

Conic – Parabola

Q.1. Find the equation of the parabola having focus at (3, - 4) and directrix as $x + y = 2$.

Solution : 1

The focus is at F (3, - 4). Let P(x, y) be any point on the parabola and | MP | be perpendicular distance from P to the directrix, then by the definition of parabola,
| FP | = | MP | [for a parabola, $e = 1$.]

$$\text{Or, } \sqrt{(x - 3)^2 + (y + 4)^2} = |x + y - 2|/\sqrt{1^2 + 1^2}$$

$$\text{Or, } 2\{(x - 3)^2 + (y + 4)^2\} = (x + y - 2)^2$$

$$\text{Or, } 2(x^2 - 6x + 9 + y^2 + 8y + 16) = x^2 + y^2 + 4 + 2xy - 4x - 4y$$

$$\text{Or, } x^2 + y^2 - 2xy - 8x + 20y + 46 = 0.$$

This is the required equation of the parabola.

Q.2. The equation of the directrix of the parabola is $3x + 2y + 1 = 0$. The focus is (2, 1). Find the equation of the parabola.

Solution : 2

$$= 4x^2 - 12xy + 9y^2 - 58x - 30y + 64 = 0]$$

Q.3. A straight line $2x + y + p = 0$ is a focal chord of the parabola $y^2 = - 8x$. Find the value of p.

Solution : 3

Equation of the parabola is

$$y^2 = - 8x \Rightarrow 4a = - 8,$$

Therefore, $a = -2$ and focus is $(-2, 0)$.

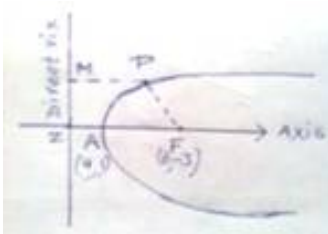
Line $2x + y + p = 0$, passes through $(-2, 0)$.

Therefore, $2 \cdot (-2) + 0 + p = 0 \Rightarrow p = 4$

Q.4. The points $(0, 4)$ and $(0, 2)$ are the vertex and focus of a parabola. Find the equation of the parabola.

Solution : 4

Fig.



The vertex is $A(0, 4)$ and focus is $F(0, 2)$. The axis of the parabola is the line joining A and F .

Let the axis meet the directrix at $Z(\alpha, \beta)$. A is the mid-point of the line ZF .

Therefore, $(\alpha + 2)/2 = 0 \Rightarrow \alpha = 0$, and $(\beta + 2)/2 = 4 \Rightarrow \beta = 6$.

Or, co-ordinate of Z is $(0, 6)$. Slope of the axis is $(4 - 2)/(0 - 0) = 2/0$.

Therefore, slope of the directrix $= -1/(2/0) = 0$.

Thus the equation of the directrix is $y - 6 = 0 (x - 0)$.

Or, $y - 6 = 0$. Let $P(x, y)$ be any point on the parabola.

Therefore, $|FP| = |MP|$

Or, $\sqrt{[x^2 + (y - 2)^2]} = |y - 6|/\sqrt{(1^2 + 0^2)} = |y - 6|$

Or, $x^2 + (y - 2)^2 = (y - 6)^2$

Or, $x^2 + y^2 - 4y + 4 = y^2 - 12y + 36$

Or, $x^2 + 8y - 32 = 0$.

Q.5. The equation $y^2 - 4y - 4x + 16 = 0$ represents a parabola. Find its vertex and focus.

Solution : 5

The equation of the parabola is

$$y^2 - 4y - 4x + 16 = 0.$$

$$\text{Or, } (y - 2)^2 = 4x - 12.$$

$$\text{Or, } (y - 2)^2 = 4(x - 3).$$

Let $Y^2 = 4X$, where, $Y = y - 2$ and $X = x - 3$.

$$\text{Here } 4a = 4 \Rightarrow a = 1.$$

Therefore, vertex is $V(0, 0)$.

$$\text{When } Y = 0 \Rightarrow y - 2 = 0, \text{ or, } y = 2.$$

$$\text{When } X = 0 \Rightarrow x - 3 = 0, \text{ or, } x = 3.$$

Therefore, vertex is $V(3, 2)$.

$$\text{Focus is } F(a, 0) = F(1, 0).$$

$$\text{Or, } X = a = 1 \Rightarrow x - 3 = 1 \Rightarrow x = 4 \text{ and } Y = 0 \Rightarrow y - 2 = 0 \Rightarrow y = 2.$$

Therefore, focus is $F(4, 2)$.

Q.6. The two lines $ty = x + t^2$ and $y + tx = 2t + t^2$ intersect at the point P. Show that P lies on the curve whose equation is $y^2 = 4x$.

Solution : 6

$$\text{Given lines are : } ty = x + t^2 \text{ ----- (i)}$$

$$\text{and } y + tx = 2t + t^2 \text{ ----- (ii)}$$

Multiplying equation (i) by t and adding to (ii) we get

$$t^2y + y + tx = tx + t^3 + 2t + t^3$$

$$\text{Or, } y(t^2 + 1) = 2t(t^2 + 1)$$

$$\text{Or, } y = 2t. \text{ ----- (iii)}$$

Putting $y = 2t$ in equation (i) we get

$$2t^2 = x + t^2 \Rightarrow x = t^2. \text{ ----- (iv)}$$

$$\text{From (iii) and (iv) } y^2 = 4t^2 = 4x$$

Hence, P lies on $y^2 = 4x$. **[Proved.]**