

Derivatives of Logarithmic Function

Q.1. If $y = e^x \log x$, find dy/dx at $x = 1$.

Solution : 1

We have, $y = e^x \log x$

Therefore, $dy/dx = e^x \cdot d/dx(\log x) + \log x \cdot d/dx(e^x)$

$$= e^x \cdot 1/x + \log x \cdot e^x$$

$$= e^x (1/x + \log x)$$

Therefore, $[dy/dx]_{x=1} = e^1 (1/1 + \log 1)$

$$= e.$$

Q.2. If $y = e^x \log \tan 2x$, find dy / dx .

Solution : 2

We have, $y = e^x \cdot \log (\tan 2x)$,

$$dy / dx = e^x \cdot d/dx\{\log (\tan 2x)\} + \log (\tan 2x) \cdot d/dx (e^x)$$

$$= e^x \{1/\tan 2x \cdot \sec^2 2x \cdot 2\} + \log (\tan 2x) \cdot e^x$$

$$= e^x \{(2 \sec^2 2x) / \tan 2x\} + e^x \cdot \log (\tan 2x)$$

$$= 2 e^x \{1 / (\sin 2x \cos 2x)\} + e^x \log (\tan 2x)$$

$$= 2 e^x \{2 / (2 \sin 2x \cos 2x)\} + e^x \log (\tan 2x)$$

$$= 4 e^x / \sin 4x + e^x \log (\tan 2x)$$

$$= e^x \{4 \operatorname{cosec} 4x + \log (\tan 2x)\}.$$

Q.3. If $y = \log \sqrt{\{(1 - \cos x) / (1 + \cos x)\}}$, find dy / dx .

Solution : 3

$$y = \log \sqrt{\{(1 - \cos x) / (1 + \cos x)\}}$$

$$= (1/2) \log \{(1 - \cos x) / (1 + \cos x)\}$$

$$= (1/2) \{\log (1 - \cos x) - \log (1 + \cos x)\}$$

$$\text{Hence, } dy/dx = (1/2) [\sin x / (1 - \cos x) + \sin x / (1 + \cos x)]$$

$$= (\sin x / 2) [1 / (1 - \cos x) + 1 / (1 + \cos x)]$$

$$= (\sin x / 2) [(1 + \cos x + 1 - \cos x) / (1 - \cos^2 x)]$$

$$= (\sin x / 2) [2 / (1 - \cos^2 x)]$$

$$= (\sin x / 2) \cdot (2 / \sin^2 x)$$

$$= 1 / \sin x = \operatorname{cosec} x .$$

Q.4. If $y = -\cot^2 (x/2) - 2 \log \sin (x/2)$, prove that : $dy/dx = \cot^3 (x/2)$.

Solution : 4

$$y = -\cot^2 (x/2) - 2 \log \sin (x/2)$$

$$= -\{\cot (x/2)\}^2 - 2 \log \sin (x/2)$$

$$\text{Hence , } dy / dx = -2 \cot (x/2) \{-\operatorname{cosec}^2 (x/2)\} \times 1/2 - 2 / \sin (x/2) \times \cos (x/2) \times 1/2$$

$$= \cot (x/2) \operatorname{cosec}^2 (x/2) - \cot (x/2)$$

$$= \cot (x/2) [\operatorname{cosec}^2 (x/2) - 1]$$

$$= \cot (x/2) \cdot \cot^2 (x/2) = \cot^3 (x/2) . \text{ [Proved.]}$$

Q.5. If $y = \log (\tan x)$, find dy / dx .

Solution : 5

$$y = \log (\tan x) \Rightarrow dy / dx = (1/\tan x) \cdot d / dx (\tan x)$$

$$= (1 / \tan x) \cdot \sec^2 x$$

$$= \operatorname{cosec} x \times \sec x.$$

Q.6. If $y = \log (\sec x + \tan x)$, find dy / dx .

Solution : 6

$$y = \log (\sec x + \tan x) \Rightarrow dy / dx = 1 / (\sec x + \tan x) \cdot d / dx (\sec x + \tan x)$$

$$= (\sec x \tan x + \sec^2 x) / (\sec x + \tan x)$$

$$= \{\sec x (\sec x + \tan x)\} / (\sec x + \tan x)$$

$$= \sec x.$$

Q.7. If $y = \log \tan (\pi / 4 + x / 2)$, find dy / dx .

Solution : 7

$$y = \log \tan (\pi / 4 + x / 2)$$

$$\text{Then } dy / dx = d / dx [\log \tan (\pi / 4 + x / 2)]$$

$$= [1 / \tan (\pi / 4 + x / 2)] \cdot d / dx [\tan (\pi / 4 + x / 2)]$$

$$= [1 / \tan (\pi / 4 + x / 2)] \cdot \sec^2 (\pi / 4 + x / 2) \cdot d / dx (\pi / 4 + x / 2)$$

$$= [\sec^2 (\pi / 4 + x / 2) / \tan (\pi / 4 + x / 2)] \cdot (1/2)$$

$$= 1 / [2 \sin (\pi / 4 + x / 2) \cos (\pi / 4 + x / 2)]$$

$$= 1 / \sin (\pi / 2 + x) = 1 / \cos x = \sec x .$$

Q.8. Differentiate $x \tan x \log_5 x$ w.r.t. x .

Solution : 8

Let $y = x \tan x \log_5 x$,

Then $dy/dx = x \tan x \cdot d/dx (\log_5 x) + \log_5 x \cdot d/dx (x \tan x)$

$$= x \tan x \cdot \frac{1}{x \log_5} + \log_5 x (x \cdot \sec^2 x + \tan x \cdot 1)$$

$$= \tan x / \log_5 + (x \sec^2 x + \tan x) \log_5 x.$$