Chapter 2

RELATIONS AND FUNCTIONS

2.1 Overview

This chapter deals with linking pair of elements from two sets and then introduce relations between the two elements in the pair. Practically in every day of our lives, we pair the members of two sets of numbers. For example, each hour of the day is paired with the local temperature reading by T.V. Station's weatherman, a teacher often pairs each set of score with the number of students receiving that score to see more clearly how well the class has understood the lesson. Finally, we shall learn about special relations called functions.

2.1.1 *Cartesian products of sets*

Definition : Given two non-empty sets A and B, the set of all ordered pairs (x, y), where $x \in$ A and $y \in$ B is called Cartesian product of A and B; symbolically, we write

$$A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$$

If $A = \{1, 2, 3\}$ and $B = \{4, 5\}$, then

$$A \times B = \{(1, 4), (2, 4), (3, 4), (1, 5), (2, 5), (3, 5)\}$$

and $B \times A = \{(4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3)\}$

(i) Two ordered pairs are equal, if and only if the corresponding first elements are equal and the second elements are also equal, i.e. $(x, y) = (u, v)$ if and only if $x = u$, $y = v$.

(ii) If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = p \times q$.

(iii) $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$. Here (a, b, c) is called an ordered triplet.

2.1.2 *Relations* A Relation R from a non-empty set A to a non empty set B is a subset of the Cartesian product set A × B. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B.

The set of all first elements in a relation R, is called the domain of the relation R, and the set of all second elements called images, is called the range of R.

For example, the set $R = \{(1, 2), (-2, 3), (\frac{1}{2}, 3)\}$ is a relation; the domain of $R = \{1, -2, \frac{1}{2}\}$ and the range of $R = \{2, 3\}$.

(i) A relation may be represented either by the Roster form or by the set builder form, or by an arrow diagram which is a visual representation of a relation.

(ii) If $n(A) = p$, $n(B) = q$; then the $n(A \times B) = pq$ and the total number of possible relations from the set A to set B = 2^{pq}.

2.1.3 *Functions* A relation f from a set A to a set B is said to be **function** if every element of set A has one and only one image in set B.

In other words, a function f is a relation such that no two pairs in the relation has the same first element.

The notation $f : X \to Y$ means that f is a function from X to Y. X is called the **domain** of f and Y is called the **co-domain** of f. Given an element $x \in X$, there is a unique element y in Y that is related to x. The unique element y to which f relates x is denoted by $f(x)$ and is called f of x, or the **value of** f **at** x, or the *image of x under f*.

The set of all values of $f(x)$ taken together is called the **range of *f*** or image of X under f. Symbolically.

$$\text{range of } f = \{ y \in Y \mid y = f(x), \text{ for some } x \text{ in } X\}$$

Definition : A function which has either **R** or one of its subsets as its range, is called a real valued function. Further, if its domain is also either **R** or a subset of **R**, it is called a real function.

2.1.4 *Some specific types of functions*

(i) **Identity function:**

The function $f : \mathbf{R} \to \mathbf{R}$ defined by $y = f(x) = x$ for each $x \in \mathbf{R}$ is called the **identity function.** Domain of $f = \mathbf{R}$

Range of $f = \mathbf{R}$

(ii) **Constant function:** The function $f : \mathbf{R} \to \mathbf{R}$ defined by $y = f(x) = C$, $x \in \mathbf{R}$, where C is a constant $\in \mathbf{R}$, is a **constant function**.

Domain of $f = \mathbf{R}$

Range of $f = \{C\}$

(iii) **Polynomial function:** A real valued function $f : \mathbf{R} \to \mathbf{R}$ defined by $y = f(x) = a_0 + a_1 x + ... + a_n x^n$, where $n \in \mathbf{N}$, and $a_0, a_1, a_2 ... a_n \in \mathbf{R}$, for each $x \in \mathbf{R}$, is called Polynomial functions.

(iv) **Rational function:** These are the real functions of the type $\dfrac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomial functions of x defined in a domain, where $g(x) \neq 0$. For example $f : \mathbf{R} - \{-2\} \rightarrow \mathbf{R}$ defined by $f(x) = \dfrac{x+1}{x+2}, \ \forall x \in \mathbf{R} - \{-2\}$ is a rational function.

(v) **The Modulus function:** The real function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by $f(x) = |x| =$

$$\begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$\forall x \in \mathbf{R}$ is called the modulus function.

Domain of $f = \mathbf{R}$
Range of $f = \mathbf{R}^{+} \cup \{0\}$

(vi) **Signum function:** The real function
$f : \mathbf{R} \rightarrow \mathbf{R}$ defined by

$$f(x) = \begin{cases} \dfrac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

is called the **signum function**. Domain of $f = \mathbf{R}$, Range of $f = \{1, 0, -1\}$

(vii) **Greatest integer function:** The real function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by $f(x) = [x], x \in \mathbf{R}$ assumes the value of the greatest integer less than or equal to x, is called the **greatest integer function**.

Thus
$f(x) = [x] = -1$ for $-1 \leq x < 0$
$f(x) = [x] = 0$ for $0 \leq x < 1$
$[x] = 1$ for $1 \leq x < 2$
$[x] = 2$ for $2 \leq x < 3$ and so on

2.1.5 *Algebra of real functions*

(i) ***Addition of two real functions***

Let $f : \mathrm{X} \rightarrow \mathbf{R}$ and $g : \mathrm{X} \rightarrow \mathbf{R}$ be any two real functions, where $\mathrm{X} \in \mathbf{R}$. Then we define $(f + g) : \mathrm{X} \rightarrow \mathbf{R}$ by $(f + g)(x) = f(x) + g(x)$, for all $x \in \mathrm{X}$.

(ii) ***Subtraction of a real function from another***

Let $f : \mathrm{X} \rightarrow \mathbf{R}$ and $g : \mathrm{X} \rightarrow \mathbf{R}$ be any two real functions, where $\mathrm{X} \subseteq \mathbf{R}$. Then, we define $(f - g) : \mathrm{X} \rightarrow \mathbf{R}$ by $(f - g)(x) = f(x) - g(x)$, for all $x \in \mathrm{X}$.

(iii) ***Multiplication by a Scalar***

Let $f : \mathrm{X} \rightarrow \mathbf{R}$ be a real function and α be any scalar belonging to $\mathbf{R}$. Then the product αf is function from X to $\mathbf{R}$ defined by $(\alpha f)(x) = \alpha f(x), x \in \mathrm{X}$.

(iv) ***Multiplication of two real functions***

Let $f : X \to \mathbf{R}$ and $g : x \to \mathbf{R}$ be any two real functions, where $X \subseteq \mathbf{R}$. Then product of these two functions i.e. $f\,g : X \to \mathbf{R}$ is defined by $(f\,g)(x) = f(x)\; g(x) \;\forall\; x \in X$.

(v) ***Quotient of two real function***

Let f and g be two real functions defined from $X \to \mathbf{R}$. The quotient of f by g denoted by $\frac{f}{g}$ is a function defined from $X \to \mathbf{R}$ as

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{ provided } g(x) \neq 0,\ x \in X.$$

Note Domain of sum function $f + g$, difference function $f – g$ and product function fg.

$$= \{x : x \in D_f \cap D_g\}$$

where D_f = Domain of function f

D_g = Domain of function g

Domain of quotient function $\frac{f}{g}$

$$= \{x : x \in D_f \cap D_g \text{ and } g(x) \neq 0\}$$

2.2 Solved Examples

Short Answer Type

Example 1 Let A = {1, 2, 3, 4} and B = {5, 7, 9}. Determine

(i) A × B (ii) B × A

(iii) Is A × B = B × A ? (iv) Is n (A × B) = n (B × A) ?

Solution Since A = {1, 2, 3, 4} and B = {5, 7, 9}. Therefore,

(i) A × B = {(1, 5), (1, 7), (1, 9), (2, 5), (2, 7), (2, 9), (3, 5), (3, 7), (3, 9), (4, 5), (4, 7), (4, 9)}

(ii) B × A = {(5, 1), (5, 2), (5, 3), (5, 4), (7, 1), (7, 2), (7, 3), (7, 4), (9, 1), (9, 2), (9, 3), (9, 4)}

(iii) No, A × B ≠ B × A. Since A × B and B × A do not have exactly the same ordered pairs.

(iv) n (A × B) = n (A) × n (B) = 4 × 3 = 12

n (B × A) = n (B) × n (A) = 4 × 3 = 12

Hence n (A × B) = n (B × A)

Example 2 Find x and y if:

(i) $(4x + 3, y) = (3x + 5, -2)$ (ii) $(x - y, x + y) = (6, 10)$

Solution

(i) Since $(4x + 3, y) = (3x + 5, -2)$, so

$4x + 3 = 3x + 5$

or $x = 2$

and $y = -2$

(ii) $x - y = 6$

$x + y = 10$

∴ $2x = 16$

or $x = 8$

$8 - y = 6$

∴ $y = 2$

Example 3 If A = {2, 4, 6, 9} and B = {4, 6, 18, 27, 54}, $a \in$ A, $b \in$ B, find the set of ordered pairs such that 'a' is factor of 'b' and $a < b$.

Solution Since A = {2, 4, 6, 9}

B = {4, 6, 18, 27, 54},

we have to find a set of ordered pairs (a, b) such that a is factor of b and $a < b$.

Since 2 is a factor of 4 and 2 < 4.

So (2, 4) is one such ordered pair.

Similarly, (2, 6), (2, 18), (2, 54) are other such ordered pairs. Thus the required set of ordered pairs is

{(2, 4), (2, 6), (2, 18), (2, 54), (6, 18), (6, 54,), (9, 18), (9, 27), (9, 54)}.

Example 4 Find the domain and range of the relation R given by

$$R = \{(x, y) : y = x + \frac{6}{x};\ \text{where } x, y \in \mathbf{N} \text{ and } x < 6\}.$$

Solution When $x = 1$, $y = 7 \in$ **N**, so $(1, 7) \in$ R. Again for,

$$x = 2.\ y = 2 + \frac{6}{2} = 2 + 3 = 5 \in \mathbf{N},\ \text{so } (2, 5) \in \text{R. Again for}$$

$x = 3,\ y = 3 + \frac{6}{3} = 3 + 2 = 5 \in \mathbf{N}$, $(3, 5) \in$ R. Similarly for $x = 4$

$y = 4 + \frac{6}{4} \notin \mathbf{N}$ and for $x = 5$, $y = 5 + \frac{6}{5} \notin \mathbf{N}$

Thus R = {(1, 7), (2, 5), (3, 5)}, where Domain of R = {1, 2, 3}

Range of R = {7, 5}

Example 5 Is the following relation a function? Justify your answer

(i) $R_1 = \{(2, 3), (\frac{1}{2}, 0), (2, 7), (-4, 6)\}$

(ii) $R_2 = \{(x, |x|) \mid x \text{ is a real number}\}$

Solution

Since $(2, 3)$ and $(2, 7) \in R_1$

$\Rightarrow$ $R_1(2) = 3$ and $R_1(2) = 7$

So $R_1(2)$ does not have a unique image. Thus R_1 is not a function.

(iii) $R_2 = \{(x, |x|) / x \in \mathbf{R}\}$

For every $x \in \mathbf{R}$ there will be unique image as $|x| \in \mathbf{R}$.

Therefore R_2 is a function.

Example 6 Find the domain for which the functions

$f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal.

Solution

For $f(x) = g(x)$

$\Rightarrow$ $2x^2 - 1 = 1 - 3x$

$\Rightarrow$ $2x^2 + 3x - 2 = 0$

$\Rightarrow$ $2x^2 + 4x - x - 2 = 0$

$\Rightarrow$ $2x(x + 2) - 1(x + 2) = 0$

$\Rightarrow$ $(2x - 1)(x + 2) = 0$

Thus domain for which the function $f(x) = g(x)$ is $\left\{\frac{1}{2}, -2\right\}$.

Example 7 Find the domain of each of the following functions.

(i) $f(x) = \dfrac{x}{x^2 + 3x + 2}$ (ii) $f(x) = [x] + x$

Solution

(i) f is a rational function of the form $\frac{g(x)}{h(x)}$, where $g(x) = x$ and $h(x) = x^2 + 3x + 2$.

Now $h(x) \neq 0 \Rightarrow x^2 + 3x + 2 \neq 0 \Rightarrow (x + 1)(x + 2) \neq 0$ and hence domain of the given function is R – {– 1, – 2}.

(ii) $f(x) = [x] + x$, i.e., $f(x) = h(x) + g(x)$

where $h(x) = [x]$ and $g(x) = x$

The domain of $h = \mathbf{R}$

and the domain of $g = \mathbf{R}$. Therefore

Domain of $f = \mathbf{R}$

Example 8 Find the range of the following functions given by

(i) $\frac{|x-4|}{x-4}$ (ii) $\sqrt{16-x^2}$

Solution

(i) $$f(x) = \frac{|x-4|}{x-4} = \begin{cases} \frac{x-4}{x-4} = 1, & x > 4 \\ \frac{-(x-4)}{x-4} = -1, & x < 4 \end{cases}$$

Thus the range of $\frac{|x-4|}{x-4} = \{1, -1\}$.

(ii) The domain of f, where $f(x) = \sqrt{16-x^2}$ is given by [– 4, 4].

For the range, let $y = \sqrt{16-x^2}$

then $y^2 = 16 - x^2$

or $x^2 = 16 - y^2$

Since $x \in [-4, 4]$

Thus range of $f = [0, 4]$

Example 9 Redefine the function which is given by

$$f(x) = |x-1| + |1+x|, \quad -2 \leq x \leq 2$$

Solution $f(x) = |x-1| + |1+x|$, $-2 \le x \le 2$

$$= \begin{cases} -x+1 \ -1-x, & -2 \le x < -1 \\ -x+1 \ +x+1, & -1 \le x < 1 \\ x-1 \ +1+x, & 1 \le x \le 2 \end{cases}$$

$$= \begin{cases} -2x, & -2 \le x < -1 \\ 2, & -1 \le x < 1 \\ 2x, & 1 \le x \le 2 \end{cases}$$

Example 10 Find the domain of the function f given by $f(x) = \dfrac{1}{\sqrt{[x]^2 - [x] - 6}}$

Solution Given that $f(x) = \dfrac{1}{\sqrt{[x]^2 - [x] - 6}}$, f is defined if $[x]^2 - [x] - 6 > 0$.

or $([x]-3)([x]+2) > 0$,

$\Rightarrow$ $[x] < -2$ or $[x] > 3$

$\Rightarrow$ $x < -2$ or $x \ge 4$

Hence Domain = $(-\infty, -2) \cup [4, \infty)$.

Objective Type Questions

Choose the correct answer out of the four given possible answers (M.C.Q.)

Example 11 The domain of the function f defined by $f(x) = \dfrac{1}{\sqrt{x - |x|}}$ is

(A) $\mathbf{R}$ (B) $\mathbf{R}^+$

(C) $\mathbf{R}^-$ (D) None of these

Solution The correct answer is (D). Given that $f(x) = \dfrac{1}{\sqrt{x - |x|}}$

where $x - |x| = \begin{cases} x - x = 0 & \text{if } x \ge 0 \\ 2x & \text{if } x < 0 \end{cases}$

Thus $\frac{1}{\sqrt{x-|x|}}$ is not defined for any $x \in \mathbf{R}$.

Hence f is not defined for any $x \in \mathbf{R}$, i.e. Domain of f is none of the given options.

Example 12 If $f(x) = x^3 - \frac{1}{x^3}$, then $f(x) + f(\frac{1}{x})$ is equal to

(A) $2x^3$ (B) $2\frac{1}{x^3}$ (C) 0 (D) 1

Solution The correct choice is C.

Since
$$f(x) = x^3 - \frac{1}{x^3}$$

$$f\left(\frac{1}{x}\right) = \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}} = \frac{1}{x^3} - x^3$$

Hence,
$$f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3 = 0$$

Example 13 Let A and B be any two sets such that $n(B) = p$, $n(A) = q$ then the total number of functions $f : A \to B$ is equal to __________.

Solution Any element of set A, say x_i can be connected with the element of set B in p ways. Hence, there are exactly p^q functions.

Example 14 Let f and g be two functions given by

$f = \{(2, 4), (5, 6), (8, -1), (10, -3)\}$

$g = \{(2, 5), (7, 1), (8, 4), (10, 13), (11, -5)\}$ then. Domain of $f + g$ is __________

Solution Since Domain of $f = D_f = \{2, 5, 8, 10\}$ and Domain of $g = D_g = \{2, 7, 8, 10, 11\}$, therefore the domain of $f + g = \{x \mid x \in D_f \cap D_g\} = \{2, 8, 10\}$

2.3 EXERCISE

Short Answer Type

1. Let $A = \{-1, 2, 3\}$ and $B = \{1, 3\}$. Determine

(i) $A \times B$ (ii) $B \times A$

(iii) $B \times B$ (iv) $A \times A$

2. If $P = \{x : x < 3, x \in \mathbf{N}\}$, $Q = \{x : x \le 2, x \in \mathbf{W}\}$. Find $(P \cup Q) \times (P \cap Q)$, where **W** is the set of whole numbers.

3. If $A = \{x : x \in \mathbf{W}, x < 2\}$ $B = \{x : x \in \mathbf{N}, 1 < x < 5\}$ $C = \{3, 5\}$ find

(i) $A \times (B \cap C)$ (ii) $A \times (B \cup C)$

4. In each of the following cases, find a and b.

(i) $(2a + b, a - b) = (8, 3)$ (ii) $\left(\frac{a}{4}, a-2b\right) = (0, 6 + b)$

5. Given $A = \{1, 2, 3, 4, 5\}$, $S = \{(x, y) : x \in A, y \in A\}$. Find the ordered pairs which satisfy the conditions given below:

(i) $x + y = 5$ (ii) $x + y < 5$ (iii) $x + y > 8$

6. Given $R = \{(x, y) : x, y \in \mathbf{W}, x^2 + y^2 = 25\}$. Find the domain and Range of R.

7. If $R_1 = \{(x, y) \mid y = 2x + 7$, where $x \in \mathbf{R}$ and $-5 \le x \le 5\}$ is a relation. Then find the domain and Range of R_1.

8. If $R_2 = \{(x, y) \mid x$ and y are integers and $x^2 + y^2 = 64\}$ is a relation. Then find R_2.

9. If $R_3 = \{(x, |x|) \mid x$ is a real number$\}$ is a relation. Then find domain and range of R_3.

10. Is the given relation a function? Give reasons for your answer.

(i) $h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$

(ii) $f = \{(x, x) \mid x$ is a real number$\}$

(iii) $g = \left\{\left(n, \frac{1}{n}\right) \mid n \text{ is a positive integer}\right\}$

(iv) $s = \{(n, n^2) \mid n$ is a positive integer$\}$

(v) $t = \{(x, 3) \mid x$ is a real number.

11. If f and g are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$, find each of the following

(a) $f(3) + g(-5)$ (b) $f\left(\frac{1}{2}\right) \times g(14)$

(c) $f(-2) + g(-1)$ (d) $f(t) - f(-2)$

(e) $\frac{f(t) - f(5)}{t - 5}$, if $t \ne 5$

12. Let f and g be real functions defined by $f(x) = 2x + 1$ and $g(x) = 4x - 7$.

(a) For what real numbers x, $f(x) = g(x)$?

(b) For what real numbers x, $f(x) < g(x)$?

13. If f and g are two real valued functions defined as $f(x) = 2x + 1$, $g(x) = x^2 + 1$, then find.

(i) $f + g$ (ii) $f - g$ (iii) fg (iv) $\frac{f}{g}$

14. Express the following functions as set of ordered pairs and determine their range.
$f : X \to \mathbf{R}$, $f(x) = x^3 + 1$, where $X = \{-1, 0, 3, 9, 7\}$

15. Find the values of x for which the functions
$f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal

Long Answer Type

16. Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? Justify. If this is described by the relation, $g(x) = \alpha x + \beta$, then what values should be assigned to α and β?

17. Find the domain of each of the following functions given by

(i) $f(x) = \frac{1}{\sqrt{1-\cos x}}$ (ii) $f(x) = \frac{1}{\sqrt{x+|x|}}$

(iii) $f(x) = x\,|x|$ (iv) $f(x) = \frac{x^3 - x + 3}{x^2 - 1}$

(v) $f(x) = \frac{3x}{2x-8}$

18. Find the range of the following functions given by

(i) $f(x) = \frac{3}{2 - x^2}$ (ii) $f(x) = 1 - |x-2|$

(iii) $f(x) = |x-3|$ (iv) $f(x) = 1 + 3\cos 2x$

(**Hint :** $-1 \le \cos 2x \le 1 \Rightarrow -3 \le 3\cos 2x \le 3 \Rightarrow -2 \le 1 + 3\cos 2x \le 4$)

19. Redefine the function $f(x) = |x-2| + |2+x|$, $-3 \le x \le 3$

20. If $f(x) = \dfrac{x-1}{x+1}$, then show that

(i) $f\left(\dfrac{1}{x}\right) = -f(x)$ (ii) $f\left(-\dfrac{1}{x}\right) = \dfrac{-1}{f(x)}$

21. Let $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined in the domain $R^+ \cup \{0\}$. Find

(i) $(f + g)(x)$ (ii) $(f - g)(x)$

(iii) $(fg)(x)$ (iv) $\left(\dfrac{f}{g}\right)(x)$

22. Find the domain and Range of the function $f(x) = \dfrac{1}{\sqrt{x-5}}$.

23. If $f(x) = y = \dfrac{ax-b}{cx-a}$, then prove that $f(y) = x$.

Objective Type Questions

Choose the correct answers in Exercises from 24 to 35 (M.C.Q.)

24. Let $n(A) = m$, and $n(B) = n$. Then the total number of non-empty relations that can be defined from A to B is

(A) m^n (B) $n^m - 1$

(C) $mn - 1$ (D) $2^{mn} - 1$

25. If $[x]^2 - 5[x] + 6 = 0$, where [.] denote the greatest integer function, then

(A) $x \in [3, 4]$ (B) $x \in (2, 3]$

(C) $x \in [2, 3]$ (D) $x \in [2, 4)$

26. Range of $f(x) = \dfrac{1}{1-2\cos x}$ is

(A) $\left[\dfrac{1}{3}, 1\right]$ (B) $\left[-1, \dfrac{1}{3}\right]$

(C) $(-\infty, -1] \cup \left[\dfrac{1}{3}, \infty\right)$ (D) $\left[-\dfrac{1}{3}, 1\right]$

27. Let $f(x) = \sqrt{1+x^2}$, then

(A) $f(xy) = f(x) \cdot f(y)$
(B) $f(xy) \geq f(x) \cdot f(y)$
(C) $f(xy) \leq f(x) \cdot f(y)$
(D) None of these

[**Hint :** find $f(xy) = \sqrt{1+x^2y^2}$, $f(x) \cdot f(y) = \sqrt{1+x^2y^2+x^2+y^2}$]

28. Domain of $\sqrt{a^2-x^2}$ $(a > 0)$ is

(A) $(-a, a)$
(B) $[-a, a]$
(C) $[0, a]$
(D) $(-a, 0]$

29. If $f(x) = ax + b$, where a and b are integers, $f(-1) = -5$ and $f(3) = 3$, then a and b are equal to

(A) $a = -3, b = -1$
(B) $a = 2, b = -3$
(C) $a = 0, b = 2$
(D) $a = 2, b = 3$

30. The domain of the function f defined by $f(x) = \sqrt{4-x} + \dfrac{1}{\sqrt{x^2-1}}$ is equal to

(A) $(-\infty, -1) \cup (1, 4]$
(B) $(-\infty, -1] \cup (1, 4]$
(C) $(-\infty, -1) \cup [1, 4]$
(D) $(-\infty, -1) \cup [1, 4)$

31. The domain and range of the real function f defined by $f(x) = \dfrac{4-x}{x-4}$ is given by

(A) Domain = **R**, Range = $\{-1, 1\}$
(B) Domain = **R** – $\{1\}$, Range = **R**
(C) Domain = **R** – $\{4\}$, Range = $\{-1\}$
(D) Domain = **R** – $\{-4\}$, Range = $\{-1, 1\}$

32. The domain and range of real function f defined by $f(x) = \sqrt{x-1}$ is given by

(A) Domain = $(1, \infty)$, Range = $(0, \infty)$
(B) Domain = $[1, \infty)$, Range = $(0, \infty)$
(C) Domain = $[1, \infty)$, Range = $[0, \infty)$
(D) Domain = $[1, \infty)$, Range = $[0, \infty)$

33. The domain of the function f given by $f(x) = \dfrac{x^2+2x+1}{x^2-x-6}$

(A) $\mathbf{R} - \{3, -2\}$ (B) $\mathbf{R} - \{-3, 2\}$

(C) $\mathbf{R} - [3, -2]$ (D) $\mathbf{R} - (3, -2)$

34. The domain and range of the function f given by $f(x) = 2 - |x-5|$ is

(A) Domain = $\mathbf{R}^+$, Range = $(-\infty, 1]$

(B) Domain = $\mathbf{R}$, Range = $(-\infty, 2]$

(C) Domain = $\mathbf{R}$, Range = $(-\infty, 2)$

(D) Domain = $\mathbf{R}^+$, Range = $(-\infty, 2]$

35. The domain for which the functions defined by $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal is

(A) $\left\{-1, \dfrac{4}{3}\right\}$ (B) $\left[-1, \dfrac{4}{3}\right]$

(C) $\left(-1, \dfrac{4}{3}\right)$ (D) $\left[-1, \dfrac{4}{3}\right)$

Fill in the blanks :

36. Let f and g be two real functions given by

$f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 1)\}$

$g = \{(1, 0), (2, 2), (3, -1), (4, 4), (5, 3)\}$

then the domain of $f \cdot g$ is given by __________.

37. Let $f = \{(2, 4), (5, 6), (8, -1), (10, -3)\}$

$g = \{(2, 5), (7, 1), (8, 4), (10, 13), (11, 5)\}$

be two real functions. Then Match the following :

(a) $f - g$ (i) $\left\{\left(2, \dfrac{4}{5}\right), \left(8, \dfrac{-1}{4}\right), \left(10, \dfrac{-3}{13}\right)\right\}$

(b) $f + g$ (ii) $\{(2, 20), (8, -4), (10, -39)\}$

(c) $f \cdot g$ (iii) $\{(2, -1), (8, -5), (10, -16)\}$

(d) $\frac{f}{g}$ (iv) {(2, 9), (8, 3), (10, 10)}

State True or False for the following statements given in Exercises 38 to 42 :

38. The ordered pair (5, 2) belongs to the relation R = $\{(x, y) : y = x - 5, x, y \in \mathbf{Z}\}$

39. If P = {1, 2}, then P × P × P = {(1, 1, 1), (2, 2, 2), (1, 2, 2), (2, 1, 1)}

40. If A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}, then (A × B) ∪ (A × C)
= {(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)}.

41. If $(x - 2, y + 5) = \left(-2, \frac{1}{3}\right)$ are two equal ordered pairs, then $x = 4,\ y = \frac{-14}{3}$

42. If A × B = $\{(a, x), (a, y), (b, x), (b, y)\}$, then A = $\{a, b\}$, B = $\{x, y\}$