Binomial Distribution

Q.1. The probability that a teacher will give an unannounced test during any class meeting is 1/5. If a student is absent twice , find the probability that the student will miss at least one test.

Solution : 1

Here, p = 1/5 , q = 4/5 , n = 2 = no. of days the student is absent.

Required probability = ${}^{2}C_{1}$ (1/5)1(4/5) + ${}^{2}C_{2}$ $(1/5)^{2}$.

= 2 × (4/25) + 1/25 = 9/25 .

Q.2. Eight coins are thrown simultaneously.

i. Show that the probability of getting at least 6 heads is 37/256.
ii. What is the probability of getting at least 3 heads ?

Solution : 2

We have , p = 1/2 , q = 1/2 , n = 8 .

i. Probability (at least 6 heads) = ${}^{8}C_{6}$ $(1/2)^{6}$ $(1/2)^{2}$ + ${}^{8}C_{7}$ $(1/2)^{8}$
   = [(8 × 7)/ (1 × 2) + 8 + 1] $(1/2)^{8}$
   = 37 × (1/256) = 37/256 .

ii. Probability (at least 3 heads) = 1 – [${}^{8}C_{0}$ $(1/2)^{8}$ + ${}^{8}C_{1}$ $(1/2)^{8}$ + ${}^{8}C_{2}$ $(1/2)^{8}$]
   = 1 – $(1/2)^{8}$
   [1 + 8 + 28]
   = 1 – 37/256 = 219/256 .

Q.3. Assuming that on an average one telephone out of 10 is busy, seven telephone numbers are randomly selected and called. Find the probability that three of them will be busy.

Solution : 3

Let p = P(telephone being busy) = 1/10. Then q = 9/10.

Required probability = 7C3(1/10)3(9/10)4

= $[(7\times6\times5)/(1\times2\times3)]\times(1/103)\times(9)^4\times(1/10)^4$.

= $[35 \times(9)^4]/10^7$.

Q.4. In a binomial distribution, the sum of its mean and the variance is 1.8. Find the probability of two successes if the event was conducted 5 times.

Solution : 4

We have ,
mean + variance = np + npq = 1.8

Therefore, p + pq = 1.8/5 = 0.36 ------------- (i)

But p + q = 1 -------------------------- (ii)

Therefore, 1 – q + (1 – q)q = 0.36 [putting p = 1 – q in (i)]

Or, $1 - q^2 = 0.36$

Or, $q^2 = 1 - 0.36 = 0.64$

Or, q = 0.8 and p = 1 – 0.8 = 0.2.

The probability of 2 successes = ${}^5C_2\ (0.2)^2(0.8)^3$

= 10×0.04×0.512 = 0.2048.

Q.5. Four dice are thrown simultaneously. If the occurrence of an odd number in a single dice is considered a success, find the probability of at most 2 successes.

Solution : 5

No. of dices = 4, n = 4

Probability of getting odd no. in a dice = 3/6 = 1/2

Therefore, p = 1/2 and q = 1/2

Using Binomial distribution gives , $\sum {}^nC_r q^{n-r} p^r$ [r from 0 to n] ,

for at most 2 success $r \le 2$.

Therefore, required probability = ${}^4C_0q^4 + {}^4C_1q^3p^1 + {}^4C_2q^2p^2$

$= {}^4C_0 (1/2)^4 + {}^4C_1 (1/2)^3(1/2)^1 + 4C^2 (1/2)^2(1/2)^2$

$= (1/2)^4\{{}^4C_0 + {}^4C_1 + {}^4C_2\}$

= 1/16 {1 + 4 + 6} = 11/16 .

Q.6. A coin is tossed 5 times. What is the probability of getting at least three heads.

Solution : 6

We have n = 5, p = 1/2 and q = 1/2 .

Therefore , probability of at least 3 heads

$= {}^5C_3(1/2)^3(1/2)^2 + {}^5C_4(1/2)^4(1/2) + {}^5C_5(1/2)^5$

= (5×4)/(2×32) + (5×1)/32 + 1/32

= 16/32 = 1/2 .

Q.7. A dice is tossed three times. Getting a '3' or a '5' is considered a success. Find the probability of at least two successes.

Solution : 7

Probability of 3 or 5 = P(3 or 5) = 2/6 = 1/3 = p = Probability of success.

Probability of failure = q = 1 − 1/3 = 2/3.

Therefore, probability of at least 2 successes = ${}^3C_0(p)^3 + {}^3C_1(p)^2(q)^1$

$= (1/3)^3 + 3(1/3)^2(2/3)^1$

= 1/27 + 3×1/9×2/3 = 7/27.

Q.8. A and B play a game in which A's chance of winning the game is 3/5. In a series of 6 games, find the probability that A will win at least 4 games.

Solution : 8

We are given, p = 3/5 then q = 1 – p = 1 – 3/5 = 2/5.

Probability of winning at least 4 games out of 6 games

$= {}^6C_0p^6 + {}^6C_1p^5q_1 + {}^6C_2p^4q_2$

$= (3/5)^6 + 6(3/5)^5(2/5)^1 + 15(3/5)^4(2/5)^2$

= 1/56[729 + 2916 + 4860] = 8505/15625

= 1701/3125.

Q.9. The mean and variance of a binomial distribution are 4 and 2 respectively. Find the probability of at least 6 successes.

Solution : 9

We have, np = 4, npq = 2.

Dividing we get q = 1/2 and then p = 1/2 [As, p + q = 1]

Thus n = 8.

Therefore, required probability $= {}^8C_6\ (1/2)^8 + {}^8C_7\ (1/2)^8 + {}^8C_8\ (1/2)^8$

$= (1/2)^8[28 + 8 + 1]$

= 37/256.