Ellipse

Q.1. Find the equation of the normal to the ellipse $5x^2 + 3y^2 = 137$ at the point where the ordinate is 2.

Solution : 1

Equation of the ellipse is :

$5x^2 + 3y^2 = 137$ ---------- (1)

If $y = 2$, $5x^2 + 3 (2)^2 = 137$

Or, $5x^2 = 125 \Rightarrow x2 = 25$

Or, $x = \pm 5$.

Differentiating (1) we get $dy/dx = 10x/6y = 50/12 = 25/6$ [at (5,2)] = m .

Equation of normal at (5,2) is , $y - 2 = 6/25 (x - 5)$

Or, $6x - 25y + 20 = 0$

And equation at (– 5 ,2) is , $6x + 25y - 20 = 0$

Q.2. Find the equation of the ellipse whose minor axis is 4 and which has a distance of 6 units between its foci.

Solution : 2

We have, $2b = 4 \Rightarrow b = 2$. FF’ = 6.

$2ae = 6 \Rightarrow a e = 3$ ---------- (i)

and $b = a \sqrt{(1 - e^2} = 2$ ---------- (ii)

dividing (i) by (ii) we get

$a e/a\sqrt{(1 - e^2)} = 3/2$

Or, $4e^2 = 9 (1 - e^2)$

Or, 13 e^2 = 9 => e = 3/√13

From (ii) a√(1 – 9/13) = 2

Or, 2a/√13 = 2 => a = √13.

Therefore, equation of the ellipse is x2/a^2 + y^2/b^2 = 1

Or, x^2 /13 + y^2/4 = 1.

Q.3. Find the equation of the ellipse whose eccentricity is 1/2 and whose foci are the points (±2, 0).

Solution : 3

We have, e = 1/2, and focus is (±2, 0).

Thus, ae = ± 2

Or, a × 1/2 = ± 2 => a = ± 4.

Also, b = a √(1 – e^2) = ± 4 √(1 – 1/4) = ±4 × √3/2 = ± 2√3.

Equation of the ellipse is x2/a^2 + y^2/b^2 = 1.

Or, x^2/(± 4)^2 + y^2/(±2√3)^2 = 1

Or, x^2/16 + y^2/1^2 = 1.

Q.4. The directrix of a conic section is 3x + 4y = 1 and focus is (2, 3). Find the equation of the conic section if its eccentricity is 1/√2.

Solution : 4

We have, focus F (2, 3), e = 1/√2, and directrix 3x + 4y = 1.

Let P(x, y) be any point on the ellipse and | MP | be the perpendicular

distance from the directrix 3x + 4y = 1, then by definition,

| FP | = e| MP |

$\sqrt{[(x-2)^2 + (y-3)2]} = (1/\sqrt{2}).|\ 3x + 4y - 1|/\sqrt{[3^2 + 4^2]} = |\ 3x + 4y\ |/5\sqrt{2}$.

Squaring both sides we get,

$50\ \{(x-2)^2 + (y-3)^2\} = (3x + 4y - 1)^2$

Or, $50\ (x^2 - 4x + 4 + y^2 - 6y + 9) = 9x^2 + 16y^2 + 1 + 24\ x\ y - 6x - 8y$

Or, $41x^2 - 24xy + 34y^2 - 194x - 292y + 649 = 0$.

Which is the required equation of the conic.

Here $a = 41$, $h = -12$, $b = 34$

and $h^2 - a\ b = (-12)^2 - 41 \times 34 = 144 - 1394 = -1250 < 0$

The conic section is an ellipse.

Q.5. Find the eccentricity of the ellipse $(x-3)^2/8 + (y-4)^2/6 = 1$.

Solution : 5

We have, $(x-3)^2/8 + (y-4)^2/6 = 1$.

Shifting origin to the point (3, 4), we have the ellipse as

$X^2/8 + Y^2/6 = 1$.

$a^2 = 8 \Rightarrow a = 2\sqrt{2}$; $b^2 = 6 \Rightarrow b = \sqrt{6}$.

Now, $b^2 = a2\ (1 - e^2)$

Or, $6 = 8\ (1 - e^2) \Rightarrow 6/8 = 1 - e^2 \Rightarrow e^2 = 1 - 6/8 = 2/8 = 1/4$

Therefore, $e = 1/2$.

Q.6. Find the equation of the ellipse whose latus rectum is 10 and whose eccentricity is 2/3.

Solution : 6

We have, latus rectum $= 2b^2/a = 10$, and $e = 2/3$.

Now, $b^2 = a^2 (1 – e2) = a^2 (1 – 4/9)$

Or, $b^2/a = a^2/a\{(9 – 4)/9\}$

Or, $10/^2 = a . 5/9$

Or, $1 = a/9 => a = 9.$

And $b^2 = a^2 (1 – e^2) = 81 (1 – 4/9) = 45$

Therefore, $b = 3\sqrt{5}$.

Thus the equation of ellipse is

$x^2/9^2 + y^2/(3\sqrt{5})^2 = 1$

Or, $x^2/81 + y^2/45 = 1.$

Q.7. Find the equation of the ellipse whose foci are at (– 2, 4) and (4, 4) and whose major and minor axes are 10 and 8 respectively. Also find the eccentricity of the ellipse.

Solution : 7

$2ae = 4 – (– 2) = 6 => a e = 3 => e = 3/a.$

Again $2a = 10 => a = 5$ and $2b = 8 => b = 4.$

Thus $e = 3/a = 3/5$. Centre of the ellipse is $[(– 2 + 4)/2, (4 + 4)/2] = (1, 4)$ T

herefore, equation of the ellipse is

$(x – 1)^2/25 + (y – 4)^2/16 = 1.$

Q.8. Find the equation of the ellipse whose foci are (– 1, 5) and (5, 5) and whose major axis is 10.

Solution : 8

$2ae = 5 – (– 1) = 6 => a e = 3 => e =3/a$

Also $2a = 10 => a = 5$

Thus $e = 3/a = 3/5,$

Using $b^2 = a^2 (1 – e^2)$ we get, $b^2 = 25 (1 – 9/25) = 16 \Rightarrow b = 4$.

Centre of the ellipse is $[(– 1+ 5)/^2, (5 + 5)/^2] = (2, 5)$

Therefore, equation of the ellipse is

$(x – 2)^2/25 + (y – 5)^2/16 = 1$.