Playing With numbers

Factors and multiples:

- Factor of a number is an exact divisor of that number
  6 = 1 x 6 = 2 x 3 : factors of 6 are 1, 2, 3 and 6
- Every number is a factor of itself and 1 is a factor of every number
- Every factor of a number is less than or equal to the given number
- Every number is a multiple of each of its factors
  Factors of 12 are 1, 2, 3, 4, 6 and 12 : 3 x 4 =12, 2 x 6 =12, 12 x 1 = 12
- Every multiple of a given number is greater than or equal to that number.
- Every number is a multiple of itself
- Factors of a given number are finite while its multiples are infinite
- If two given numbers are divisible by a number, then their sum (or difference) is also divisible by that number : 6 is divisible by 3, 18 is divisible by 3. Hence 24 (= 18+ 6) and 12 (= 18-6) are also divisible by 3
- A number for which sum of all its factors is equal to twice the number is called a perfect number :Ex. 6 and 28

Prime and composite numbers:

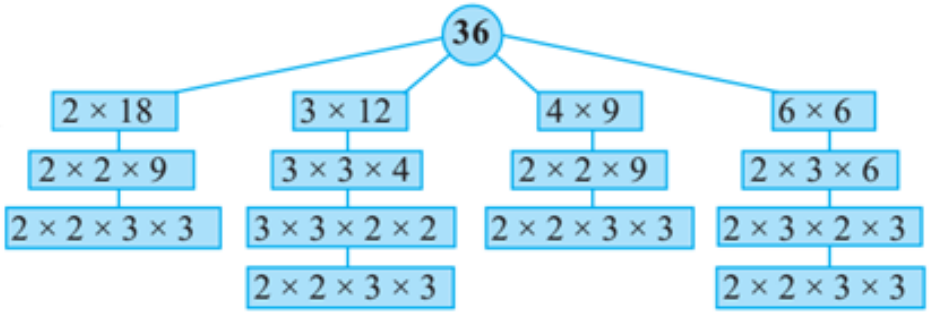
- A number having only 2 factors, namely 1 and the number itself, is a prime number : 2, 3, 5, 7, 11 etc
- Numbers that have more than two factors are called composite numbers : 4, 6, 8, 9, 10, 12 etc
- Number 1 is neither prime nor composite.
- The number 2 is the smallest prime number and is even. Every prime number other than 2 is odd.
- Two numbers with only 1 as a common factor are called co-prime numbers : 15 and 16 , 8 and 9 , 25 and 33 etc
- If a number is divisible by another number then it is divisible by each of the factors of that number
  24 is divisible by 8: it is also divisible by 2 and 4 which are factors of 8
- A number divisible by two co-prime numbers is divisible by their product also 8 and 9 are co-primes and 8 x 9 =72
  144 is divisible 9 : 9 x 16 =144 , 144 is divisible by 8 : 8 x 18 =144
  144 is also divisible by 72 (product of 8 and 9) : 72 x 2 =144

Test of divisibility:

By :	Criteria/ nature of the given number	examples
2	Even numbers (0, 2, 4, 6 or 8 in its ones place)	12, 148, 36, 100, 14
3	sum of the digits is a multiple of 3	24: 2 + 4 = 6 1566: 1 + 5 + 6 + 6 =18
4	Last two digits should be both zeroes or divisible by 4	100, 2400, 1936, 7848
5	Number should have 0 or 5 in the ones place	15, 1125, 4560
6	Number should be divisible by 2 and 3	48 : even and 48 : 4 + 8 =12
8	Last three digits should be divisible by 8 or should be all zeroes	1000, 5000, 7024, 3144
9	sum of the digits is a multiple of 9	144 : 1 + 4 + 4 = 9 8631: 8 + 6 + 3 + 1=18
10	Number should have 0 in the ones place	10 20, 30, 56000, 900
11	difference between the sum of the digits at odd places (from the right) and the sum of the digits at even places (from the right) of the number is either 0 or divisible by 11	61809 : 9 + 8 + 6 = 23 and 0 + 1 = 1 23 – 1 = 22 22 is divisible by 11 Other egs : 275, 1331, 6006

Prime factorization:

Expressing a number as a product of its prime factors



The prime factorisation of 36 is = 2 × 2 × 3 × 3. i.e. the only prime factorisation of 36.

The Highest Common Factor(HCF)

- The HCF of two or more given numbers is the highest (or greatest) of their common factors.
- It is also known as Greatest Common Divisor (GCD)

2	20	2	28	20 = 2 x 2 x 5 28 = 2 x 2 x 7 The common factors of 20 and 28 are 2, 2 Hence HCF (20, 28) = 2 x 2 = 4
2	10	2	14	
5	5	7	7	
	1		1	

The Lowest Common Multiple (LCM)

The LCM of two or more given numbers is the lowest (or smallest or least) of their common multiples

Method 1

2	12	16	24	36
2	6	8	12	18
2	3	4	6	9
2	3	2	3	9
3	3	1	3	9
3	1	1	1	3
	1	1	1	1

LCM = 2 × 2 × 2 × 2 × 3 × 3 = 144

2	20	25	30
2	10	25	15
3	5	25	15
5	5	25	5
5	1	5	1
	1	1	1

LCM = 2 × 2 × 3 × 5 × 5

Method 2 : Finding the LCM of 24 and 90

The prime factorisations of 24 is given by : 24 = 2 × 2 × 2 × 3	The prime factorisations of 90 is given by : 90 = 2 × 3 × 3 × 5

- In these prime factorisations, the maximum number of times the prime factor 2 occurs is three; this happens for 24.
- Similarly, the maximum number of times the prime factor 3 occurs is two; this happens for 90
- The prime factor 5 occurs only once in 90
- Thus, LCM = (2 × 2 × 2) × (3 × 3) × 5 = 360