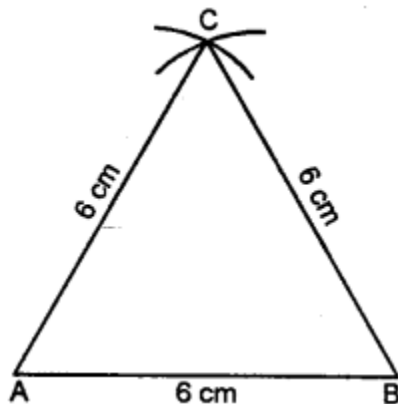
(iv) Draw the perpendicular bisectors of sides AC and BC which intersect each other at P-

(v) Join PA, PB and PC.

On measuring, each is 2.8 cm.

(ii) Steps of Construction :

(i) Draw a line segment AB = 6 cm.



(ii) At A and B as centre and 6 cm as radius draw two arcs intersecting each other at C.

(iii) Join AC and BC.

$\triangle ABC$ is the required triangle.

Question 7.

(i) Construct a $\triangle ABC$ such that AB = 6 cm, BC = 4.5 cm and AC = 5.5 cm.

Construct a circumcircle of this triangle.

(ii) Construct an isosceles $\triangle PQR$ such that PQ = PR = 6.5 cm and $\angle PQR = 75^\circ$.

Using ruler and compasses only construct a circumcircle to this triangle.

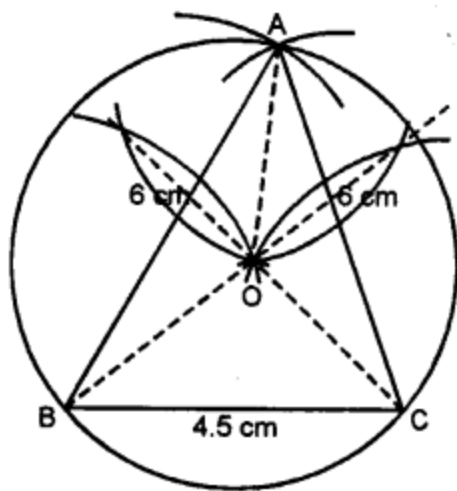
(iii) Construct an equilateral triangle ABC such that its one side = 5.5 cm.

Construct a circumcircle to this triangle.

Solution:

(i) Steps of Construction :

- (i) Draw a line segment $BC = 4.5$ cm



- (ii) With centre B and radius 6 cm, draw an arc

- (iii) With centre C and radius 5.5 cm, draw another arc intersecting the first arc at A.

- (iv) Join AB and AC.

ΔABC is the required triangle.

- (v) Draw the perpendicular bisectors of AB and AC. Which intersect each other at O.

- (vi) Join OB, OC and OA.

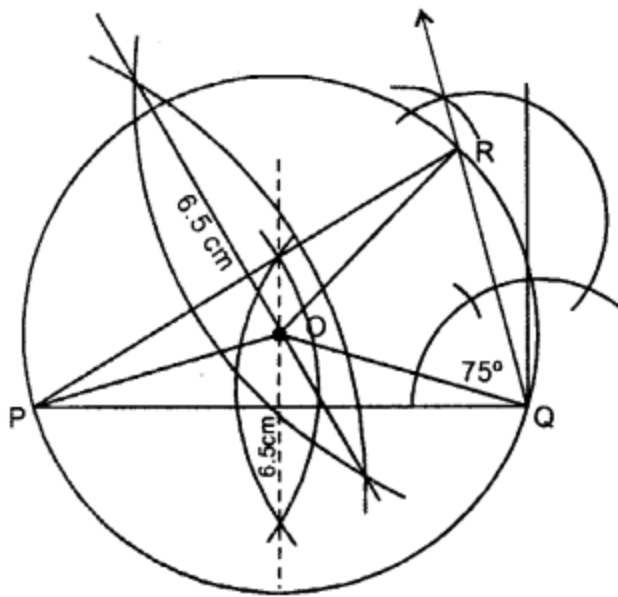
- (vii) With centre O, and radius OA, draw a circle which passes through A, B and C.

This is the required circum circle of ΔABC .

(ii) Steps of Construction :

- (i) Draw a line segment $PQ = 6.5$ cm

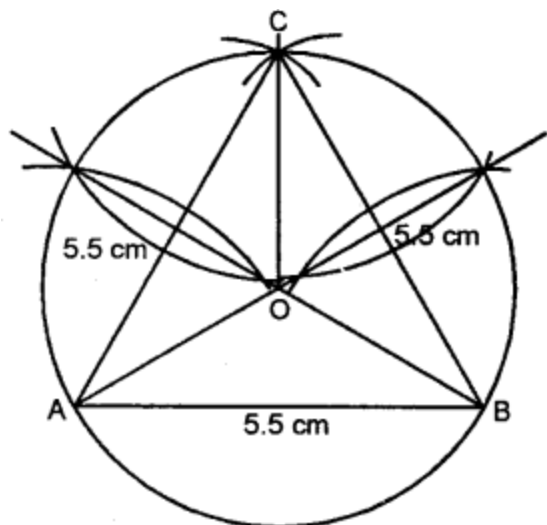
- (ii) At Q, draw a ray making an angle of 75° .



- (iii) Through P, with a radius of 6.5 cm, draw an arc which intersects the angle ray at R.
- (iv) Join PR,
 ΔPQR is the required triangle.
- (v) Draw the perpendicular bisectors of sides PQ and PR intersecting each other at O.
- (vi) Join OP, OQ and OR.
- (vii) With centre O and radius equal to OP or OQ or OR draw a circle which passes through P, Q and R. This is the required circum circle of ΔPQR

(iii) Steps of Construction :

- (i) Draw a line segment $AB = 5.5$ cm



- (ii) With centres A and B and radius 5.5 cm, draw two arcs intersecting each other at C.
- (iii) Join AC and BC.
 $\triangle ABC$ is the required triangle.
- (iv) Draw perpendicular bisectors of sides AC and BC which intersect each other at O.
- (v) Join OA, OB and OC.
- (vi) With centre O and radius OA or OB or OC, draw a circle which passes through A, B and C. This is the required circumcircle.

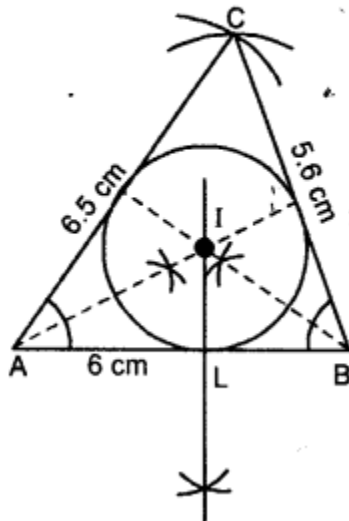
Question 8.

- (i) Construct a $\triangle ABC$ such that $AB = 6$ cm, $BC = 5.6$ cm and $CA = 6.5$ cm. Inscribe a circle to this triangle and measure its radius.
- (ii) Construct an isosceles $\triangle MNP$ such that base $MN = 5.8$ cm, base angle $MNP = 30^\circ$. Construct an incircle to this triangle and measure its radius.
- (iii) Construct an equilateral $\triangle DEF$ whose one side is 5.5 cm. Construct an incircle to this triangle.
- (iv) Construct a $\triangle PQR$ such that $PQ = 6$ cm, $\angle QPR = 45^\circ$ and angle $PQR = 60^\circ$. Locate its incentre and then draw its incircle.

Solution:

(i) Steps of Construction :

- (i) Draw a line segment $AB = 6$ cm.

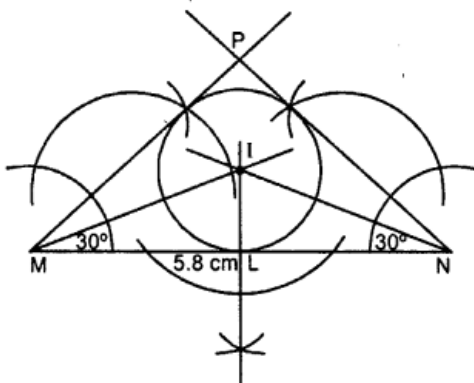


- (ii) With centre A and radius 6.5 cm and with centre B and radius 5.6 cm, draw arcs intersecting each other at C.
- (iii) Join AC and BC.
- (iv) Draw the angle bisector of $\angle A$ and $\angle B$ intersecting each other at I.
- (v) From I, draw $IL \perp AB$
- (vi) With centre I and radius IL, draw a circle which touches the sides of $\triangle ABC$ internally.

On measuring the required incircle whose radius is 1.6 cm.

(ii) Steps of Construction :

- (i) Draw a line segment $MN = 5.8$ cm.

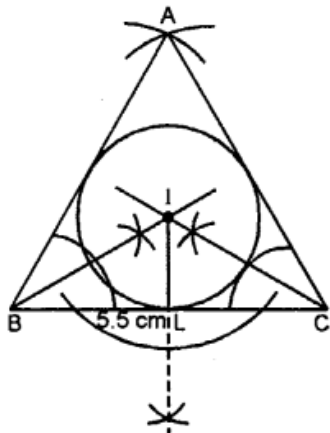


- (ii) At M and N, draw two rays making an angle of 30° each which intersect each other at P.
- (iii) Now draw the angle bisectors of $\angle M$ and $\angle N$ which intersect each other at I.
- (iv) From I, draw perpendicular IL on MN.
- (v) With centre I and radius IL, draw a circle which touches the sides of the ΔPMN internally.

On measuring the required incircle and its radius is 0.6 cm.

(iii) Steps of Construction :

- (i) Draw a line segment $BC = 5.5$ cm.

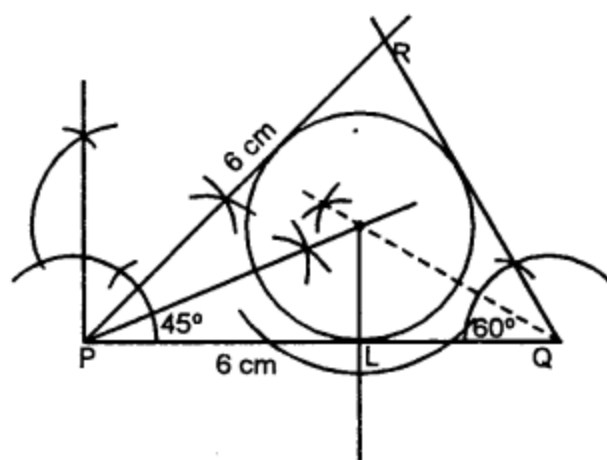


- (ii) With centres B and C and radius 5.5 cm each draw two arcs intersecting each other at A.
- (iii) Join AB and AC.
- (iv) Draw the perpendicular bisectors of $\angle B$ and $\angle C$ intersecting each other at I.
- (v) From I, draw $IL \perp BC$
- (vi) With centre I and radius IL, draw a circle which touches the sides of the ΔABC internally.

This is the required incircle.

(iv) Steps of Construction :

- (i) Draw a line segment $PQ = 6$ cm.
- (ii) At P draw a rays making an angle of 45° and at Q, making an angle of 60° , intersecting each other at R.
- (iii) Draw the bisectors of $\angle P$ and $\angle Q$ intersecting each other at I.
- (iv) From I, draw $IL \perp PQ$.



- (v) With centre I and radius IL, draw a circle which touches the sides of ΔPQR internally. This is the required incircle whose I is incentre.