

Boolean Algebra

Q.1. Show that in a Boolean algebra, the zero element 0 and the unit element 1 are unique.

Solution : 1

Let two zero elements in a Boolean algebra, B, be 0 and 0'.

As, 0 is an additive identity, then by definition,

$$a + 0 = a \text{ for all } a \in B.$$

Therefore, $0' + 0 = 0'$ ----- (i) [taking $a = 0'$]

Also as 0' is an additive identity, then by definition,

$$a + 0' = a \text{ for all } a \in B,$$

Therefore, $0 + 0' = 0$ ----- (ii) [taking $a = 0$]

However, by commutativity, we have $0' + 0 = 0 + 0' \Rightarrow 0' = 0$ [using (i) and (ii)]

Hence, we see zero element in a Boolean algebra is unique.

Let 1 and 1' be two unit elements in B. Then by definition

$$a.1 = a \text{ and } a.1' = a \text{ for all } a \in B.$$

Therefore, $1'.1 = 1'$ and $1.1' = 1$

However by commutativity, we have $1'.1 = 1.1'$.

Hence, $1' = 1$.

Hence, unit element in B is unique.

Q.2. Prove the following :

- i. If $x + y = 0$, then $x = 0 = y$,
- ii. $x.y' = 0$ if and only if $x.y = x$,
- iii. $x = 0$ if and only if $y = x.y' + x'.y$ for all y .

Solution : 2

- i. We have, $x + y = 0$
Now $x = x.(x + y)$ [By absorption law]
 $= x.0$ [given, $x + y = 0$]
 $= 0$.
Similarly, $y = y.(y + x) = y.(x + y) = y.0 = 0$.
- ii. Let us suppose, $x.y' = 0 \Rightarrow (x.y')' = 0' \Rightarrow x' + y = 1$ [using $x' + y = 1$]
Now $x = x.1 = x.(x' + y)$
 $= x.x' + x.y$
 $= 0 + x.y$
 $= x.y$.
Then we have, $x.y' = (x.y).y' = x.(y.y') = x.0 = 0$
- iii. Let us suppose that $x = 0$
Then $x.y' + x'.y = 0.y' + 0'.y$
 $= 0 + 1.y$
 $= 0 + y = y$ for all y .
Now suppose, $y = x.y' + x'.y$ for all y .
It means it is true for $y = 0$.
Therefore, $0 = x.0' + x'.0 = x.1 + x'.0 = x + 0 = x$.
Hence, $x = 0$.

Q.3. If $x + y = x + z$ and $x' + y = x' + z$, then prove that $y = z$.

Solution : 3

We have, $x + y = x + z$ ----- (i)

and $x' + y = x' + z$ ----- (ii)

Now, $y = y + 0 = y + x.x'$ [As, $x.x' = 0$]

$= (y + x).(y + x')$ [by distributive law]

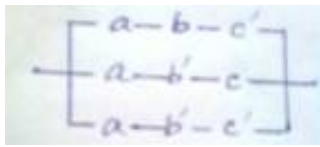
$= (x + y).(x' + y)$ [by commutative law]

$= (x + z).(x' + z)$ [using (i) and (ii)]

$= x.x' + z$ [by distributive law]

$= 0 + z = z$.

Q.4. Write the Boolean expression for the following switching circuit :



Fig

Simplify the expression. Construct the switching circuit for the simplified expression.

Solution : 4

The Boolean expression for the given switching circuit is :

$$abc' + ab'c + ab'c' = abc' + ab'(c + c')$$

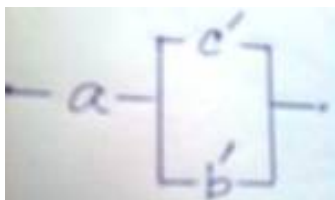
$$= abc' + ab' [As, c + c' = 1]$$

$$= a(bc' + b')$$

$$= a(b + b')(c' + b')$$

$$= a(c' + b').$$

The simplified switching circuit is :



Fig

Q.5. If a, b are elements of Boolean algebra, prove that : $(a + b)' = a' b'$.

Solution : 5

$$(a + b)' \cdot a' b' = a a' b' + b a' b'$$

$$= 0 \cdot b' + a' \cdot 0 [As, aa' = 0; bb' = 0]$$

$$= 0 + 0 = 0 [identity for +]$$

$$\text{and } (a + b)' + a' b' = (a + b + a').(a + b + b')$$

$$= (1 + b).(a + 1) \text{ [As, } a + a' = 1, b + b' = 1]$$

$$= 1.1 = 1 \text{ [identity for .]}$$

Therefore, $(a + b)$ and $a' b'$ are complement of each other.

Hence, $(a + b)' = a' b'$.

Q.6. P, Q and R represent switches in 'on' position and P', Q' and R' represent switches in 'off' positions. Construct a switching circuit representing the polynomial

$$PR + Q(Q' + R)(P + QR).$$

Use Boolean algebra to prove that the above circuit can be simplified to an expression in which, when P and R are 'on' or Q and R are 'on', the light is on. Construct an equivalent circuit.

Solution : 6

Fig. [sol 3(b)/page 359, 10 yrs.]

$$PR + Q(Q' + R)(P + QR) = PR + (Q.Q' + QR)(P + QR)$$

$$= PR + (0 + QR)(P + QR) \text{ [As, } QQ' = 0]$$

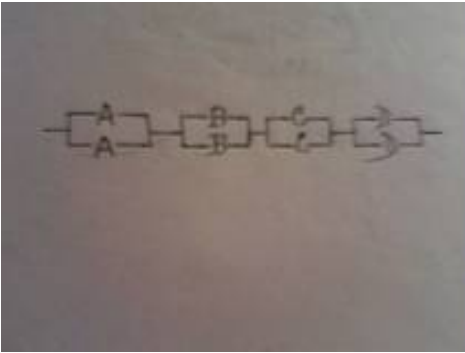
$$= PR + PQR + QR \text{ [As, } QQRR = 0]$$

$$= PR + (P + 1)QR$$

$$= PR + QR \text{ [As, } P + 1 = 1]$$

$$= (P + Q)R$$

Q.7. (i) Write down the Boolean Expression corresponding to the switching circuit given below :



Fig

(ii) Simplify the expression and construct the switching circuit for the simplified expression.

Solution : 7

i. $(A + A)(B + D)(C + A)(C + D)$

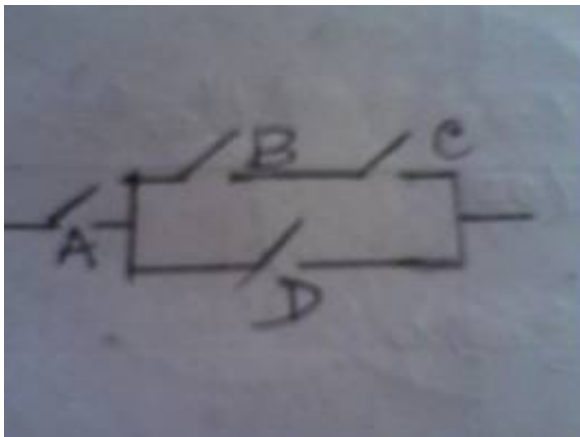
$$\Rightarrow A(C + A)(B + D)(C + D)$$

$$\Rightarrow (AC + A)(BC + D)$$

$$\Rightarrow A(C + 1)(BC + D) \text{ [Distributive Law]}$$

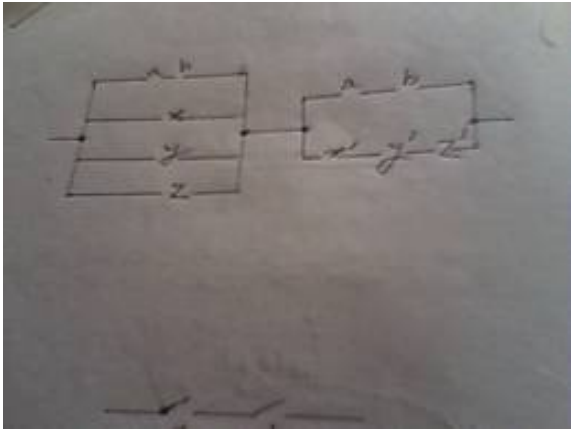
$$\Rightarrow A.(BC + D) [C + 1 = 1.]$$

ii.



Fig

Q.8. Write the Boolean expression for the following given circuit :



Fig

Using the laws of Boolean Algebra , simplify the circuit and construct an equivalent switching circuit.

Solution : 8

Given : $(a b + x + y + z).(a b + x' y' z')$

$$\Rightarrow a b . a b + a b . x' y' z' + x . a b + x . x' y' z' + y . a b + y . x' y' z' + z . a b + z . x' y' z'$$

$$\Rightarrow a b + a b . x' y' z' + x . a b + y . a b$$

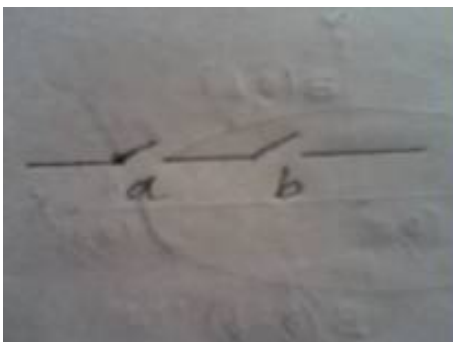
$$+ z . a b \text{ [As } a . a' = 0]$$

$$\Rightarrow a b (1 + x' y' z') + x . a b + y . a b + z . a b [1 + a = 1]$$

$$\Rightarrow a b . 1 + x . a b + y . a b + z . a b$$

$$\Rightarrow a b . (1 + x + y + z)$$

$$\Rightarrow a b . 1 = a b .$$



Fig

Q.9. If a, b, c are elements of Boolean Algebra, prove that :

$$a b + c(a' + b') = a b + c.$$

Solution : 9

$$a b + c (a' + b')$$

$$\Rightarrow (a b + c).(a b + a' + b') \text{ [As, } a + b c = (a + b).(a + c)\text{]}$$

$$\Rightarrow (a b + c).[(a + a').(b + a') + b'] \text{ [As, } a + a' = 1\text{]}$$

$$\Rightarrow (a b + c).[1.(b + a') + b']$$

$$\Rightarrow (a b + c).[b + b' + a]$$

$$\Rightarrow (a b + c).[1 + a']$$

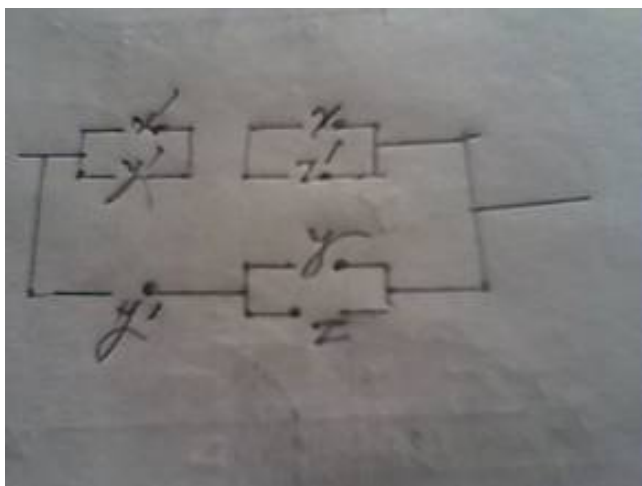
$$\Rightarrow (a b + c).1 = a b + c.$$

Q.10. If x, y and z represent three switches in an on position and x', y' and z' represent the three switches in an off position. Construct a switching circuit representing the polynomial

$$(x' + y')(x + z') + y'(y + z)$$

Using the law of Boolean Algebra , show that the above polynomial is equivalent to $x' z' + y'$ and construct an equivalent switching circuit.

Solution : 10



Fig

$$\text{Given : } (x' + y')(x + z') + y'(y + z)$$

$$\Rightarrow x'x + x'z' + y'x + y'z' + y'y + y'z$$

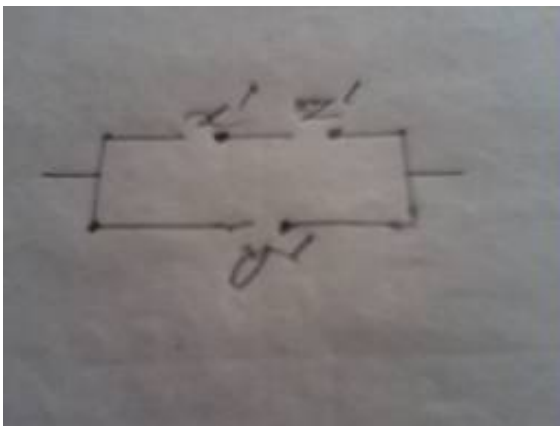
$$\Rightarrow x'z' + y'x + y'z' + y'z \text{ [As, } x'x = 0, y'y = 0]$$

$$\Rightarrow x'z' + y'x + y'(z' + z)$$

$$\Rightarrow x'z' + y'x + y'.1 \text{ [As, } z' + z = 1]$$

$$\Rightarrow x'z' + y'(x + 1)$$

$$\Rightarrow x'z' + y'. \text{ [As, } x + 1 = 1]$$



Fig

Q.11. Prove that every element in a Boolean Algebra has a unique inverse.

Solution : 11

Let a' and a'' be two inverses of a .

$$\text{Then } a' = a'.1$$

$$= a'.(a + a'') \text{ [1 = } a + a'']$$

$$= a'.a + a'.a''$$

$$= 0 + a'.a'' \text{ [a'.a = 0]}$$

$$= a.a'' + a'.a'' \text{ [a.a'' = 0]}$$

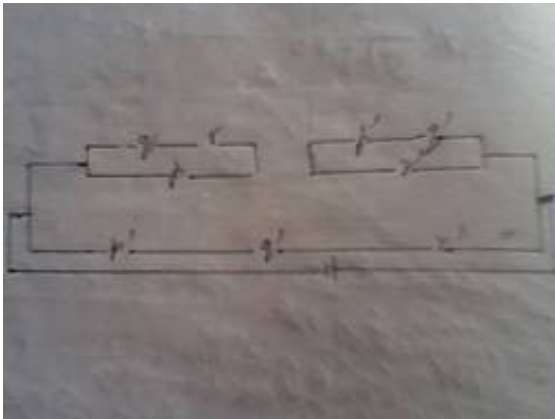
$$= (a + a').a''$$

$$= 1.a'' \text{ [a + a' = 1]}$$

$$= a''.$$

Thus we see that a' and a'' are the same. Hence there is an unique inverse.

Q.12. Write the Boolean function corresponding to the following switching circuit network:



Fig

Use the laws of Boolean Algebra to simplify the circuit. Construct the network for the simplified circuit.

Solution : 12

$$\text{Given : } (q r + p).(p' q' + r') + p' q' r'$$

$$\Rightarrow q r. p' q' + q r r' + p p' q' + p r' + p' q' r'$$

$$\Rightarrow (q q')(r p') + q(r r') + (p p') q' + p r' + p' q' r'$$

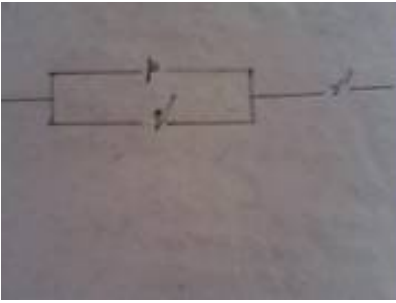
$$\Rightarrow p r' + p; q' r' \text{ [As, } q q' = r r' = p p' = 0]$$

$$\Rightarrow (p + p' q').r'$$

$$\Rightarrow (p + p')(p + q')r'$$

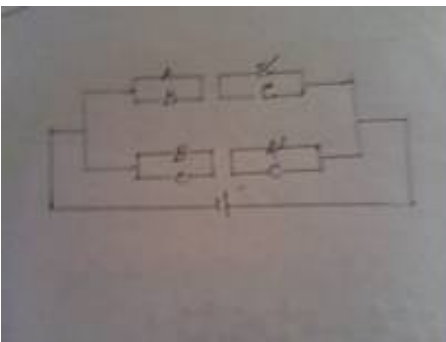
$$\Rightarrow 1.(p + q')r' \text{ [p + p' = 1]}$$

$$\Rightarrow (p + q')r'$$



Fig

Q.13. Write the Boolean function corresponding to the following switching circuit network:



Fig

Use the laws of Boolean Algebra to simplify the circuit. Construct the network for the simplified circuit.

Solution : 13

$$\text{Given : } (A + B)(B' + C) + (B + C)(A' + C)$$

$$= AB' + AC + BB' + BC + BA' + BC + CA' + CC$$

$$= AB' + AC + BC + BA' + CA' + C [BB' = 0, BC + BC = BC, CC = C]$$

$$= AB' + AC + BC + CA' + C + BA'$$

$$= AB' + (A + B + A' + 1)C + BA'$$

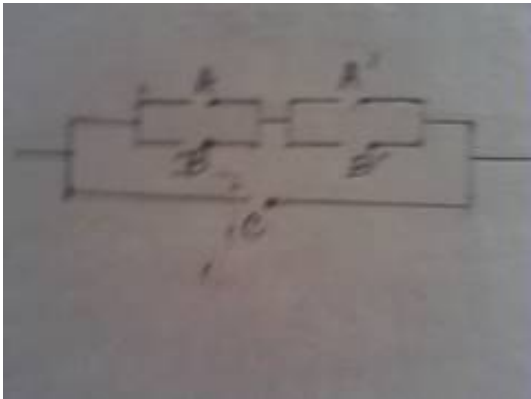
$$= AB' + C + BA' [B + B' = 1, B + 1 = 1]$$

$$= AB' + BA' + C$$

$$= (AB' + B)(AB' + A') + C$$

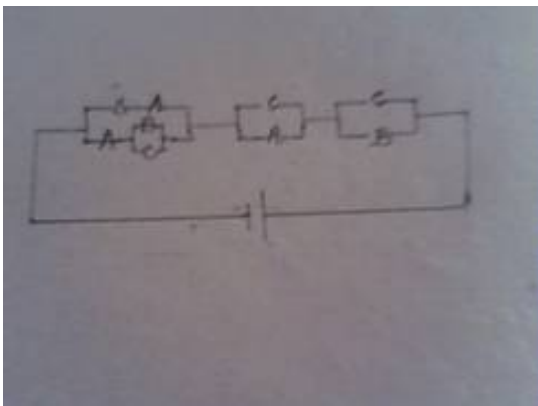
$$= (A + B)(B' + B).(A + A') (B' + A') + C$$

$$= (A + B)(B' + A') + C.$$



Fig

Q.14. Write the Boolean function corresponding to the following switching circuit network:



Fig

Use the Laws of Boolean Algebra to simplify the circuit. Construct the network for the simplified circuit.

Solution : 14

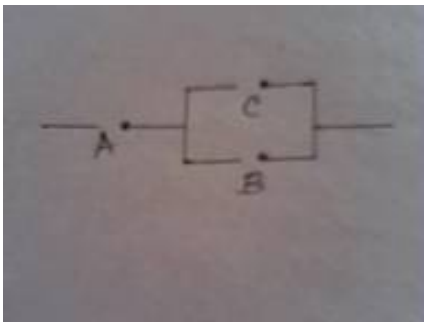
$$\text{Given : } [CA + A.(B + C)].(C + A)(C + B)$$

$$= (CA + AB + AC).(C + CB + AC + AB) [C.C = C]$$

$$= (AC + AB).[C(1 + B) + AC + AB] [1 + B = 1]$$

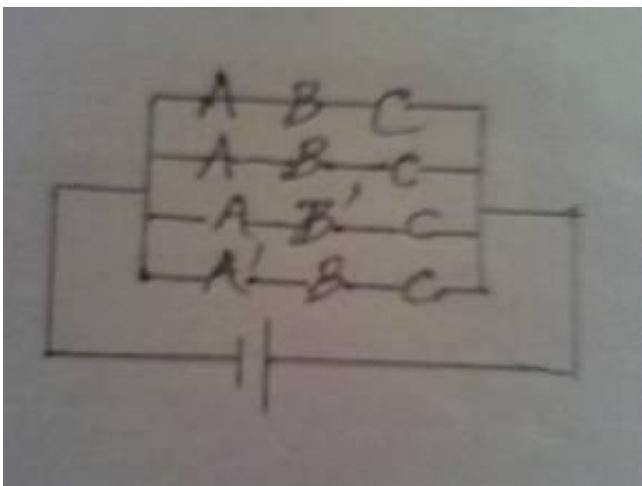
$$= (AC + AB).(C + AC + AB)$$

$$\begin{aligned}
 &= (AC + AB) \cdot [(A + 1)C + AB] \\
 &= (AC + AB) \cdot (C + AB) \\
 &= AC + ACB + ABC + AB \quad [ABC + ABC = ABC] \\
 &= AC(1 + B) + AB(C + 1) \\
 &= AC + AB \quad [1 + B = 1 + C = 1] \\
 &= A \cdot (C + B)
 \end{aligned}$$



Fig

Q.15. Write the statement for the following switching circuit :



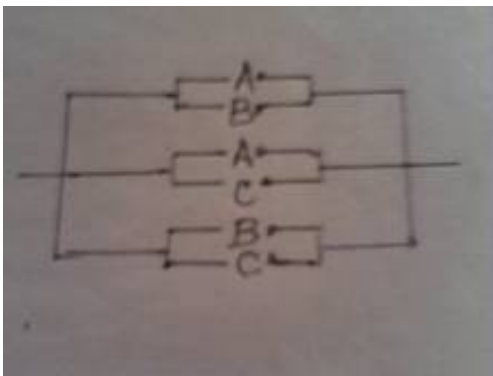
Fig

Simplify the statement. Construct the switching circuit for the simplified form.

Solution : 15

The Boolean function is :

$$\begin{aligned}
& ABC + ABC' + AB'C + A'BC \\
&= AB(C + C') + AB'C + A'BC \quad [C + C' = 1] \\
&= AB.1 + AB'C + A'BC \\
&= (AB + AB')(AB + C) + A'BC \\
&= A(B + B')(AB + C) + A'BC \\
&= A(AB + C) + A'BC \\
&= AB + AC + A'BC \\
&= AB + (A + A'B)C \\
&= AB + (A + A')(A + B)C \quad [A + A' = 1] \\
&= AC + AC + BC .
\end{aligned}$$



Fig

Q.16. Prove that the current will flow through the network represented by the function :

$$[AB(A'B + AB')]'$$

Irrespective of whether x and y are closed or open.

Solution : 16

$$\text{Given : } [AB(A'B + AB')]' = [AB.A'B + AB.AB']'$$

$$= [B(AA')B + A(BB')A]'$$

$$= [0 + 0]'$$

$$= [0]' = 1.$$

Hence, the current will flow.