

Rank Correlation

Q.1. Calculate Spearman's rank correlation coefficient between the advertisement cost and sales from the following data :

Advertisement cost Rs. (in thousand)	39	65	62	90	82	75	25	98	36	78
Sales Rs. (in lakhs)	47	53	58	86	62	68	60	91	51	84

Solution : 1

Advertisement cost Rs. (in thousand) X	Sales Rs. (in lakhs) Y	R_x	R_y	$D = R_x - R_y$	D^2
39	47	8	10	- 2	4
65	53	6	8	- 2	4
62	58	7	7	0	0
90	86	2	2	0	0
82	62	3	5	- 2	4
75	68	5	4	1	1
25	60	10	6	4	16
98	91	1	1	0	0
36	51	9	9	0	0
78	84	4	3	1	1
Total →					30

Spearman's Rank Correlation Coefficient

$$= r = 1 - [6 \sum D^2] / [N(N^2 - 1)]$$

$$= 1 - [6 \times 30] / [10(10^2 - 1)]$$

$$= 1 - [180 / (10 \times 99)]$$

$$= 1 - (2/11) = 9/11$$

$$= 0.818 = 0.82.$$

Q.2. Internal and External assessment were conducted on a group of 10 students who were studying in a postgraduate class in a college. The following marks were obtained in the assessment :

Roll no. of	1	2	3	4	5	6	7	8	9	10
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students:										
Internal Assessment:	45	62	67	32	12	38	47	68	42	85
External Assessment:	39	48	65	32	20	35	45	77	30	62

Find the Spearman's Rank Correlation Coefficient and comment on the result.

Solution : 2

Roll No.	Internal x	External y	Rank of x	Rank of y	d = x - y	d ²
1	45	39	6	6	0	0
2	62	48	4	4	0	0
3	67	65	3	2	1	1
4	32	32	9	8	1	1
5	12	20	10	10	0	0
6	38	35	8	7	1	1
7	47	45	5	5	0	0
8	68	77	2	1	1	1
9	42	30	7	9	-2	4
10	85	62	1	3	-2	4
Total →						12

Here, $n = 10$, $r = 1 - [6\sum d^2]/[n(n^2 - 1)]$

$= 1 - (6 \times 12)/[10(99)]$

$= 1 - 0.07 = 0.93$.

Comment : There is a high positive correlation between the internal and external assessment.

Q.3. The mathematical aptitude score (MAS) of ten computer programmers with job performance rating (JPR) is given below. Calculate Spearman's rating of rank correlation and state whether those who have aptitude for maths are likely to be better programmers.

Person	A	B	C	D	E	F	G	H	I	J
MAS	2	5	0	4	3	1	6	8	7	9
JPR	8	16	8	9	5	4	3	17	8	12

Solution : 3

Person	MAS	JPR	R_1	R_2	$d = R_1 - R_2$	d^2
A	2	8	8	6	2	4
B	5	16	5	2	3	9
C	0	8	10	6	4	16
D	4	9	6	4	2	4
E	3	5	7	8	- 1	1
F	1	4	9	9	0	0
G	6	3	4	10	- 6	36
H	8	17	2	1	1	1
I	7	8	3	6	- 3	9
J	9	12	1	3	- 2	4
Total →						84

Now, $r = 1 - 6 [\sum d^2 + 1/12(32 - 3)]/n(n^2 - 1)$

$$= 1 - 6 (84 + 2)/[10(100) - 1]$$

$$= 1 - (6 \times 86)/990$$

$$= (990 - 516)/990$$

$$= 474/990 = 0.478 = 0.48.$$

Q.4. The marks obtained by nine students in Physics and Mathematics are given below :

Physics	35	23	47	17	10	43	9	6	28
Mathematics	30	33	45	23	8	49	12	4	31

Calculate Spearman's coefficient of rank correlation and interpret the result.

Solution : 4

Physics	Mathematics	R_1	R_2	$R_1 - R_2 = d$	D^2
35	30	3	5	-2	4
23	33	5	3	2	4
47	45	1	2	-1	1
17	23	6	6	0	0
10	8	7	8	-1	1

43	49	2	1	1	1
9	12	8	7	1	1
6	4	9	9	0	0
28	31	4	4	0	0
Total →					12

Here, $n = 9$, $r = 1 - [6\sum d^2]/[n(n^2 - 1)]$

$$= 1 - [6 \times 12]/[9(81 - 1)]$$

$$= 1 - 72/[9 \times 80]$$

$$= 1 - 1/10 = 9/10 = +0.9.$$

Q.5. The following table gives the two kinds of assessment of ten post-graduate students performance :

Students	Marks in Internal Assessment	Marks in External Assessment
1	45	39
2	62	48
3	67	65
4	32	32
5	12	20
6	38	35
7	47	45
8	67	77
9	42	30
10	85	62

Find Spearman's coefficient of rank correlation and interpret the result.

Solution : 5

Students S.No.	Marks in Internal Assessment x	Marks in External Assessment y	Rank of x	Rank of y	$x - y = d$	d^2
1	45	39	6	6	0	0
2	62	48	4	4	0	0
3	67	65	2.5	2	0.5	0.25

4	32	32	9	8	1	1
5	12	20	10	10	0	0
6	38	35	8	7	1	1
7	47	45	5	5	0	0
8	67	77	2.5	1	1.5	2.25
9	42	30	7	9	- 2	4
10	85	62	1	3	- 2	4
Total →						12.50

Here $n = 10$, $r = 1 - [6\sum d^2]/[\sum n(n^2 - 1)]$

$$= 1 - [6 \times 12.50]/[10 \times 99]$$

$$= 1 - 0.008 = 0.992$$

Q.6. A psychologist selected a random sample of 22 students. He grouped them in 11 pairs so that students in a pair have nearly equal scores in an intelligent test. In each pair one student was taught by method A and the other by method B and examined after the course. The marks obtained by them are tabulated below :

Pair	1	2	3	4	5	6	7	8	9	10	11
Method A(marks)	24	29	19	14	30	19	27	30	20	28	11
Method B(marks)	37	35	16	26	23	27	19	20	16	11	21

Find the rank correlation coefficient.

Solution : 6

Pair	Method A	Method B	Rank x	Rank y	$d = x - y$	d^2
1	24	37	6	1	5	25
2	29	35	3	2	1	1
3	19	16	8.5	9.5	-1	1
4	14	26	10	4	6	36
5	30	23	1.5	5	-3.5	12.2
6	19	27	8.5	3	5.5	30.2
7	27	19	5	8	-3	9.2
8	30	20	1.5	7	-5.5	30.2
9	20	16	7	9.5	-2.5	6.2

10	28	11	4	11	-7	49
11	11	21	11	6	5	25
Total →						225

$$\text{Correlation Coefficient } r = 1 - (6\sum d^2)/[n(n^2 - 1)]$$

$$= 1 - (6 \times 225)/(11 \times 120)$$

$$= 1 - 45/44 = 1 - 1.0227 = -0.0227.$$

Q.7. An examination of 8 applicants for a clerical post was taken by a firm. The marks obtained by the applicants in the Reasoning and Aptitude tests are given below :

Applicants	A	B	C	D	E	F	G	H
Reasoning Test	20	28	15	60	40	80	20	12
Aptitude Test	30	50	40	20	10	60	30	30

Calculate the Spearman's coefficient of rank correlation from the data given above.

Solution : 7

Reasoning Test	Aptitude Test	R_1	R_2	$D = R_1 - R_2$	D^2
20	30	5.5	5	0.5	0.25
28	50	4	2	2	4
15	40	7	3	4	16
60	20	2	7	-5	25
40	10	3	8	-5	25
80	60	1	1	0	0
20	30	5.5	5	0.5	0.25
12	30	8	5	3	9
					$\sum D^2 = 79.5$

$$\text{Using } r = 1 - 6[\sum D^2 + 1/12(m^3 - m) + 1/12(m^3 - m)]/[n(n^2 - 1)]$$

$$= 1 - 6[79.5 + 6/12 + 24/12]/(8 \times 63)$$

$$= 1 - 6[82]/(8 \times 63)$$

$$= 1 - 3/126 = 0.02.$$