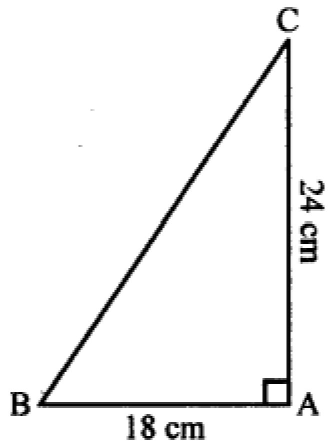
Pythagoras Theorem

EXERCISE 16

Question 1.
Triangle ABC is right-angled at vertex A. Calculate the length of BC, if AB = 18 cm and AC = 24 cm.

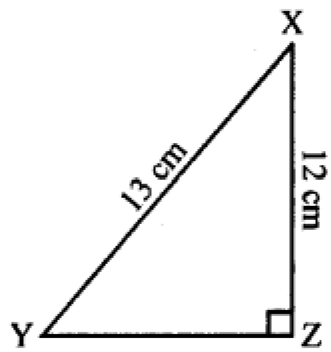
Solution:
Given : ΔABC right angled at A and AB = 18 cm, AC = 24 cm.



To find : Length of BC.
According to Pythagoras Theorem,
$BC^2 = AB^2 + AC^2$
$= 18^2 + 24^2 = 324 + 576 = 900$
$\therefore BC = \sqrt{900} = \sqrt{30x30} = 30$ cm

Question 2.
Triangle XYZ is right-angled at vertex Z. Calculate the length of YZ, if XY = 13 cm and XZ = 12 cm.

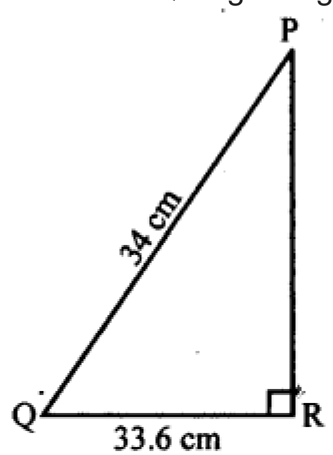
Solution:
Given : ΔXYZ right angled at Z and XY = 13 cm, XZ = 12 cm.
To find : Length of YZ.
According to Pythagoras Theorem,
$XY^2 = XZ^2 + YZ^2$
$13^2 = 12^2 + YZ^2$



$169 = 144 + YZ^2$
$169 - 144 = YZ^2$
$25 = YZ^2$
$\therefore YZ = \sqrt{25}$cm $\sqrt{5x5}$ = 5 cm

Question 3.
Triangle PQR is right-angled at vertex R. Calculate the length of PR, if:
PQ = 34 cm and QR = 33.6 cm.

Solution:
Given : ΔPQR right angled at R and PQ = 34 cm, QR = 33.6 cm.



To find : Length of PR.
According to Pythagoras Theorem,
$PR^2 + QR^2 = PQ^2$
$PR^2 + 33.6^2 = 34^2$
$PR^2 + 1128.96 = 1156$
$PR^2 = 1156 - 1128.96$
$\therefore PR = \sqrt{27.04}$ = 5.2 cm

Question 4.

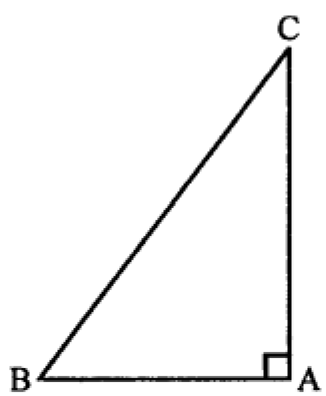
The sides of a certain triangle are given below. Find, which of them is right-triangle
(i) 16 cm, 20 cm and 12 cm
(ii) 6 m, 9 m and 13 m

Solution:

(i) 16 cm, 20 cm and 12 cm
The given triangle will be a right-angled triangle if square of its largest side is equal to the sum of the squares on the other two sides.
i.e., If $(20)^2 = (16)^2 = (12)^2$
$(20)^2 = (16)^2 + (12)^2$
$400 = 256 + 144$
$400 = 400$
So, the given triangle is right angled.
(ii) 6 m, 9 m and 13 m
The given triangle will be a right-angled triangle if square of its largest side is equal to the sum of the squares on the other two sides.
i.e., If $(13)^2 = (9)^2 + (6)^2$
$169 = 81+36$ $169 \neq 117$
So, the given triangle is not right angled.

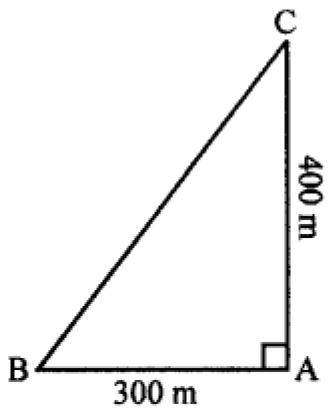
Question 5.

In the given figure, angle BAC = 90°, AC = 400 m and AB = 300 m. Find the length of BC.



Solution:

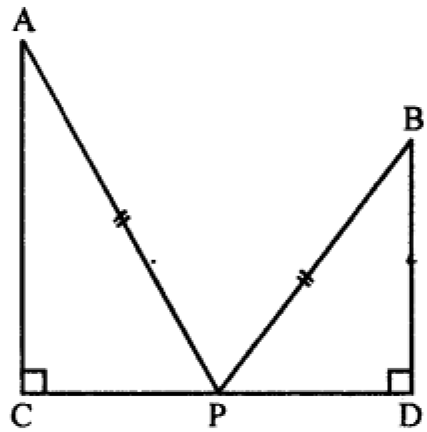
AC = 400 m
AB = 300 m
BC = ?



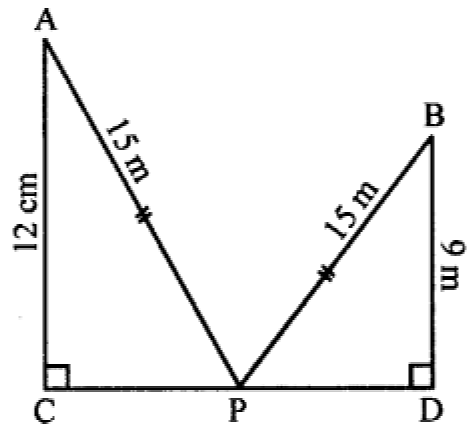
According to Pythagoras Theorem,
$BC^2 = AB^2 + AC^2$
$BC^2 = (300)^2 + (400)^2$
$BC^2 = 90000 + 160000$
$BC^2 = 250000$
$BC = \sqrt{250000} = 500$ m

Question 6.
In the given figure, angle ACP = ∠BDP = 90°, AC = 12 m, BD = 9 m and PA= PB = 15 m. Find:

(i) CP
(ii) PD
(iii) *CD*



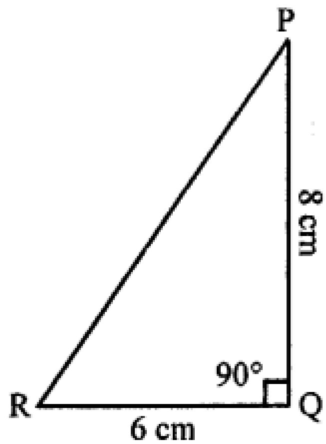
Solution:
Given : AC = 12 m
BD = 9 m
PA = PB= 15 m



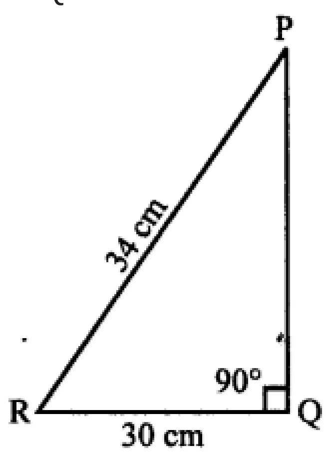
(i) In right angle triangle ACP
$(AP)^2 = (AC)^2 + (CP)^2$
$15^2 = 12^2 + CP^2$
$225 = 144 + CP^2$
$225 – 144 = CP^2$
$81 = CP$
$\sqrt{81} = CP$
$\therefore$ CP = 9 m
(ii) In right angle triangle BPD
$(PB)^2 = (BD)^2 + (PD)^2$
$(15)^2 = (9)^2 + PD^2$
$225 = 81 + PD^2$
$225-81 = PD^2$
$144 = PD^2$
$\sqrt{144} = PD$
$\therefore$ PD = 12 m
(iii) CP = 9 m
PD = 12 m
$\therefore$ CD = CP + PD
= 9+ 12 = 21 m

Question 7.
In triangle PQR, angle Q = 90°, find :
(i) PR, if PQ = 8 cm and QR = 6 cm
(ii) PQ, if PR = 34 cm and QR = 30 cm

Solution:
(i) Given:
PQ = 8 cm
QR = 6 cm
PR = ?
$\angle PQR = 90°$



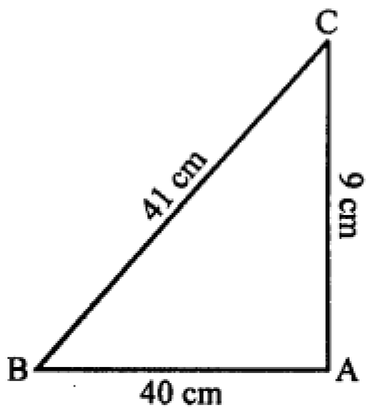
According to Pythagoras Theorem,
$(PR)^2 = (PQ)^2 + (QR)^2$
$PR^2 = 8^2 + 6^2$
$PR^2 = 64 + 36$
$PR^2 = 100$
∴ $PR = \sqrt{100} = 10$ cm
(ii) Given :
PR = 34 cm
QR = 30 cm
PQ = ?
∠PQR = 90°



According to Pythagoras Theorem,
$(PR)^2 = (PQ)^2 + (QR)^2$
$(34)^2 = PQ^2 + (30)^2$
$1156 = PQ^2 + 900$
$1156-900 = PQ^2$
$256 = PQ^2$
∴ PQ = 16 cm

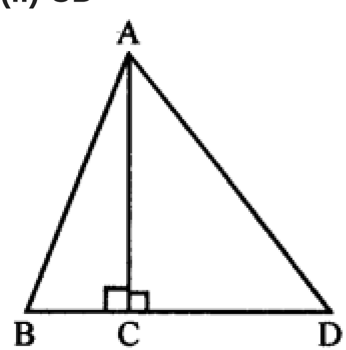
Question 8.
Show that the triangle ABC is a right-angled triangle; if:
AB = 9 cm, BC = 40 cm and AC = 41 cm

Solution:
AB = 9 cm
CB = 40 cm
AC = 41 cm

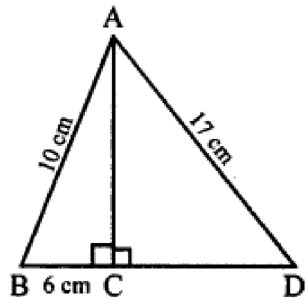


The given triangle will be a right angled triangle if square of its largest side is equal to the sum of the squares on the other two sides.
According to Pythagoras Theorem,
$(AC)^2 = (BC)^2 + (AB)^2$
$(41)^2 = (40)^2 + (9)^2$
1681 = 1600 + 81
1681 = 1681
Hence, it is a right-angled triangle ABC.

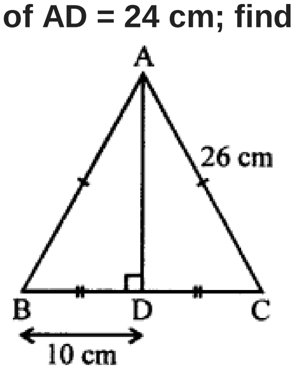
Question 9.
In the given figure, angle ACB = 90° = angle ACD. If AB = 10 m, BC = 6 cm and AD = 17 cm, find :
(i) AC
(ii) CD



Solution:
Given:

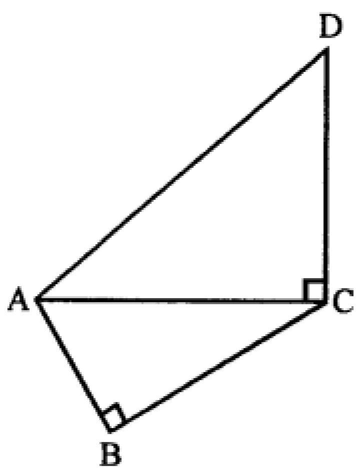
$\triangle ABD$
$\angle ACB = \angle ACD = 90°$
and AB = 10 cm, BC = 6 cm and AD = 17 cm


To find:
(i) Length of AC
(ii) Length of CD
Proof:
(i) In right-angled triangle ABC
BC = 6 cm, AB = 110 cm
According to Pythagoras Theorem,
$AB^2 = AC^2 + BC^2$
$(10)^2 = (AC)^2 + (6)^2$
$100 = (AC)^2 + 36$
$AC^2 = 100 - 36 = 64$ cm
$AC^2 = 64$ cm
$\therefore$ AC = $\sqrt{8x8}$ = 8 cm
(ii) In right-angle triangle ACD
AD = 17 cm, AC = 8 cm
According to Pythagoras Theorem,
$(AD)^2 = (AC)^2 + (CD)^2$
$(17)^2 = (8)^2 + (CD)^2$
$289 – 64 = CD^2$
$225 = CD^2$
CD =$\sqrt{15x15}$ = 15 cm

Question 10.
In the given figure, angle ADB = 90°, AC = AB = 26 cm and BD = DC. If the length of AD = 24 cm; find the length of BC.


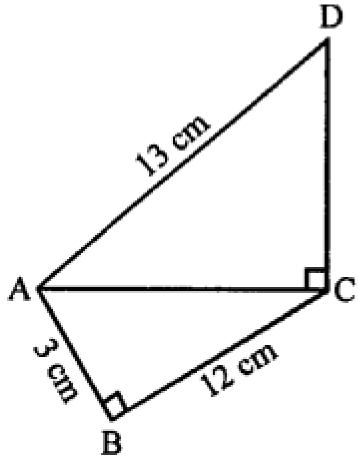
Solution:
Given:
ΔABC
$\angle ADB = 90°$ and AC = AB = 26 cm
AD = 24 cm
To find : Length of BC In right angled ΔADC
AB = 26 cm, AD = 24 cm
According to Pythagoras Theorem,
$(AC)^2 = (AD)^2 + (DC)^2$
$(26)^2 = (24)^2 + (DC)^2$
$676 = 576 + (DC)^2$
$\Rightarrow (DC)^2 = \underline{100}$
$\Rightarrow DC = \sqrt{100} = 10$ cm
$\therefore$ Length of BC = BD + DC
= 10 + 10 = 20 cm

Question 11.
In the given figure, AD = 13 cm, BC = 12 cm, AB = 3 cm and angle ACD = angle ABC = 90°. Find the length of DC.



Solution:
Given :
$\Delta ACD = \angle ABC = 90°$
and AD = 13 cm, BC = 12 cm, AB = 3 cm

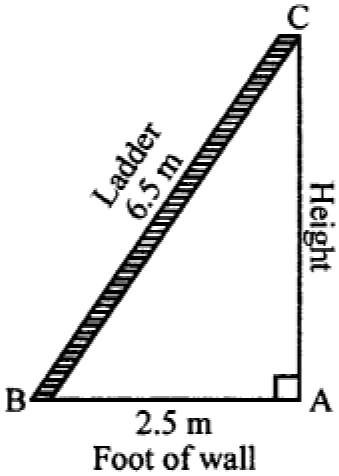
To find : Length of DC.



(i) In right angled $\triangle ABC$
AB = 3 cm, BC = 12 cm
According to Pythagoras Theorem,
$(AC)^2 = (AB)^2 + (BC)^2$
$(AC)^2 = (3)^2 + (12)^2$
$(AC) = \sqrt{9+144} = \sqrt{153}$ cm
(ii) In right angled triangle ACD
AD = 13 cm, AC = $\sqrt{153}$
According to Pythagoras Theorem,
$DC^2 = AB^2 - AC^2$
$DC^2 = 169 - 153$
DC = $\sqrt{16}$ = 4 cm
∴ Length of DC is 4 cm

Question 12.
A ladder, 6.5 m long, rests against a vertical wall. Ifthe foot of the ladcler is 2.5 m from the foot of the wall, find upto how much height does the ladder reach?

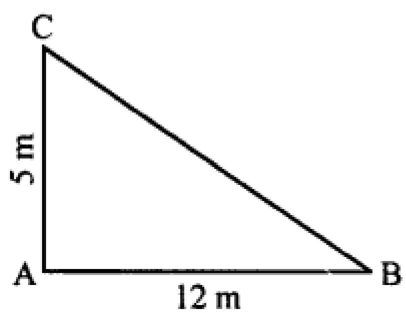
Solution:
Given :
Length of ladder = 6.5 m
Length of foot of the wall = 2.5 m
To find : Height AC According to Pythagoras Theorem,
$(BC)^2 = (AB)^2 + (AC)^2$



$(6.5)^2 = (2.5)^2 + (AC)^2$
$42.25 = 6.25 + AC^2$
$AC^2 = 42.25 – 6.25 = 36$ m
AC = $\sqrt{6x6}$ = 6 m
∴ Height of wall = 6 m

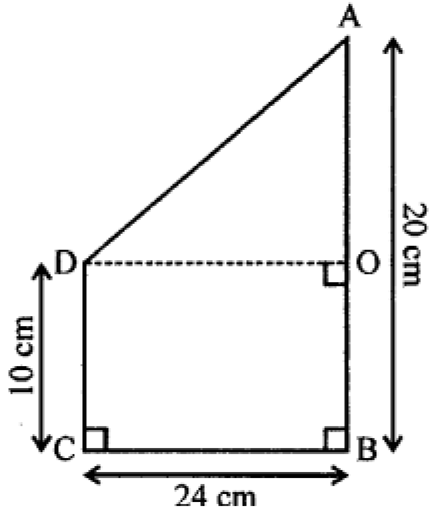
Question 13.
A boy first goes 5 m due north and then 12 m due east. Find the distance between the initial and the final position of the boy.

Solution:
Given : Direction of north = 5 m i.e. AC Direction of east = 12 m i.e. AB



To find: BC
According to Pythagoras Theorem,
In right angled AABC
$(BC)^2 = (AC)^2 + (AB)^2$
$(BC)^2 = (5)^2 + (12)^2$
$(BC)^2 = 25 + 144$
$(BC)^2 = 25 + 144$
$(BC)^2= 169$
∴ BC = $\sqrt{169}$ = $\sqrt{13x13}$ = 13 m

Question 14.
Use the information given in the figure to find the length AD.



Solution:
Given :
AB = 20 cm
$\therefore$AO $= \frac{AB}{2} = \frac{20}{2} =$10cm
BC = OD = 24 cm
To find : Length of AD
In right angled triangle
AOD $(AD)^2 = (AO)^2 + (OD)^2$
$(AD)^2 = (10)^2 + (24)^2$
$(AD)^2 = 100 + 576$
$(AD)^2 = 676$
$\therefore$ AD = $\sqrt{26x26}$
AD = 26 cm