

Cube Roots of Unity

Q.1. Simplify : $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^8)$.

Solution : 1

$$\begin{aligned} \text{We have } & (1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^8) \\ &= (1 - \omega)(1 - \omega^2)(1 - \omega^3 \cdot \omega)(1 - \omega^6 \cdot \omega^2) \\ &= (1 - \omega)(1 - \omega^2)(1 - \omega)(1 - \omega^2) \quad [\omega^3 = 1, \omega^6 = (\omega^3)^2 = 1] \\ &= (1 - \omega)(1 - \omega)(1 - \omega^2)(1 - \omega^2) \\ &= (1 + \omega^2 - 2\omega)(1 + \omega^4 - 2\omega^2) \\ &= (1 + \omega^2 - 2\omega)(1 + \omega^3 \cdot \omega - 2\omega^2) \\ &= (1 + \omega^2 - 2\omega)(1 + \omega - 2\omega^2) \quad [\omega^3 = 1] \\ &= (-\omega - 2\omega)(-\omega^2 - 2\omega^2) \quad [1 + \omega + \omega^2 = 0] \\ &= (-3\omega)(-3\omega^2) \\ &= 9\omega^3 = 9 \times 1 = 9 . \end{aligned}$$

Q.2. Simplify : $(1 - 3\omega + \omega^2)(1 + \omega - 3\omega^2)$.

Solution : 2

$$\begin{aligned} \text{We have } & (1 - 3\omega + \omega^2)(1 + \omega - 3\omega^2) = (1 + \omega^2 - 3\omega)(1 + \omega - 3\omega^2) \\ &= (1 + \omega^2 + \omega - 3\omega - \omega)(1 + \omega + \omega^2 - 3\omega^2 - \omega^2) \\ &= (0 - 4\omega)(0 - 4\omega^2) \\ &= (-4\omega)(-4\omega^2) \\ &= 16\omega^3 = 16 \times 1 \quad [\text{As, } \omega^3 = 1] \end{aligned}$$

$$= 16.$$

Q.3. If ω and ω^2 are cube roots of unity, prove that $(2 - \omega + 2\omega^2)(2 + 2\omega - \omega^2) = 9$.

Solution : 3

$$\begin{aligned} \text{L.H.S.} &= (2 - \omega + 2\omega^2)(2 + 2\omega - \omega^2) \\ &= 4 + 4\omega - 2\omega^2 - 2\omega - 2\omega^2 + \omega^3 + 4\omega^2 + 4\omega^3 - 2\omega^4 \\ &= 4 + 2\omega + 5\omega^3 - 2\omega^4 \\ &= 4 + 2\omega + 5 - 2\omega \\ &= 9 = \text{R.H.S.} \end{aligned}$$

Q.4. If 1, ω and ω^2 are cube of unity , prove that : $(x + y)^2 + (x\omega + y\omega^2)^2 + (x\omega^2 + y\omega)^2 = 6xy$.

Solution : 4

$$\begin{aligned} \text{L.H.S.} &= (x + y)^2 + (x\omega + y\omega^2)^2 + (x\omega^2 + y\omega)^2 \\ &= (x^2 + 2xy + y^2) + (x^2\omega^2 + 2xy\omega^3 + y^2\omega^4) + (x^2\omega^4 + 2xy\omega^3 + y^2\omega^2) \\ &= x^2(1 + \omega^2 + \omega) + y^2(1 + \omega + \omega^2) + 6xy \\ &= 0 + 0 + 6xy \text{ [As, } 1 + \omega + \omega^2 = 0, \omega^3 = 1] \\ &= 6xy = \text{R.H.S.} \end{aligned}$$