Probability

Q.1. In a single throw of two dice, find the probability of getting a total of at most 9.

Solution : 1

Favourable ways are : (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)

(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)

(3,1), (3,2), (3,3), (3,4), (3,5) ,(3,6)

(4,1), (4,2), (4,3), (4,4), (4,5)

(5,1), (5,2),(5,3), (5,4)

(6,1), (6,2), (6,3).

Therefore total number of favourable ways = 30.

Total number of ways = 36.

Therefore required probability = 30/36 = 5/6 .

Q.2. A pair of dice is thrown. If the two numbers appearing on them are different, find the probability that the sum of the numbers appearing on dice is 6.

Solution : 2

Favourable pairs of events out of 36 are : (1, 5), (2, 4), (3, 3), (4, 2), (5, 1).

Number of favourable events = 5.

Therefore, P(Sum of numbers appearing on the two dice is 6)

= No. of favourable outcomes/total no. of outcomes

= 5/36.

Q.3. There are 3 urns A, B and C. Urn A contains 4 red balls and 3 black balls. Urn B contains 5 red and 4 black balls. Urn C contains 4 red balls and 4 black balls. One ball is

drawn from each of these urns. What is the probability that the 3 balls drawn consist of 2 red balls and 1 black ball ?

Solution : 3

The possible cases are : RRB , RBR and BRR.

The probability is = [(4/7) × (5/9) × (4/8) + (4/7) × (4/9) × (4/8) + (3/7) × (5/9) × (4/8)]

= (80 + 64 + 60)/ 504 = 204/504 = 51/126.

Q.4. A bag has 4 red and 5 black balls, a second bag has 3 red and 7 black balls. One ball is drawn from the first bag and two from the second. Find the probability that two balls are black and one is red.

Solution : 4

Bag one = B_1	Bag second = B_2
4 Red	3 Red
5 Black	7 Black

i. We can draw 1 black ball from B1 and 1 red and 1 black ball from B2.

   P1 = 5C1/9C1 × [3C1×7C1]/10C2 = (5/9)[(3 × 7)/45] = 7/27

ii. We can draw 1 red ball from B1 and 2 black balls from B2.

   P2 = [4C1/9C1]×[7C2/10C2] = (4/9)×(21/45) = 28/135.

   Here event (i) and event (ii) are mutually exclusive.

   Probability of event of drawing 2 black balls and 1 red ball from the bag

   = P1 + P2 = 7/27 + 28/135 = 7/15.

Q.5. Kamal and Monika appear for an interview for two vacancies. The probability of Kamal's selection is 1/3 and that of Monika's selection is 1/5. Find the probability that only one of them will be selected.

Solution : 5

Probability of selection of Kamal = P(K) = 1/3 .

and probability of selection of Monika = P(M) = 1/5 .

Probability of not selection of Kamal = 1 – P(K) = 2/3 .

and Probability of not selection of Monika = 1 – P(M) = 4/5 .

Therefore , P(only one will be selected) = P(K) . {1 – P(M)}+ P(M) . {1 – P(K)}

= 1/3 × 4/5 + 1/5 × 2/3

= 4/15 + 2/15 = 6/15 = 2/5.

Q.6. Two horses are considered for a race. The probability of selection of the first horse is 1/4 and that of the second is 1/3. What is the probability that :

i. both of them will be selected.
ii. only one of them will be selected.
iii. none of them will be selected.

Solution : 6

i. P(both of them will be selected) = 1/4 × 1/3 = 1/12.

ii. P(only one of them will be selected) = P[(1^{st} selected and 2^{nd} not selected) or (1^{st} not selected and 2^{nd} selected)] = 1/4 ×(1 – 1/3) + (1 – 1/4)× 1/3 = 1/6 + 1/4 = 5/12.

iii. P(none of them will be selected) = (1 – 1/4)(1 – 1/3) = 1/2.

Q.7. The bag 'A' contains 3 white and 2 black balls while the bag 'B' contains 2 white and 5 black balls. One of the bag is selected at random and a ball is drawn from it. What is the probability that the balls is white.

Solution : 7

If bag A is selected, then the probability of being white ball

= P(A) = 1/2×3/5 = 3/10.

If bag B is selected , then P(B) = 1/2 ×2/7 = 1/7.

Therefore, required probability = 3/10 + 1/7 = 31/70.

Q.8. The probability that A,B and C solving a problem are 1/3, 2/7 and 3/8 respectively. If all the three try and solve the problem simultaneously , find the probability that only one of them will solve it.

Solution : 8

We have P(A) = 1/3 => P(not A) = 2/3,

P(B) = 2/7 => P(not B) = 5/7,

P(C) =3/8 => P(not C) = 5/8.

Probability that only one will solve the problem

= P(A).P(not B).P(not C) + P(not A).P(B).P(not C)+ P(not A).P(not B).P(C)

= 1/3×5/7×5/8 + 2/3×2/7×5/8 + 2/3×5/7×3/8

= (25/168) + (20/168) + (30/168)

= 75/168 = 25/56.

Q.9. A and B throw two dice each. If A gets a sum of 9 on his two dice, then find the probability of B getting a higher sum.

Solution : 9

Possible sum of 9 in two dices : (3, 6), (6, 3), (4, 5), (5, 4)

Therefore, Prbability P(A) = 4/36 = 1/9

Possible sum of more than 9 in two dices : (4, 6),(6, 4), (5, 5), (5, 6), (6, 5), (6, 6)

Probability P(B) = 6/36 = 1/6

Required probability = P(A)P(B) = (1/9)×(1/6) = 1/54 .

Q.10. A card is drawn at random from a pack of 52 playing cards. What is the probability that the card drawn is neither a spade nor a queen?

Solution : 10

Here the case is of non-mutually exclusive event.

Let P(A) = Probability of card being spade = 13/52,

P(B) = Probability of card being queen = 4/52, then

P(A∩B) = Probability of card being queen of spade = 1/52.

Probability of card being either a spade or a queen = P(AUB)

= P(A) + P(B) – P(A∩B)

= 13/52 + 4/52 – 1/52

= 16/52 = 4/13

Thus probability of card being neither a spade nor a queen = 1 – P(AUB)

= 1 – 4/13

= 9/13

Q.11. Tickets numbered from 1 to 20 are mixed up together and then a ticket is drawn at random. What is the probability that the ticket has a number which is a multiple of 3 or 7.

Solution : 11

Total number of cases , n = 20.

Multiples of 3 are : 3, 6, 9, 12, 15, 18. [6 in number]

Multiples of 7 are : 7, 14. [2 in number]

Let A be the events of multiple of 3 and B be the events of multiple of 7.

P(A) = 6/20 = 3/10 and P(B) = 2/20 = 1/10,

Total Probability = P(AUB) = P(A) + P(B)

= 3/10 + 1/10 = 4/10 = 2/5 .

Q.12. In a certain city, the probability of not reading the morning newspaper by the residents is 1/2 and that of not reading the evening newspaper is 2/5. The probability of reading both the newspapers is 1/5. Find the probability that a resident reads either the morning or evening or both the papers.

Solution : 12

Probability of not reading morning newspaper (A') = 1/2

Probability of reading morning newspaper (A) = 1 – 1/2 = 1/2 = P(A)

Probability of not reading evening newspaper (B') = 2/5

Probability of reading evening news paper (B) = 1 – 2/5 = 3/5 = P(B)

Therefore, P(A∩B) = 1/5 [P(A∩B) = P(A).P(B), as A and B are independent.]

Probability of reading either of news papers = P(AUB)

= P(A) + P(B) – P(A∩B)

= 1/2 + 3/5 – 1/5

= 1/2 + 2/5 = 9/10.

Q.13. The probability that a contractor will get a plumbing contract is 2/3 and an electric contract is 4/9. If the probability of getting at least one contract is 4/5, find the probability that he will get both the contracts.

Solution : 13

Let P(A) = 2/3, P(B) = 5/9 and P(AUB) = 4/5. We have to find P(A∩B).

Using P(AUB) = P(A) + P(B) – P(A∩B)

Or, 4/5 = 2/3 + 5/9 – P(A∩B)

Or, P(A∩B) = 11/9 – 4/5 = (55 – 36)/45 = 19/45.

Q.14. A candidate is selected for interview of management trainees for 3 companies. For the first company, there are 12 candidates, for the second there are 15 candidates and for the third, there are 10 candidates. Find the probability that he is selected in at least one of the companies.

Solution : 14

Probability of selection in 1^{st} company = 1/12 = P(A)

Probability of selection in 2^{nd} company = 1/15 = P(B)

Probability of selection in 3^{rd} company = 1/10 = P(C)

Therefore, probability of not selected = P(A').P(B').P(C')

= [1 – P(A)].[1 – P(B)].[1 – P(C)]

= (1 – 1/12).(1 – 1/15).(1 – 1/10)

= (11/12)(14/15)(9/10) = 77/100

Therefore, probability of selection in at least one company = 1 – P(A')P(B')P(C')

= 1 – 77/100 = 23/100 .

Q.15. There are 10 persons who are to be seated around a circular table. Find the probability that two particular persons will always sit together.

Solution : 15

If 'n' objects are arranged in a circle, the total number of ways in which they can be arranged are (n – 1)!. We have n = 10.

Therefore, number of ways in which 10 person can be seated = (10 – 1)! = 9!

The two particular persons are always together. Hence we have to arrange only 9 person around a circular table.

Therefore, number of ways = (9 – 1)! = 8!

The two particular person can interchange their places,the favourable cases are = 2×8!

Therefore, the probability that two particular persons always sit together

= No. of favourable cases/Total cases

= (2 × 8!)/9! = (2 × 8!)/(9 × 8!) = 2/9.

Q.16. Bag A contains 5 white and 4 black balls and bag B contains 7 white and 6 black balls. One bag is drawn from the bag A and without noticing its colour, is put in the bag B. If a ball is then drawn from bag B, find the possibility that it is black in colour.

Solution : 16

We have, Bag A : – 5W, 4B & Bag B : – 7W, 6B.

Probability of black ball = P(whit form 1st).P(black from 2nd) + P(black from 1st).P(black from 2nd) = (5/9)(6/14) + (4/9)(7/14)

= (30 + 28)/126 = 58/126 = 29/63.

Q.17. An article manufactured by a company consists of two parts A and B. In the process of manufacture of part A, 9 out of 104 parts may be defective. Similarly, 5 out of 100 are likely to be defective in the manufacture of part B. Calculate the probability that the article manufactured will not be defective.

Solution : 17

P(non-defective) = P(non-defective part A).P(non-defective part B)

= [1 – (9/104)][1 – (5/100)]

= (95/104)(95/100) = 361/416.

Q.18. A firm produces steel pipes in three plants A, B and C, with daily production of 500, 1000 and 2000 units respectively. It is known that fractions of defective output produced by the three plants are respectively 0.005, 0.008 and 0.010. A pipe is selected at random from a day's total production and found to be defective. What is the probability that it came from the first plant?

Solution : 18

We have, P(Defective pipe from A) = 0.005,

P(Defective pipe from B) = 0.008,

P(Defective pipe from C) = 0.010.

P(A) = Probability of selecting pipe from A = 500/3500 = 1/7,

P(B) = Probability of selecting pipe from B = 1000/3500 = 2/7,

P(C) = Probability of selecting pipe from C = 2000/3500 = 4/7.

Let P(D) = Probability of defective pipe.

$P(A \cap C) = P(A) \times P(D) = 1/ 7 \times 0.005 = 1/1400$,

$P(B \cap C) = P(B) \times P(D) = 2/ 7 \times 0.008 = 4/1750$,

$P(C \cap D) = P(C) \times P(D) = 4/ 7 \times 0.010 = 1/175$.

If a pipe is defective then certainly it may be from either $A \cap D$, or $B \cap D$, or $C \cap D$.

$P(\text{Defective pipe from plant A}) = P(A \cap D)/[P(A \cap D) + P(B \cap D) + P(C \cap D)]$

$= (1/1400)/[(1/1400) + (4/1750) + (1/175)]$

$= 5/61$.

Q.19. What is the probability that a leap year has 53 Sundays?

Solution : 19

In a leap year there are 366 days out of which there are 52 full weeks and 2 days. Last two days can be Sunday and Monday, Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday, Thursday and Friday, Friday and Saturday, Saturday and Sunday. Thus we see there are 7 possible outcomes out of which first and last are favourable.

Hence, P(53 Sundays) = 2/7.

Q.20. A problem in Mathematics is given to four students A, B, C and D. Their chances of solving the problem respectively are 1/2 , 1/3 , 1/4 and 1/5. What is the probability that the problem will be solved?

Solution : 20

We have, probability that A can solve the problem = P(A) = 1/2 ,

And in this way P(B) = 1/3, P(C) = 1/4 and P(D) = 1/5.

P(A cannot solve the problem) = 1 – P(A) = 1/2 , 1 – P(B) = 1 – 1/3 = 2/3, 1 – P(C) = 1 – 1/4 = 3/4 and 1 – P(D) = 1 – 1/5 = 4/5.

P(A, B, C and D cannot solve the problem) = 1/2 × 2/3 × 3/4 × 4/5 = 1/5.

Therefore , P(Problem will be solve) = 1 – P(Problem is not solved by any of them)

= 1 – 1/5 = 4/5.

Q.21. The probability that a boy will not pass M.B.A. examination is 2/5 and that a girl will not pass is 4/5. Calculate the probability that at least one of them passes the examination.

Solution : 21

Let p1 = P(Boy will not pass) = 2/5 ; p2 = P(Girl will not pass) = 4/5 ;

Then q1 = 1 – p1 = 3/5.

Therefore, P(At least one of them will pass) = p1p2 + p1q2 + p2q1

= 2/5×1/5 + 2/5×4/5 + 1/5×3/5

= 2/25 + 8/25 + 3/25 = 13/25.