Derivative of Exponential Function

Q.1. Differentiate $3^x/(2 + \sin x)$.

Solution : 1

Let $y = 3x/(2 + \sin^{\mathbf{x}})$

Then $dy/dx = \{(2 + \sin x).d/dx(3^{\mathbf{x}}) - 3^{\mathbf{x}}.d/dx(2 + \sin x)\}/(2 + \sin x)^{\mathbf{2}}$

$= \{(2 + \sin x).\ 3^{\mathbf{x}} \log 3 - 3x.(0 + \cos x)\}/(2 + \sin x)^{\mathbf{2}}$

$= 3^{\mathbf{x}}\{(2 + \sin x) \log 3 - \cos x\}/(2 + \sin x)^{\mathbf{2}}$.

Q.2. If $e^{x + y} = x y$, show that $dy / dx = [y (1 - x)] / [x (y - 1)]$.

Solution : 2

We have, $e^{\mathbf{x + y}} = x y$,

Taking log of both the sides,

$(x + y) \log e_e = \log e\ x + \log_e y$

Or, $x + y = \log e\ x + \log_e y$ [As, $\log e\ e = 1$]

Differentiating both sides with respect to x we get,

$1 + dy / dx = 1 / x + (1/y) . dy / dx$

Or, $[1 - (1 / y)]\ dy / dx = (1 / x) - 1$

Or, $dy / dx = (1 - x) / x \times y / (y - 1)$

$= [y (1 - x)] / [x (y - 1)]$ **[Proved.]**

Q.3. If $y = e^\wedge \sin x^2$, find dy / dx.

Solution : 3

We have , $y = e^{\sin x^2}$,

$dy / dx = e^{\sin x^2} \times d/dx (\sin x^2)$

$= e^{\sin x^2} \times \cos x^2 \times d / dx (x^2)$

$= e^{\sin x^2} \times \cos x^2 \times (2x)$

$= 2 x \cos x^2 e^{\sin x^2}$.

Q.4. If $y = [e^x + e^{-x}] / [e^x - e^{-x}]$, find dy / dx.

Solution : 4

$y = [e^x + e^{-x}] / [e^x - e^{-x}]$

$= [e^x + 1 / e x] / [e^x - 1 / e^x]$

$= [e^{2x} + 1] /[e^{2x} -1]$

$dy / dx = [(e^{2x} -1) \times d / dx (e^{2x} + 1) - (e^{2x} + 1) \times d / dx (e^{2x} -1)] / [e^{2x} -1]2$

$= [(e^{2x} -1) \times 2 e^{2x} - (e^{2x} + 1) \times 2e^{2x}] / [e^{2x} -1]^2$

$= [\{e^{2x} -1 - e^{2x}-1\}\times 2e^{2x}] / [e^{2x} -1]^2$

$= - 4 e^{2x}/[e^{2x} -1]^2$.

Q.5. If y = e^ e x, find dy / dx.

Solution : 5

Let $y = e ^ {e^x} = e^u$, where $u = e^x$,

$du / dx = e^x$ and $dy / du = e u = e ^ {e^x}$,

Hence, $dy / dx = dy / du \times du / dx = e ^ {e^x} . e^x$.

Q.6. If $y = \sqrt{(1 + e^{x}) / (1 - e^{x})}$, find dy / dx.

Solution : 6

$dy / dx = d / dx [\sqrt{\{(1 + e^{x}) / (1 - e^{x})\}}]$

$= [\sqrt{(1 - e^{x})}\, d / dx \{\sqrt{(1 + e^{x})}\} - \sqrt{(1 + e^{x})}\, d / dx \{\sqrt{(1 - e^{x})}] / \{\sqrt{(1 - e^{x})}\}^{2}$

$= [\sqrt{(1 - e^{x})} \times 1 / 2\sqrt{(1 + e^{x})} \times e^{x} - \sqrt{(1 + e^{x})} \times 1/2\sqrt{(1 - e^{x})} \times (- e^{x})] / (1 - e^{x})$

$= [(e^{x}) / 2] [\{\sqrt{(1 - e^{x})}\}/\{\sqrt{(1 + e^{x})}\} + \{\sqrt{(1 + e^{x})}\}/\{\sqrt{(1 - e^{x})}\}] / (1 - e^{x})$

$= [e^{x}\{(1 - e^{x}) + (1 + e^{x})\}/ 2 \sqrt{(1 - e^{2x})}] / (1 - e^{x})$

$= e^{x}/ [(1 - e^{x}) \sqrt{(1 - e^{2x})}]$.

Q.7. If $y = e^{ax} / \sin (bx + c)$, find dy/dx .

Solution : 7

$dy / dx = [\sin (bx + c) . d / dx (e^{ax}) - e\, ax . d / dx \{\sin (bx + c)\}] / \sin^{2} (bx + c)$

$= [\sin (bx + c) . ae^{ax} - e^{ax} . \cos (bx + c) . b] / \sin2 (bx + c)$

$= e^{ax} [a \sin (bx + c) - b \cos (bx + c)] / \sin2 (bx + c)$.

Q.8. If $y = \{x + \sqrt{(x^{2} - 1)}\}^{m}$, prove that $(x^{2} - 1)(dy/dx)^{2} = m^{2} . y^{2}$.

Solution : 8

We have , $y = \{x + \sqrt{(x^{2} - 1)}\}^{m}$,

Then, $dy/dx = m\{x + \sqrt{(x^{2} - 1)}\}^{(m -1)} \times \{1 + 1/2. 2x/\sqrt{(x^{2} - 1)}\}$

$= m \{x + \sqrt{(x^{2} - 1)}\}^{(m -1)} \times [\{\sqrt{(x^{2} - 1)} + x \}/\sqrt{(x^{2} - 1)}]$

$= m\{x + \sqrt{(x^{2} - 1)}\}^{m} /\sqrt{(x^{2} - 1)}$ Or, $\sqrt{(x^{2} - 1)} . dy/dx$

$= m \{ x + \sqrt{(x^{2} - 1)} \}^{m}$

Squaring both sides , we get

$(x^2 - 1) \cdot (dy/dx)^2 = m^2 [\{x + \sqrt{(x^2 - 1)}\}m]^2$

$= m^2 \cdot y^2$. **[Proved.]**