

2

Relations and Functions

Short Answer Type Questions

Q. 1 If $A = \{-1, 2, 3\}$ and $B = \{1, 3\}$, then determine

- (i) $A \times B$ (ii) $B \times A$ (iii) $B \times B$ (iv) $A \times A$

Sol. $A = \{-1, 2, 3\}$ and $B = \{1, 3\}$

(i) $A \times B = \{(-1, 1), (-1, 3), (2, 1), (2, 3), (3, 1), (3, 3)\}$

(ii) $B \times A = \{(1, -1), (1, 2), (1, 3), (3, -1), (3, 2), (3, 3)\}$

(iii) $B \times B = \{(1, 1), (1, 3), (3, 1), (3, 3)\}$

(iv) $A \times A = \{(-1, -1), (-1, 2), (-1, 3), (2, -1), (2, 2), (2, 3), (3, -1), (3, 2), (3, 3)\}$

Q. 2 If $P = \{x : x < 3, x \in N\}$, $Q = \{x : x \leq 2, x \in W\}$, then find $(P \cup Q) \times (P \cap Q)$, where W is the set of whole numbers.

Sol. We have,
and
 $\therefore$

$$\begin{aligned} P &= \{x : x < 3, x \in N\} = \{1, 2\} \\ Q &= \{x : x \leq 2, x \in W\} = \{0, 1, 2\} \\ P \cup Q &= \{0, 1, 2\} \text{ and } P \cap Q = \{1, 2\} \\ (P \cup Q) \times (P \cap Q) &= \{0, 1, 2\} \times \{1, 2\} \\ &= \{(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2)\} \end{aligned}$$

Q. 3 If $A = \{x : x \in W, x < 2\}$, $B = \{x : x \in N, 1 < x < 5\}$ and $C = \{3, 5\}$, then find

- (i) $A \times (B \cap C)$ (ii) $A \times (B \cup C)$

Sol. We have,
and

$$\begin{aligned} A &= \{x : x \in W, x < 2\} = \{0, 1\} \\ B &= \{x : x \in N, 1 < x < 5\} \\ &= \{2, 3, 4\} \text{ and } C = \{3, 5\} \end{aligned}$$

(i) $\therefore B \cap C = \{3\}$

$\therefore A \times (B \cap C) = \{0, 1\} \times \{3\} = \{(0, 3), (1, 3)\}$

(ii) $\therefore (B \cup C) = \{2, 3, 4, 5\}$

$\therefore A \times (B \cup C) = \{0, 1\} \times \{2, 3, 4, 5\}$
 $= \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$

Q. 4 In each of the following cases, find a and b .

(i) $(2a + b, a - b) = (8, 3)$

(ii) $\left(\frac{a}{4}, a - 2b\right) = (0, 6 + b)$

Sol. (i) We have, $(2a + b, a - b) = (8, 3)$

$$\Rightarrow 2a + b = 8 \text{ and } a - b = 3$$

[since, two ordered pairs are equal, if their corresponding first and second elements are equal]

On substituting, $b = a - 3$ in $2a + b = 8$, we get

$$2a + a - 3 = 8 \Rightarrow 3a - 3 = 8$$

$$\Rightarrow 3a = 11 \Rightarrow a = \frac{11}{3}$$

Again, substituting $a = \frac{11}{3}$ in $b = a - 3$, we get

$$b = \frac{11}{3} - 3 = \frac{11 - 9}{3} = \frac{2}{3}$$

$$\therefore a = \frac{11}{3} \text{ and } b = \frac{2}{3}$$

(ii) We have, $\left(\frac{a}{4}, a - 2b\right) = (0, 6 + b)$

$$\Rightarrow \frac{a}{4} = 0 \Rightarrow a = 0$$

$$\text{and } a - 2b = 6 + b$$

$$\Rightarrow 0 - 2b = 6 + b$$

$$\Rightarrow -3b = 6$$

$$\therefore b = -2$$

$$\therefore a = 0, b = -2$$

Q. 5 $A = \{1, 2, 3, 4, 5\}$, $S = \{(x, y) : x \in A, y \in A\}$, then find the ordered which satisfy the conditions given below.

(i) $x + y = 5$

(ii) $x + y < 5$

(iii) $x + y > 8$

Sol. We have, $A = \{1, 2, 3, 4, 5\}$ and $S = \{(x, y) : x \in A, y \in A\}$

(i) The set of ordered pairs satisfying $x + y = 5$ is,

$$\{(1, 4), (2, 3), (3, 2), (4, 1)\}.$$

(ii) The set of ordered pairs satisfying $x + y < 5$ is $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)\}$.

(iii) The set of ordered pairs satisfying $x + y > 8$ is $\{(4, 5), (5, 4), (5, 5)\}$.

Q. 6 If $R = \{(x, y) : x, y \in W, x^2 + y^2 = 25\}$, then find the domain and range of R .

Thinking Process

First, write the relation in Roaster form, then find the domain and range of R .

Sol. We have,

$$R = \{(x, y) : x, y \in W, x^2 + y^2 = 25\}$$

$$= \{(0, 5), (3, 4), (4, 3), (5, 0)\}$$

Domain of R = Set of first element of ordered pairs in R

$$= \{0, 3, 4, 5\}$$

Range of

R = Set of second element of ordered pairs in R

$$= \{5, 4, 3, 0\}$$

Q. 7 If $R_1 = \{(x, y) | y = 2x + 7, \text{ where } x \in R \text{ and } -5 \leq x \leq 5\}$ is a relation. Then, find the domain and range of R_1 .

Sol. We have,

$$R_1 = \{(x, y) | y = 2x + 7, \text{ where } x \in R \text{ and } -5 \leq x \leq 5\}$$

$$\text{Domain of } R_1 = \{-5 \leq x \leq 5, x \in R\}$$

$$= [-5, 5]$$

$\therefore$

$$y = 2x + 7$$

When $x = -5$, then

$$y = 2(-5) + 7 = -3$$

When $x = 5$, then

$$y = 2(5) + 7 = 17$$

$\therefore$

$$\text{Range of } R_1 = \{-3 \leq y \leq 17, y \in R\}$$

$$= [-3, 17]$$

Q. 8 If $R_2 = \{(x, y) | x \text{ and } y \text{ are integers and } x^2 + y^2 = 64\}$ is a relation, then find the value of R_2 .

Sol. We have, $R_2 = \{(x, y) | x \text{ and } y \text{ are integers and } x^2 + y^2 = 64\}$

Since, 64 is the sum of squares of 0 and ± 8 .

When $x = 0$, then $y^2 = 64 \Rightarrow y = \pm 8$

$$x = 8, \text{ then } y^2 = 64 - 8^2 \Rightarrow 64 - 64 = 0$$

$$x = -8, \text{ then } y^2 = 64 - (-8)^2 = 64 - 64 = 0$$

$$\therefore R_2 = \{(0, 8), (0, -8), (8, 0), (-8, 0)\}$$

Q. 9 If $R_3 = \{(x, |x|) | x \text{ is a real number}\}$ is a relation, then find domain and range of R_3 .

Sol. We have,

$$R_3 = \{(x, |x|) | x \text{ is a real number}\}$$

Clearly, domain of

$$R_3 = R$$

Since, image of any real number under R_3 is positive real number or zero.

$\therefore$

$$\text{Range of } R_3 = R^+ \cup \{0\} \text{ or } (0, \infty)$$

Q. 10 Is the given relation a function? Give reason for your answer.

(i) $h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$

(ii) $f = \{(x, x) | x \text{ is a real number}\}$

(iii) $g = \left\{ \left(x, \frac{1}{x} \right) | x \text{ is a positive integer} \right\}$

(iv) $s = \{(x, x^2) | x \text{ is a positive integer}\}$

(v) $t = \{(x, 3) | x \text{ is a real number}\}$

Sol. (i) We have, $h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$.

Since, 3 has two images 9 and 11. So, it is not a function.

(ii) We have, $f = \{(x, x) | x \text{ is a real number}\}$.

We observe that, every element in the domain has unique image. So, it is a function.

(iii) We have, $g = \left\{ \left(x, \frac{1}{x} \right) | x \text{ is a positive integer} \right\}$

For every x , it is a positive integer and $\frac{1}{x}$ is unique and distinct. Therefore, every element in the domain has unique image. So, it is a function.

(iv) We have, $s = \{(x, x^2) | x \text{ is a positive integer}\}$

Since, the square of any positive integer is unique. So, every element in the domain has unique image. Hence, it is a function.

(v) We have, $t = \{(x, 3) | x \text{ is a real number}\}$.

Since, every element in the domain has the image 3. So, it is a constant function.

Q. 11 If f and g are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$. Then, find each of the following.

(i) $f(3) + g(-5)$

(ii) $f\left(\frac{1}{2}\right) \times g(14)$

(iii) $f(-2) + g(-1)$

(iv) $f(t) - f(-2)$

(v) $\frac{f(t) - f(5)}{t - 5}$, if $t \neq 5$

Sol. Given, f and g are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$.

(i) $f(3) = (3)^2 + 7 = 9 + 7 = 16$ and $g(-5) = 3(-5) + 5 = -15 + 5 = -10$

$\therefore f(3) + g(-5) = 16 - 10 = 6$

(ii) $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + 7 = \frac{1}{4} + 7 = \frac{29}{4}$

and $g(14) = 3(14) + 5 = 42 + 5 = 47$

$\therefore f\left(\frac{1}{2}\right) \times g(14) = \frac{29}{4} \times 47 = \frac{1363}{4}$

(iii) $f(-2) = (-2)^2 + 7 = 4 + 7 = 11$ and $g(-1) = 3(-1) + 5 = -3 + 5 = 2$

$\therefore f(-2) + g(-1) = 11 + 2 = 13$

$$(iv) f(t) = t^2 + 7 \text{ and } f(-2) = (-2)^2 + 7 = 4 + 7 = 11$$

$$\therefore f(t) - f(-2) = t^2 + 7 - 11 = t^2 - 4$$

$$(v) f(t) = t^2 + 7 \text{ and } f(5) = 5^2 + 7 = 25 + 7 = 32$$

$$\therefore \frac{f(t) - f(5)}{t - 5}, \text{ if } t \neq 5$$

$$= \frac{t^2 + 7 - 32}{t - 5}$$

$$= \frac{t^2 - 25}{t - 5} = \frac{(t - 5)(t + 5)}{(t - 5)}$$

$$= t + 5$$

$$[\because t \neq 5]$$

Q. 12 Let f and g be real functions defined by $f(x) = 2x + 1$ and $g(x) = 4x - 7$.

(i) For what real numbers x , $f(x) = g(x)$?

(ii) For what real numbers x , $f(x) < g(x)$?

Sol. We have,

$$f(x) = 2x + 1 \text{ and } g(x) = 4x - 7$$

$$(i) \therefore f(x) = g(x)$$

$$\Rightarrow 2x + 1 = 4x - 7 \Rightarrow 2x = 8$$

$$\therefore x = 4$$

$$(ii) \therefore f(x) < g(x)$$

$$\Rightarrow 2x + 1 < 4x - 7$$

$$\Rightarrow 2x - 4x + 1 < 4x - 7 - 4x$$

$$\Rightarrow -2x + 1 < -7$$

$$\Rightarrow -2x < -7 - 1$$

$$\Rightarrow -2x < -8$$

$$\Rightarrow \frac{-2x}{-2} > \frac{-8}{-2}$$

$$\therefore x > 4$$

Q. 13 If f and g are two real valued functions defined as $f(x) = 2x + 1$ and $g(x) = x^2 + 1$, then find

$$(i) f + g$$

$$(ii) f - g$$

$$(iii) fg$$

$$(iv) \frac{f}{g}$$

Sol. We have, $f(x) = 2x + 1$ and $g(x) = x^2 + 1$

$$(i) (f + g)(x) = f(x) + g(x)$$

$$= 2x + 1 + x^2 + 1 = x^2 + 2x + 2$$

$$(ii) (f - g)(x) = f(x) - g(x) = (2x + 1) - (x^2 + 1)$$

$$= 2x + 1 - x^2 - 1 = 2x - x^2 = x(2 - x)$$

$$(iii) (fg)(x) = f(x) \cdot g(x) = (2x + 1)(x^2 + 1)$$

$$= 2x^3 + 2x + x^2 + 1 = 2x^3 + x^2 + 2x + 1$$

$$(iv) \frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{2x + 1}{x^2 + 1}$$

Q. 14 Express the following functions as set of ordered pairs and determine their range.

$$f : x \rightarrow R, f(x) = x^3 + 1, \text{ where } x = \{-1, 0, 3, 9, 7\}$$

Sol. We have,

$$f : X \rightarrow R, f(x) = x^3 + 1.$$

Where

$$X = \{-1, 0, 3, 9, 7\},$$

When

$$x = -1, \text{ then } f(-1) = (-1)^3 + 1 = -1 + 1 = 0$$

$$x = 0, \text{ then } f(0) = (0)^3 + 1 = 0 + 1 = 1$$

$$x = 3, \text{ then } f(3) = (3)^3 + 1 = 27 + 1 = 28$$

$$x = 9, \text{ then } f(9) = (9)^3 + 1 = 729 + 1 = 730$$

$$x = 7, \text{ then } f(7) = (7)^3 + 1 = 343 + 1 = 344$$

$$f = \{(-1, 0), (0, 1), (3, 28), (9, 730), (7, 344)\}$$

$\therefore$

$$\text{Range of } f = \{0, 1, 28, 730, 344\}$$

Q. 15 Find the values of x for which the functions $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal.

Sol.

$$\begin{aligned} \therefore & f(x) = g(x) \\ \Rightarrow & 3x^2 - 1 = 3 + x \\ \Rightarrow & 3x^2 - x - 4 = 0 \\ \Rightarrow & 3x^2 - 4x + 3x - 4 = 0 \\ \Rightarrow & x(3x - 4) + 1(3x - 4) = 0 \\ \Rightarrow & (3x - 4)(x + 1) = 0 \\ \therefore & x = -1, \frac{4}{3} \end{aligned}$$

Long Answer Type Questions

Q. 16 Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7), \}$ a function, justify. If this is described by the relation, $g(x) = \alpha x + \beta$, then what values should be assigned to α and β ?

Thinking Process

First, find the two equation by substitutions different values of x and $g(x)$.

Sol. We have,

$$g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$$

Since, every element has unique image under g . So, g is a function.

Now,

$$g(x) = \alpha x + \beta$$

When $x = 1$, then

$$g(1) = \alpha(1) + \beta$$

...(i)

$\Rightarrow$

$$1 = \alpha + \beta$$

When $x = 2$, then

$$g(2) = \alpha(2) + \beta$$

$\Rightarrow$

$$3 = 2\alpha + \beta$$

...(ii)

On solving Eqs. (i) and (ii), we get

$$\alpha = 2, \beta = -1$$

Q. 17 Find the domain of each of the following functions given by

$$(i) f(x) = \frac{1}{\sqrt{1 - \cos x}}$$

$$(ii) f(x) = \frac{1}{\sqrt{x + |x|}}$$

$$(iii) f(x) = x|x|$$

$$(iv) f(x) = \frac{x^3 - x + 3}{x^2 - 1}$$

$$(v) f(x) = \frac{3x}{28 - x}$$

Sol. (i) We have, $f(x) = \frac{1}{\sqrt{1 - \cos x}}$

$$\begin{aligned} \therefore & -1 \leq \cos x \leq 1 \\ \Rightarrow & -1 \leq -\cos x \leq 1 \\ \Rightarrow & 0 \leq 1 - \cos x \leq 2 \end{aligned}$$

So, $f(x)$ is defined, if $1 - \cos x \neq 0$

$$\begin{aligned} \cos x & \neq 1 \\ x & \neq 2n\pi - \forall n \in \mathbb{Z} \end{aligned}$$

$$\therefore \text{Domain of } f = \mathbb{R} - \{2n\pi : n \in \mathbb{Z}\}$$

(ii) We have,

$$f(x) = \frac{1}{\sqrt{x + |x|}}$$

$$\begin{aligned} \therefore & x + |x| = x - x = 0, x < 0 \\ & = x + x = 2x, x \geq 0 \end{aligned}$$

Hence, $f(x)$ is defined, if $x > 0$.

$$\therefore \text{Domain of } f = \mathbb{R}^+$$

(iii) We have,

$$f(x) = x|x|$$

Clearly, $f(x)$ is defined for any $x \in \mathbb{R}$.

$$\therefore \text{Domain of } f = \mathbb{R}$$

(iv) We have,

$$f(x) = \frac{x^3 - x + 3}{x^2 - 1}$$

$$f(x) \text{ is not defined, if } x^2 - 1 = 0$$

$$\Rightarrow (x - 1)(x + 1) = 0$$

$$\Rightarrow x = -1, 1$$

$$\therefore \text{Domain of } f = \mathbb{R} - \{-1, 1\}$$

(v) We have,

$$f(x) = \frac{3x}{28 - x}$$

Clearly, $f(x)$ is defined, if $28 - x \neq 0$

$$\Rightarrow x \neq 28$$

$$\therefore \text{Domain of } f = \mathbb{R} - \{28\}$$

Q. 18 Find the range of the following functions given by

$$(i) f(x) = \frac{3}{2 - x^2}$$

$$(ii) f(x) = 1 - |x - 2|$$

$$(iii) f(x) = |x - 3|$$

$$(iv) f(x) = 1 + 3 \cos 2x$$

💡 **Thinking Process**

First, find the value of x in terms of y , where $y = f(x)$. Then, find the values of y for which x attain real values.

Sol. (i) We have,

$$f(x) = \frac{3}{2 - x^2}$$

Let

$$y = f(x)$$

Then,

$$y = \frac{3}{2 - x^2} \Rightarrow 2 - x^2 = \frac{3}{y}$$

$\Rightarrow$

$$x^2 = 2 - \frac{3}{y} \Rightarrow x = \sqrt{\frac{2y - 3}{y}}$$

x assumes real values, if $2y - 3 \geq 0$ and $y > 0 \Rightarrow y \geq \frac{3}{2}$

$$\therefore \text{Range of } f = \left[\frac{3}{2}, \infty \right)$$

(ii) We know that,

$$|x - 2| \geq 0 \Rightarrow -|x - 2| \leq 0$$

$\Rightarrow$

$$1 - |x - 2| \leq 1 \Rightarrow f(x) \leq 1$$

$\therefore$ Range of

$$f = (-\infty, 1]$$

(iii) We know that,

$$|x - 3| \geq 0 \Rightarrow f(x) \geq 0$$

$\therefore$

$$\text{Range of } f = [0, \infty)$$

(iv) We know that,

$$-1 \leq \cos 2x \leq 1 \Rightarrow -3 \leq 3\cos 2x \leq 3$$

$\Rightarrow$

$$1 - 3 \leq 1 + 3\cos 2x \leq 1 + 3 \Rightarrow -2 \leq 1 + 3\cos 2x \leq 1 + 3$$

$\Rightarrow$

$$-2 \leq f(x) \leq 4$$

$\therefore$

$$\text{Range of } f = [-2, 4]$$

Q. 19 Redefine the function

$$f(x) = |x - 2| + |2 + x|, -3 \leq x \leq 3$$

💡 **Thinking Process**

First find the interval in which $|x - 2|$ and $|2 + x|$ is defined, then find the value of $f(x)$ in that interval.

Sol. Since,

$$|x - 2| = -(x - 2), x < 2$$

$$x - 2, x \geq 2$$

and

$$|2 + x| = -(2 + x), x < -2$$

$$(2 + x), x \geq -2$$

$\therefore$

$$f(x) = |x - 2| + |2 + x|, -3 \leq x \leq 3$$

$$= \begin{cases} -(x - 2) - (2 + x), & -3 \leq x < -2 \\ -(x - 2) + 2 + x, & -2 \leq x < 2 \\ x - 2 + 2 + x, & 2 \leq x \leq 3 \end{cases}$$

$$= \begin{cases} -2x, & -3 \leq x < -2 \\ 4, & -2 \leq x < 2 \\ 2, & 2 \leq x \leq 3 \end{cases}$$

Q. 20 If $f(x) = \frac{x-1}{x+1}$, then show that

$$(i) f\left(\frac{1}{x}\right) = -f(x)$$

$$(ii) f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$$

Sol. We have, $f(x) = \frac{x-1}{x+1}$

$$(i) f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}-1}{\frac{1}{x}+1} = \frac{(1-x)/x}{(1+x)/x} = \frac{1-x}{1+x} = \frac{-(x-1)}{x+1} = -f(x)$$

$$(ii) f\left(-\frac{1}{x}\right) = \frac{-\frac{1}{x}-1}{-\frac{1}{x}+1} = \frac{(-1-x)/x}{(-1+x)/x} \Rightarrow f\left(-\frac{1}{x}\right) = \frac{-(x+1)}{x-1}$$

$$\text{Now, } \frac{-1}{f(x)} = \frac{-1}{\frac{x-1}{x+1}} = \frac{-(x+1)}{x-1}$$

$$\therefore f\left(-\frac{1}{x}\right) = -\frac{1}{f(x)}$$

Q. 21 If $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined in the domain $R^+ \cup \{0\}$, then find the value of

$$(i) (f+g)(x)$$

$$(ii) (f-g)(x)$$

$$(iii) (fg)(x)$$

$$(iv) \left(\frac{f}{g}\right)(x)$$

Sol. We have, $f(x) = \sqrt{x}$ and $g(x) = x$ be two function defined in the domain $R^+ \cup \{0\}$.

$$(i) (f+g)(x) = f(x) + g(x) = \sqrt{x} + x$$

$$(ii) (f-g)(x) = f(x) - g(x) = \sqrt{x} - x$$

$$(iii) (fg)(x) = f(x) \cdot g(x) = \sqrt{x} \cdot x = x^{\frac{3}{2}}$$

$$(iv) \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$$

Q. 22 Find the domain and range of the function $f(x) = \frac{1}{\sqrt{x-5}}$.

Sol. We have, $f(x) = \frac{1}{\sqrt{x-5}}$

$f(x)$ is defined, if $x-5 > 0 \Rightarrow x > 5$

$\therefore$ Domain of $f = (5, \infty)$

Let $f(x) = y$

$$\therefore y = \frac{1}{\sqrt{x-5}} \Rightarrow \sqrt{x-5} = \frac{1}{y}$$

$$\Rightarrow x-5 = \frac{1}{y^2}$$

$$\therefore x = \frac{1}{y^2} + 5$$

$$\therefore x \in (5, \infty) \Rightarrow y \in R^+$$

Hence, range of $f = R^+$

Q. 23 If $f(x) = y = \frac{ax - b}{cx - a}$, then prove that $f(y) = x$.

Sol. We have, $f(x) = y = \frac{ax - b}{cx - a}$

$$\begin{aligned} \therefore f(y) &= \frac{ay - b}{cy - a} = \frac{a\left(\frac{ax - b}{cx - a}\right) - b}{c\left(\frac{ax - b}{cx - a}\right) - a} \\ &= \frac{a(ax - b) - b(cx - a)}{c(ax - b) - a(cx - a)} = \frac{a^2x - ab - bcx + ab}{acx - bc - acx + a^2} \\ &= \frac{a^2x - bcx}{a^2 - bc} = \frac{x(a^2 - bc)}{(a^2 - bc)} = x \end{aligned}$$

$$\therefore f(y) = x$$

Hence proved.

Objective Type Questions

Q. 24 Let $n(A) = m$ and $n(B) = n$. Then, the total number of non-empty relations that can be defined from A to B is

- (a) m^n (b) $n^m - 1$ (c) $mn - 1$ (d) $2^{mn} - 1$

Thinking Process

First find the number of element in $A \times B$ and then find the number of relation by using $2^{m(A \times B)} - 1$.

Sol. (d) We have,

$$\begin{aligned} n(A) &= m \text{ and } n(B) = n \\ n(A \times B) &= n(A) \cdot n(B) \\ &= mn \end{aligned}$$

$$\text{Total number of relation from } A \text{ to } B = 2^{mn} - 1 = 2^{n(A \times B)} - 1$$

Q. 25 If $[x]^2 - 5[x] + 6 = 0$, where $[\cdot]$ denote the greatest integer function, then

- (a) $x \in [3, 4]$ (b) $x \in (2, 3]$ (c) $x \in [2, 3]$ (d) $x \in [2, 4)$

Thinking Process

If a and b are two successive positive integer and $[x] = a, b$, then $x \in [a, b]$

Sol. (c) We have,

$$\begin{aligned} [x]^2 - 5[x] + 6 &= 0 \\ \Rightarrow [x]^2 - 3[x] - 2[x] + 6 &= 0 \\ \Rightarrow [x]([x] - 3) - 2([x] - 3) &= 0 \\ \Rightarrow ([x] - 3)([x] - 2) &= 0 \\ \Rightarrow [x] &= 2, 3 \\ \therefore x &\in [2, 3] \end{aligned}$$

Q. 26 Range of $f(x) = \frac{1}{1 - 2\cos x}$ is

(a) $\left[\frac{1}{3}, 1\right]$

(b) $\left[-1, \frac{1}{3}\right]$

(c) $(-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$

(d) $\left[-\frac{1}{3}, 1\right]$

Sol. (b) We know that,

$$-1 \leq -\cos x \leq 1$$

$\Rightarrow$

$$-2 \leq -2\cos x \leq 2$$

$\Rightarrow$

$$1 - 2 \leq 1 - 2\cos x \leq 1 + 2$$

$\Rightarrow$

$$-1 \leq 1 - 2\cos x \leq 3$$

$\Rightarrow$

$$-1 \leq \frac{1}{1 - 2\cos x} \leq \frac{1}{3}$$

$\Rightarrow$

$$-1 \leq f(x) \leq \frac{1}{3}$$

$\therefore$

$$\text{Range of } f = \left[-1, \frac{1}{3}\right]$$

Q. 27 Let $f(x) = \sqrt{1 + x^2}$, then

(a) $f(xy) = f(x) \cdot f(y)$

(b) $f(xy) \geq f(x) \cdot f(y)$

(c) $f(xy) \leq f(x) \cdot f(y)$

(d) None of these

Sol. (c) We have,

$$f(x) = \sqrt{1 + x^2}$$

$$f(xy) = \sqrt{1 + x^2 y^2}$$

$$f(x) \cdot f(y) = \sqrt{1 + x^2} \cdot \sqrt{1 + y^2}$$

$$= \sqrt{(1 + x^2)(1 + y^2)}$$

$$= \sqrt{1 + x^2 + y^2 + x^2 y^2}$$

$\therefore$

$$\sqrt{1 + x^2 y^2} \leq \sqrt{1 + x^2 + y^2 + x^2 y^2}$$

$\Rightarrow$

$$f(xy) \leq f(x) \cdot f(y)$$

Q. 28 Domain of $\sqrt{a^2 - x^2}$ ($a > 0$) is

(a) $(-a, a)$

(b) $[-a, a]$

(c) $[0, a]$

(d) $(-a, 0]$

Sol. (b) Let

$$f(x) = \sqrt{a^2 - x^2}$$

$f(x)$ is defined, if

$$a^2 - x^2 \geq 0$$

$\Rightarrow$

$$x^2 - a^2 \leq 0$$

$\Rightarrow$

$$(x - a)(x + a) \leq 0$$

$\Rightarrow$

$$-a \leq x \leq a$$

$\therefore$

$$\text{Domain of } f = [-a, a]$$

$$[\because a > 0]$$

Q. 29 If $f(x) = ax + b$, where a and b are integers, $f(-1) = -5$ and $f(3) = 3$, then a and b are equal to

(a) $a = -3, b = -1$

(b) $a = 2, b = -3$

(c) $a = 0, b = 2$

(d) $a = 2, b = 3$

Sol. (b) We have,

$$f(x) = ax + b$$

$$f(-1) = a(-1) + b$$

$$-5 = -a + b \quad \dots(i)$$

and,

$$f(3) = a(3) + b$$

$$3 = 3a + b \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$a = 2 \text{ and } b = -3$$

Q. 30 The domain of the function f defined by

$$f(x) = \sqrt{4-x} + \frac{1}{\sqrt{x^2-1}} \text{ is equal to}$$

(a) $(-\infty, -1) \cup (1, 4]$

(b) $(-\infty, -1] \cup (1, 4]$

(c) $(-\infty, -1) \cup [1, 4]$

(d) $(-\infty, -1) \cup [1, 4)$

Sol. (a) We have,

$$f(x) = \sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$$

$f(x)$ is defined, if

$$4-x \geq 0 \text{ or } x^2-1 > 0$$

$$x-4 \leq 0 \text{ or } (x+1)(x-1) > 0$$

$$x \leq 4 \text{ or } x < -1 \text{ and } x > 1$$

$$\therefore \text{Domain of } f = (-\infty, -1) \cup (1, 4]$$

Q. 31 The domain and range of the real function f defined by $f(x) = \frac{4-x}{x-4}$ is given by

(a) Domain = R , Range = $\{-1, 1\}$

(b) Domain = $R - \{1\}$, Range = R

(c) Domain = $R - \{4\}$, Range = $\{-1\}$

(d) Domain = $R - \{-4\}$, Range = $\{-1, 1\}$

● **Thinking Process**

A function $\frac{f(x)}{g(x)}$ is defined, if $g(x) \neq 0$.

Sol. (c) We have,

$$f(x) = \frac{4-x}{x-4}$$

$f(x)$ is defined, if $x-4 \neq 0$ i.e., $x \neq 4$

$$\therefore \text{Domain of } f = R - \{4\}$$

Let

$$f(x) = y$$

$$\therefore y = \frac{4-x}{x-4} \Rightarrow xy - 4y = 4 - x$$

$$\Rightarrow xy + x = 4 + 4y \Rightarrow x(y+1) = 4(1+y)$$

$$\therefore x = \frac{4(1+y)}{y+1}$$

x assumes real values, if $y+1 \neq 0$ i.e., $y \neq -1$.

$$\therefore \text{Range of } f = R - \{-1\}$$

Q. 32 The domain and range of real function f defined by

$$f(x) = \sqrt{x-1} \text{ is given by}$$

- (a) Domain = $(1, \infty)$, Range = $(0, \infty)$ (b) Domain = $[1, \infty)$, Range = $(0, \infty)$
 (c) Domain = $(1, \infty)$, Range = $[0, \infty)$ (d) Domain = $[1, \infty)$, Range = $[0, \infty)$

Thinking Process

A function is defined $f(x) = \sqrt{x}$ is defined $x \geq 0$.

Sol. (d) We have, $f(x) = \sqrt{x-1}$

$f(x)$ is defined, if $x-1 \geq 0$.

$$\Rightarrow x \geq 1$$

$$\therefore \text{Domain of } f = [1, \infty)$$

$$\text{Let } f(x) = y$$

$$\therefore y = \sqrt{x-1}$$

$$\Rightarrow y^2 = x-1$$

$$\therefore x = y^2 + 1$$

x assumes real values for $y \in R$.

$$\text{but } y \geq 0$$

$$\therefore \text{Range of } f = [0, \infty)$$

Q. 33 The domain of the function f given by $f(x) = \frac{x^2 + 2x + 1}{x^2 - x - 6}$.

- (a) $R - \{3, -2\}$ (b) $R - \{-3, 2\}$ (c) $R - [3, -2]$ (d) $R - (3, -2)$

Sol. (a) We have, $f(x) = \frac{x^2 + 2x + 1}{x^2 - x - 6}$

$$f(x) \text{ is defined, if } x^2 - x - 6 = 0$$

$$\Rightarrow x^2 - 3x + 2x - 6 = 0$$

$$\Rightarrow x(x-3) + 2(x-3) = 0$$

$$\Rightarrow (x-3)(x+2) = 0$$

$$\therefore x = -3, -2$$

$$\therefore \text{Domain of } f = R - \{3, -2\}$$

Q. 34 The domain and range of the function f given by $f(x) = 2 - |x-5|$ is

$$(a) \text{ Domain} = R^+, \text{ Range} = (-\infty, 1]$$

$$(b) \text{ Domain} = R, \text{ Range} = [-\infty, 2]$$

$$(c) \text{ Domain} = R, \text{ Range} = (-\infty, 2)$$

$$(d) \text{ Domain} = R^+, \text{ Range} = (-\infty, 2]$$

Sol. (b) We have, $f(x) = 2 - |x-5|$

$f(x)$ is defined for all $x \in R$

$$\therefore \text{Domain of } f = R$$

$$\text{We know that, } |x-5| \geq 0 \Rightarrow -|x-5| \leq 0$$

$$\Rightarrow 2 - |x-5| \leq 2$$

$$\therefore f(x) \leq 2$$

$$\therefore \text{Range of } f = [-\infty, 2]$$

Q. 35 The domain for which the functions defined by $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal to

(a) $\left[-1, \frac{4}{3}\right]$

(b) $\left[1, \frac{4}{3}\right]$

(c) $\left[-1, -\frac{4}{3}\right]$

(d) $\left[-2, -\frac{4}{3}\right]$

Sol. (a) We have, $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$

$$f(x) = g(x)$$

$$\Rightarrow 3x^2 - 1 = 3 + x$$

$$\Rightarrow 3x^2 - x - 4 = 0$$

$$\Rightarrow 3x^2 - 4x + 3x - 4 = 0$$

$$\Rightarrow x(3x - 4) + 1(3x - 4) = 0$$

$$\Rightarrow (3x - 4)(x + 1) = 0$$

$$\therefore x = -1, \frac{4}{3}$$

So, domain for which $f(x)$ and $g(x)$ are equal to $\left[-1, \frac{4}{3}\right]$.

Fillers

Q. 36 Let f and g be two real functions given by

$$f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 1)\}$$

$$\text{and } g = \{(1, 0), (2, 2), (3, -1), (4, 4), (5, 3)\},$$

then the domain of $f \cdot g$ is given by.....

💡 Thinking Process

First find the domain of f and domain of g . Then,

$$\text{domain of } f \cdot g = \text{domain of } f \cap \text{domain of } g.$$

Sol. We have,

$$f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 1)\}$$

and

$$g = \{(1, 0), (2, 2), (3, -1), (4, 4), (5, 3)\}$$

$\therefore$

$$\text{Domain of } f = \{0, 2, 3, 4, 5\},$$

and

$$\text{Domain of } g = \{1, 2, 3, 4, 5\}$$

$$\therefore \text{Domain of } (f \cdot g) = \text{Domain of } f \cap \text{Domain of } g = \{2, 3, 4, 5\}$$

- Q. 37** Let $f = \{(2, 4), (5, 6), (8, -1), (10, -3)\}$
 and $g = \{(2, 5), (7, 1), (8, 4), (10, 13), (11, 5)\}$
 be two real functions. Then, match the following.

Column I	Column II
(i) $f - g$	(a) $\left\{\left(2, \frac{4}{5}\right), \left(8, \frac{-1}{4}\right), \left(10, \frac{-3}{13}\right)\right\}$
(ii) $f + g$	(b) $\{(2, 20), (8, -4), (10, -39)\}$
(c) $f \cdot g$	(c) $\{(2, -1), (8, -5), (10, -16)\}$
(d) $\frac{f}{g}$	(d) $\{(2, 9), (8, 3), (10, -10)\}$

The domain of $f - g, f + g, f \cdot g, \frac{f}{g}$ is domain of $f \cap$ domain of g . Then, find their images.

Sol. We have,

$$f = \{(2, 4), (5, 6), (8, -1), (10, -3)\}$$

and

$$g = \{(2, 5), (7, 1), (8, 4), (10, 13), (11, 5)\}$$

So, $f - g, f + g, f \cdot g, \frac{f}{g}$ are defined in the domain (domain of $f \cap$ domain of g)

$$\text{i.e., } \{2, 5, 8, 10\} \cap \{2, 7, 8, 10, 11\} \Rightarrow \{2, 8, 10\}$$

$$(i) (f - g)(2) = f(2) - g(2) = 4 - 5 = -1$$

$$(f - g)(8) = f(8) - g(8) = -1 - 4 = -5$$

$$(f - g)(10) = f(10) - g(10) = -3 - 13 = -16$$

$$\therefore f - g = \{(2, -1), (8, -5), (10, -16)\}$$

$$(ii) (f + g)(2) = f(2) + g(2) = 4 + 5 = 9$$

$$(f + g)(8) = f(8) + g(8) = -1 + 4 = 3$$

$$(f + g)(10) = f(10) + g(10) = -3 + 13 = 10$$

$$\therefore f + g = \{(2, 9), (8, 3), (10, 10)\}$$

$$(iii) (f \cdot g)(2) = f(2) \cdot g(2) = 4 \times 5 = 20$$

$$(f \cdot g)(8) = f(8) \cdot g(8) = -1 \times 4 = -4$$

$$(f \cdot g)(10) = f(10) \cdot g(10) = -3 \times 13 = -39$$

$$\therefore fg = \{(2, 20), (8, -4), (10, -39)\}$$

$$(iv) \left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{4}{5}$$

$$\left(\frac{f}{g}\right)(8) = \frac{f(8)}{g(8)} = \frac{-1}{4}$$

$$\left(\frac{f}{g}\right)(10) = \frac{f(10)}{g(10)} = \frac{-3}{13}$$

$$\therefore \frac{f}{g} = \left\{\left(2, \frac{4}{5}\right), \left(8, -\frac{1}{4}\right), \left(10, \frac{-3}{13}\right)\right\}$$

Hence, the correct matches are (i) $\rightarrow$ (c), (ii) $\rightarrow$ (d), (iii) $\rightarrow$ (b), (iv) $\rightarrow$ (a).

True/False

Q. 38 The ordered pair $(5, 2)$ belongs to the relation

$$R = \{(x, y) : y = x - 5, x, y \in \mathbb{Z}\}$$

Sol. False

We have,

$$R = \{(x, y) : y = x - 5, x, y \in \mathbb{Z}\}$$

If

$$x = 5, \text{ then } y = 5 - 5 = 0$$

Hence, $(5, 2)$ does not belong to R .

Q. 39 If $P = \{1, 2\}$, then $P \times P \times P = \{(1, 1, 1), (2, 2, 2), (1, 2, 2), (2, 1, 1)\}$

Sol. False

We have,

$$P = \{1, 2\} \text{ and } n(P) = 2$$

$\therefore$

$$n(P \times P \times P) = n(P) \times n(P) \times n(P) = 2 \times 2 \times 2 = 8$$

But given $P \times P \times P$ has 4 elements.

Q. 40 If $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$, then $(A \times B) \cup (A \times C)$
 $= \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3),$
 $(3, 4), (3, 5), (3, 6)\}.$

Thinking Process

First, we find $A \times B$ and $A \times C$, then we will find $(A \times B) \cup (A \times C)$.

Sol. True

We have,

$$A = \{1, 2, 3\}, B = \{3, 4\} \text{ and } C = \{4, 5, 6\}$$

$$A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$$

$$A \times C = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$$

$$(A \times B) \cup (A \times C) = \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$$

Q. 41 If $(x - 2, y + 5) = \left(-2, \frac{1}{3}\right)$ are two equal ordered pairs, then $x = 4$,

$$y = \frac{-14}{3}$$

Sol. False

We have,

$$(x - 2, y + 5) = \left(-2, \frac{1}{3}\right)$$

$\Rightarrow$

$$x - 2 = -2, y + 5 = \frac{1}{3} \Rightarrow x = -2 + 2, y = \frac{1}{3} - 5$$

$\therefore$

$$x = 0, y = \frac{-14}{3}$$

Q. 42 If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$, then $A = \{a, b\}$ and $B = \{x, y\}$.

Sol. True

We have,

$$A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$$

$$A = \text{Set of first element of ordered pairs in } A \times B = \{a, b\}$$

$$B = \text{Set of second element of ordered pairs in } A \times B = \{x, y\}$$